Radiation Spectrum of a Massive Quark-Gluon Antenna in a QCD Medium

M. R. Calvo[ab], M. R. Moldes[a], C. Salgado[a]

[a] Departamento de Física de Partículas
Universidade de Santiago de Compostela, Galicia, Spain
[b] IPhT CEA-Saclay, France

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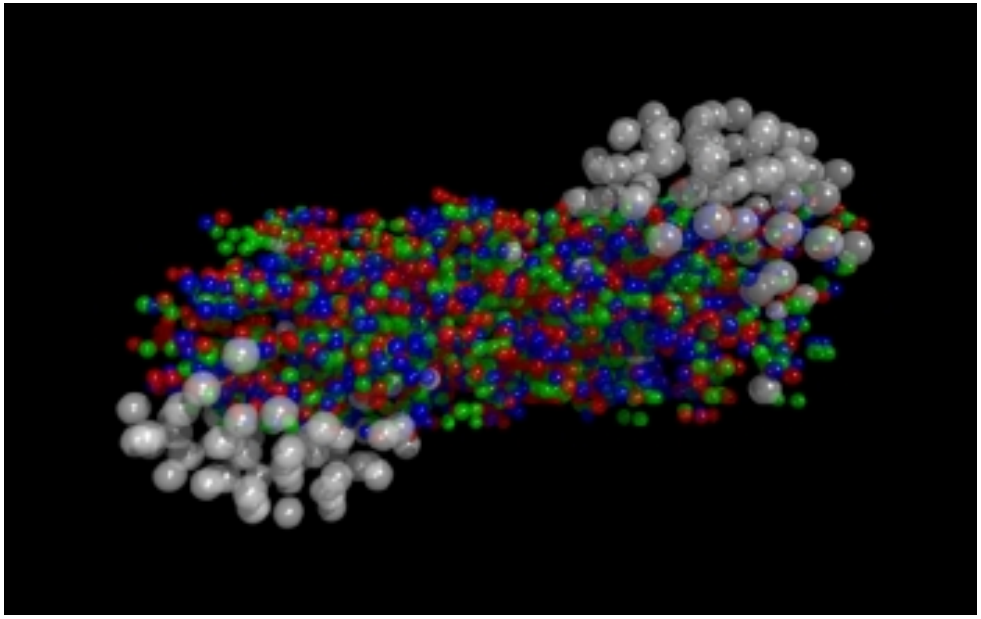
Outline

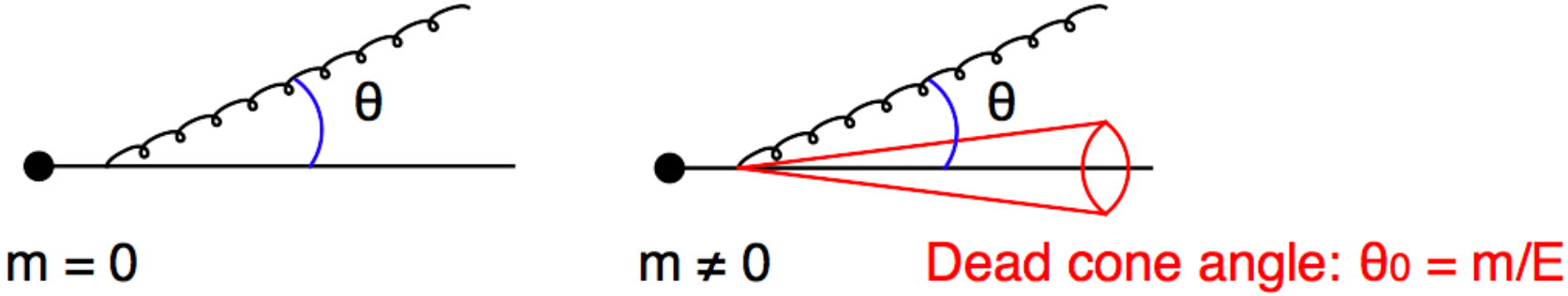
Motivation



- A state of deconfined coloured particles (QGP) is formed in heavy ion collisions.
- One of the most striking effects of QGP is the energy loss of high energy partons in heavy ion collisions.
- Is this energy loss the same for heavy quarks than for massless quarks?

Dead Cone Effect

- In vacuum, the presence of a mass leads to a suppression of gluon radiation inside the so-called dead cone.



Massive BDMPS Spectrum

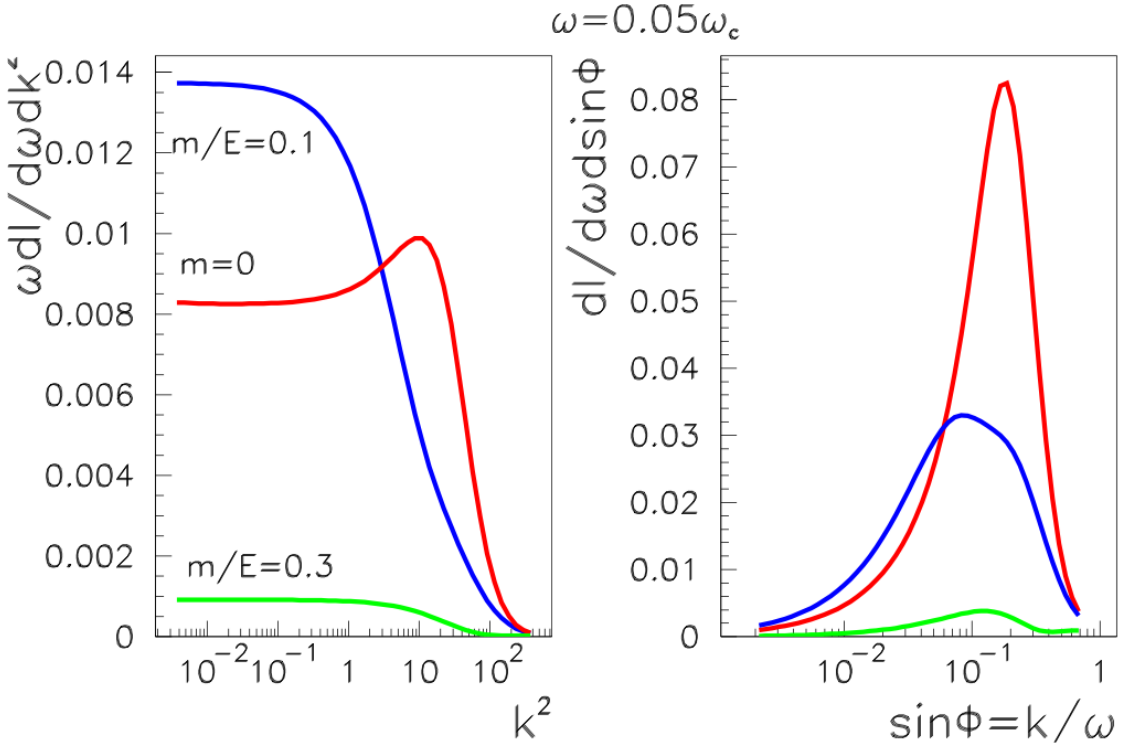
- In a medium, there is also a suppression due to mass effects.

$$k^+ \frac{\mathrm{d}N}{\mathrm{d}k^+ \mathrm{d}^2\mathbf{k}} = \frac{\alpha_s C_F}{(2\pi)^2 (k^+)^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \times \exp\left[i \frac{k^+}{2} \theta_{DC}^2 (y^+ - y'^+) \right] \int \mathrm{d}^2\mathbf{z} \exp\left[- i\mathbf{k}\cdot\mathbf{z} - \frac{1}{2} \int_{y'^+}^\infty \mathrm{d}\xi\, n(\xi) \sigma(\mathbf{z}) \right] \boldsymbol{\partial}_y \cdot \boldsymbol{\partial}_z \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y}) \Big|_{\mathbf{y}=\mathbf{0}}$$

(Armesto, Salgado, Wiedemann; Dokshitzer, Karzeev; Zhang, Wang, Wang; Djordjevic, Gyulassy, ...)

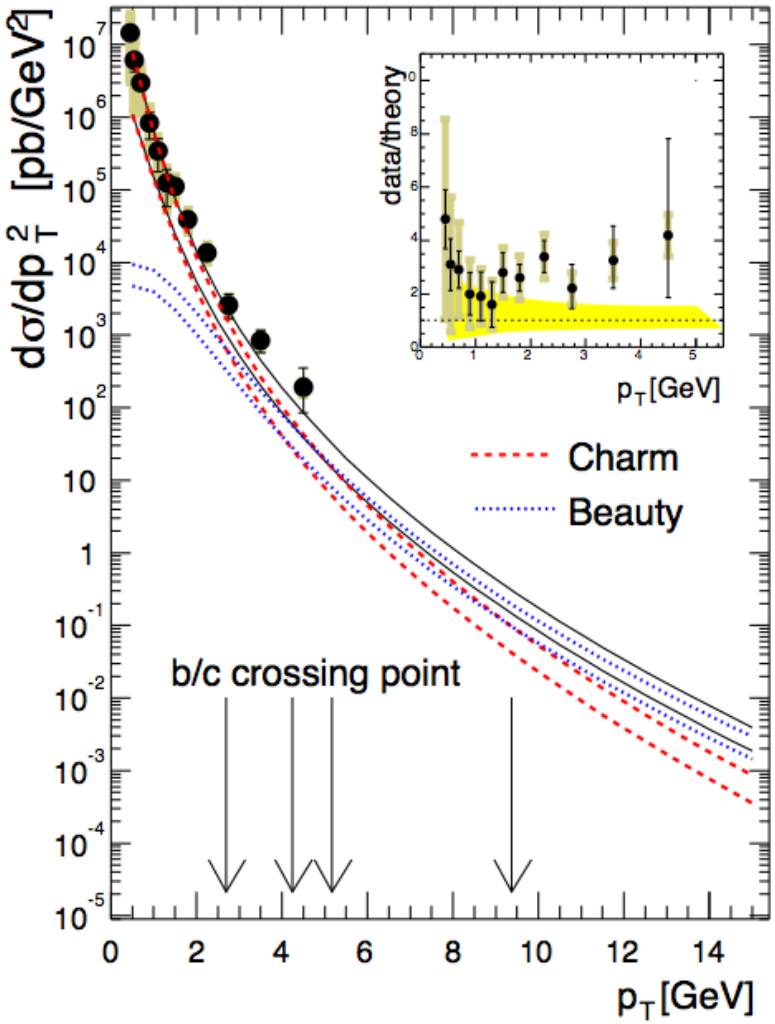
Massive BDMPS

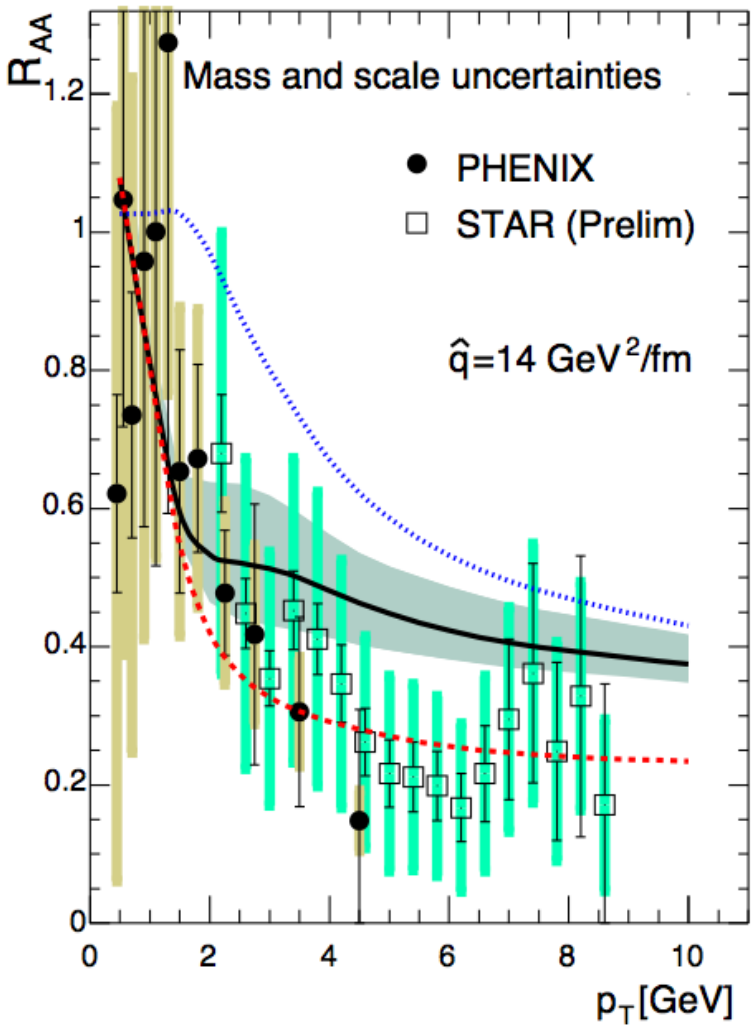
- Smaller energy loss in the massive case (dead cone).



(Armesto, Salgado, Wiedemann)

Trouble with RHIC data





Armesto, Cacciari, Dainese, Salgado, Wiedemann

- BDMPS does not fit energy loss of massive quarks.

Radiative Antenna

- The BDMPS spectrum does not deal with interference effects between emitters.

$$\Downarrow$$

  Gluon emission spectrum off an antenna is considered.
- In vacuum, interference effects lead to *angular ordering*: emission angles within the partonic cascade decrease from one emission to the next one.

$$\theta_1 > \theta_2 > \ldots > \theta_n > \ldots$$

Interference of a Massless Antenna in a QCD medium

$$\mathcal{J} = \mathrm{Re}\Bigg\{ \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \, \left(1 - \Delta_{\mathsf{med}}(y^+, 0)\right) \, e^{i\frac{k^+}{2} y^+ \delta \mathbf{n}^2}$$
$$\times \int \mathrm{d}^2\mathbf{z} \exp\left[-i\bar{\boldsymbol{\kappa}} \cdot \mathbf{z} - \frac{1}{2} \int_{y'^+}^\infty \mathrm{d}\xi \, n(\xi) \sigma(\mathbf{z}) \right]$$
$$\times \left(\boldsymbol{\partial}_y - ik^+ \delta \mathbf{n}\right) \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y}) \Big|_{\mathbf{y} = \delta \mathbf{n} y^+} \Bigg\} + \mathsf{sym.}$$

where $\mathcal{K}$ takes into account the rescattering of the emitted gluon with the medium.

$$\mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y} | k^+) = \int_{\mathbf{r}(y^+) = \mathbf{y}}^{\mathbf{r}(y'^+) = \mathbf{z}} \mathcal{D}\mathbf{r} \exp\left\{ \int_{y^+}^{y'^+} \mathrm{d}\xi \, \left(i \frac{k^+}{2} \dot{\mathbf{r}}^2(\xi) - \frac{1}{2} n(\xi) \sigma(\mathbf{r}) \right) \right\}$$

Antenna Parameters

- Decoherence parameter:

$$\Delta_{\rm med}(y^+,0) \equiv 1 - \exp\left\{-\frac{1}{2}\int_0^{y^+} {\rm d}\xi\, n(\xi)\sigma(\delta\mathbf{n}\xi)\right\}$$

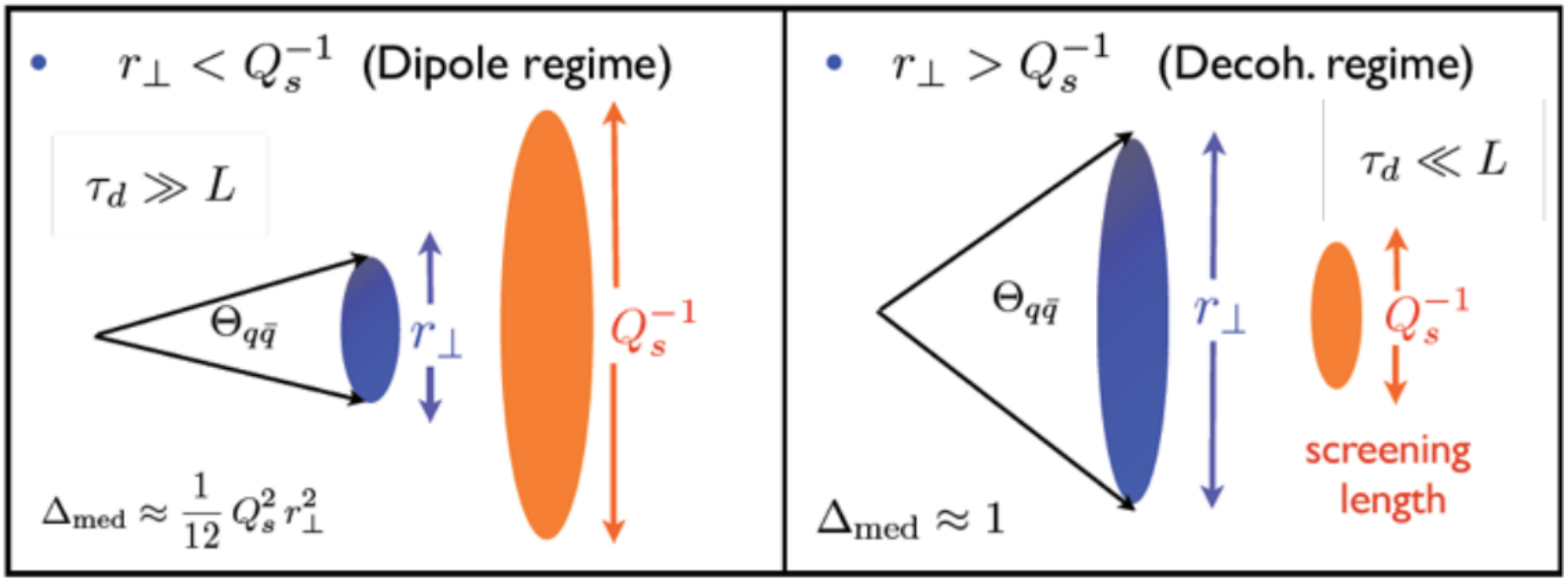
- Opening angle:

$$\delta\mathbf{n} \equiv \frac{\mathbf{p}_q}{p_q^+} - \frac{\mathbf{p}_g}{p_g^+} \to |\delta\mathbf{n}| \equiv \sin\theta_{qg} \sim \theta_{qg}$$

- $\sigma(\mathbf{r})$ is the dipole cross-section.
- Harmonical approximation: $n(\xi)\sigma(\mathbf{r}) \approx \frac{1}{2}\hat{q}(\xi)\mathbf{r}^2$
  $\hat{q}$ is the medium transport coefficient.

Antenna Scales

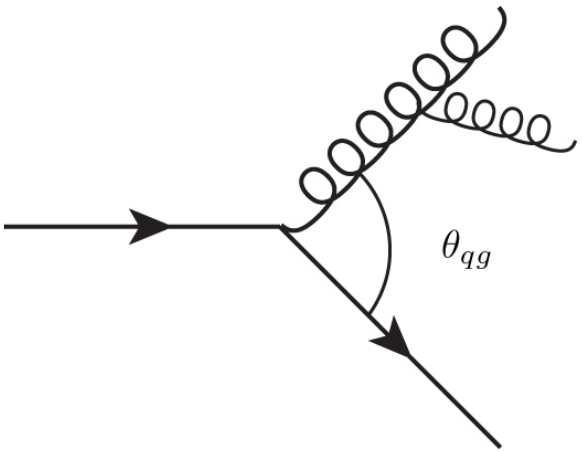
- What is the role of the parton mass in the picture below?

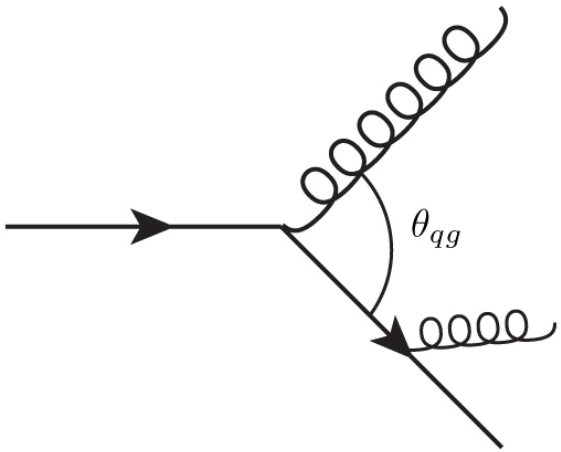


(Mehtar-Tani, Tywoniuk, Salgado, Casalderrey, Iancu)

Massive Antennae

- The case of a massive quark-antiquark antenna has been studied before. (Armesto, Ma, Mehtar-Tani, Salgado, Tywoniuk)
- Here we focus in the case of a massive quark-gluon antenna, more relevant for high-energy jets.





Framework Set-up

- BDMPS and interference spectra are calculated by solving the Classical Yang-Mills (CYM) equations

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

with the help of the continuity equation

$$[D_\mu, J^\mu] = 0$$

- Light-cone gauge ($A^+ = 0$) + linearized CYM equations.

$$\Box A^i - 2ig[A^-_{\text{med}}, \partial^+ A^i] = -\frac{\partial^i}{\partial^+} J^+ + J^i$$

Currents

- High energy partons are represented in the vacuum by the classical currents $J^{\mu}_{(0)}$

$$J^{\mu,a}_{(0)}(x) = g\frac{p^{\mu}}{E}\delta^{(3)}\left(\vec{x} - \frac{\vec{p}}{E}t\right)\theta(t)Q^{a}$$

- The classical currents get color rotated because of interactions with the medium.

$$J^{\mu}(x) = U_p(x^+, 0)J^{\mu}_{(0)}$$

- Wilson lines are responsible for in-medium color rotation.

$$U_p(x^+, 0) \equiv \mathcal{P}\exp\left\{\int_0^{x^+} \mathrm{d}\xi\; T \cdot A^-_{\mathrm{med}}(\xi, \xi\mathbf{p}_\perp/p^+)\right\}$$

Emitted Gluon

- The propagation in the medium of the emitted gluon is expressed by the following path integral, depicting its color rotation and Brownian motion in the transverse plane due to interactions with the background field.

$$\mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+) = \int_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(x^+)=\mathbf{x}} \mathcal{D}\mathbf{r} \exp\left\{\frac{ik^+}{2}\int_{y^+}^{x^+} \mathrm{d}\xi\, \dot{\mathbf{r}}^2(\xi)\right\} U(x^+, y^+; \mathbf{r})$$

Amplitude

- Applying the usual formulation to compute amplitudes, we get for the massive quark-gluon antenna:

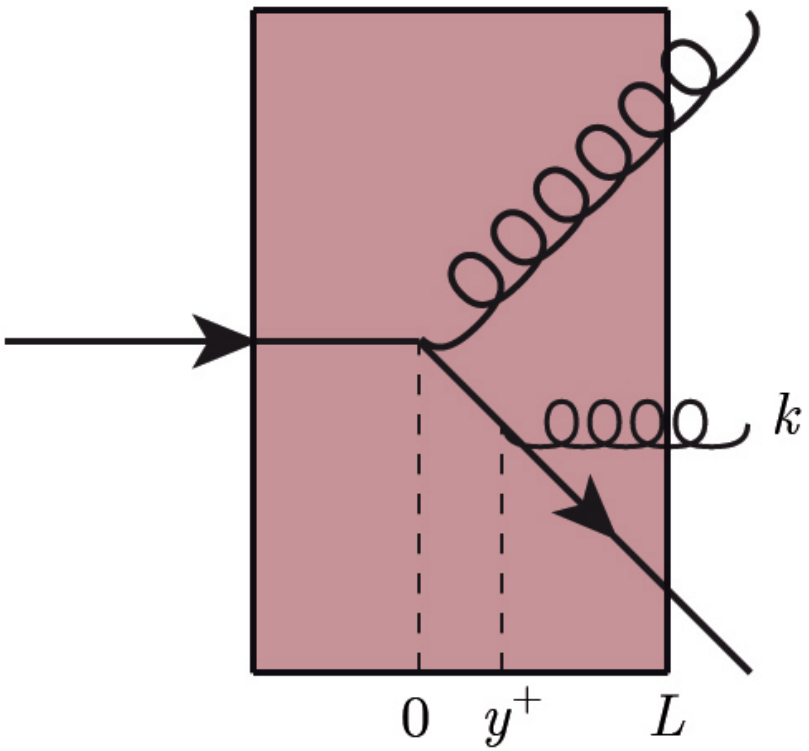
$$\mathcal{M}_\lambda(\vec{k}) = \frac{g}{k^+} \int\limits_{x^+=+\infty} \mathrm{d}^2\mathbf{x}\, e^{ik^- x^+} e^{-i\mathbf{k}\cdot\mathbf{x}} \int_0^{+\infty} \mathrm{d}y^+\, e^{ik^+ \frac{p^-}{p^+} y^+}$$

$$\times \boldsymbol{\epsilon}_\lambda(k) \cdot (i\boldsymbol{\partial}_y + k^+\mathbf{n})\, \mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+)\Big|_{\mathbf{y}=\mathbf{n}y^+} U_p(y^+, 0) Q$$

where

$$\mathbf{n} \equiv \frac{\mathbf{p}}{p^+}$$

Amplitude



$$\mathcal{M}_\lambda(\vec{k}) = \frac{g}{k^+} \int\limits_{x^+=+\infty} \mathrm{d}^2\mathbf{x}\, e^{ik^- x^+} e^{-i\mathbf{k}\cdot\mathbf{x}} \int_0^{+\infty} \mathrm{d}y^+ \, e^{ik^+ \frac{p^-}{p^+} y^+}$$

$$\times \epsilon_\lambda(k) \cdot (i\partial_{\mathbf{y}} + k^+ \mathbf{n})\, \mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+)\Big|_{\mathbf{y}=\mathbf{n}y^+} U_{\mathbf{p}}(y^+, 0) Q$$

Mass Effect

- The difference between the massive and massless cases comes from the dispersion relation:

$$2p^+p^- - \mathbf{p}^2 = M^2$$

- This leads to the appearence of a new phase:

$$\exp\left(ik^+\frac{p^-}{p^+}y^+\right) = \exp\left(i\frac{k^+}{2}\theta_{DC}^2 y^+\right)\exp\left(i\frac{k^+}{2}\mathbf{n}^2 y^+\right)$$

where

$$\boxed{\theta_{DC} \equiv \frac{M}{p^+}}$$

Radiation Spectrum of the Antenna

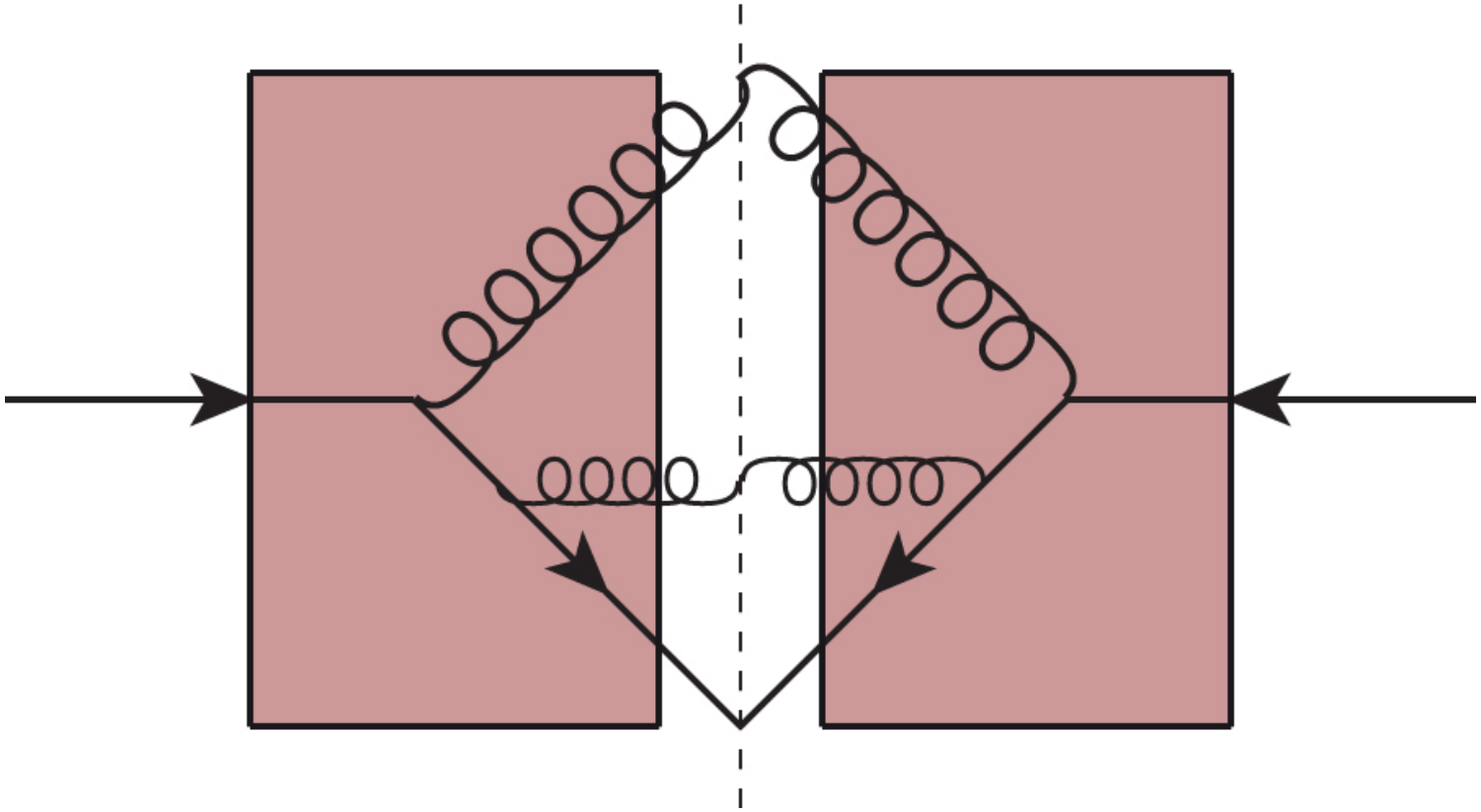
- The medium is modeled as a collection of uncorrelated, static scattering centers.
- The background field (A_{med}) is treated as a Gaussian white noise.

- The radiation spectrum for the massive quark-gluon antenna reads

$$dN = \frac{\alpha_s}{(2\pi)^2}\left[C_F\mathcal{R}_q + C_A\mathcal{R}_g - \frac{C_A}{2}\mathcal{J}\right]\frac{d^3k}{(k^+)^3}$$

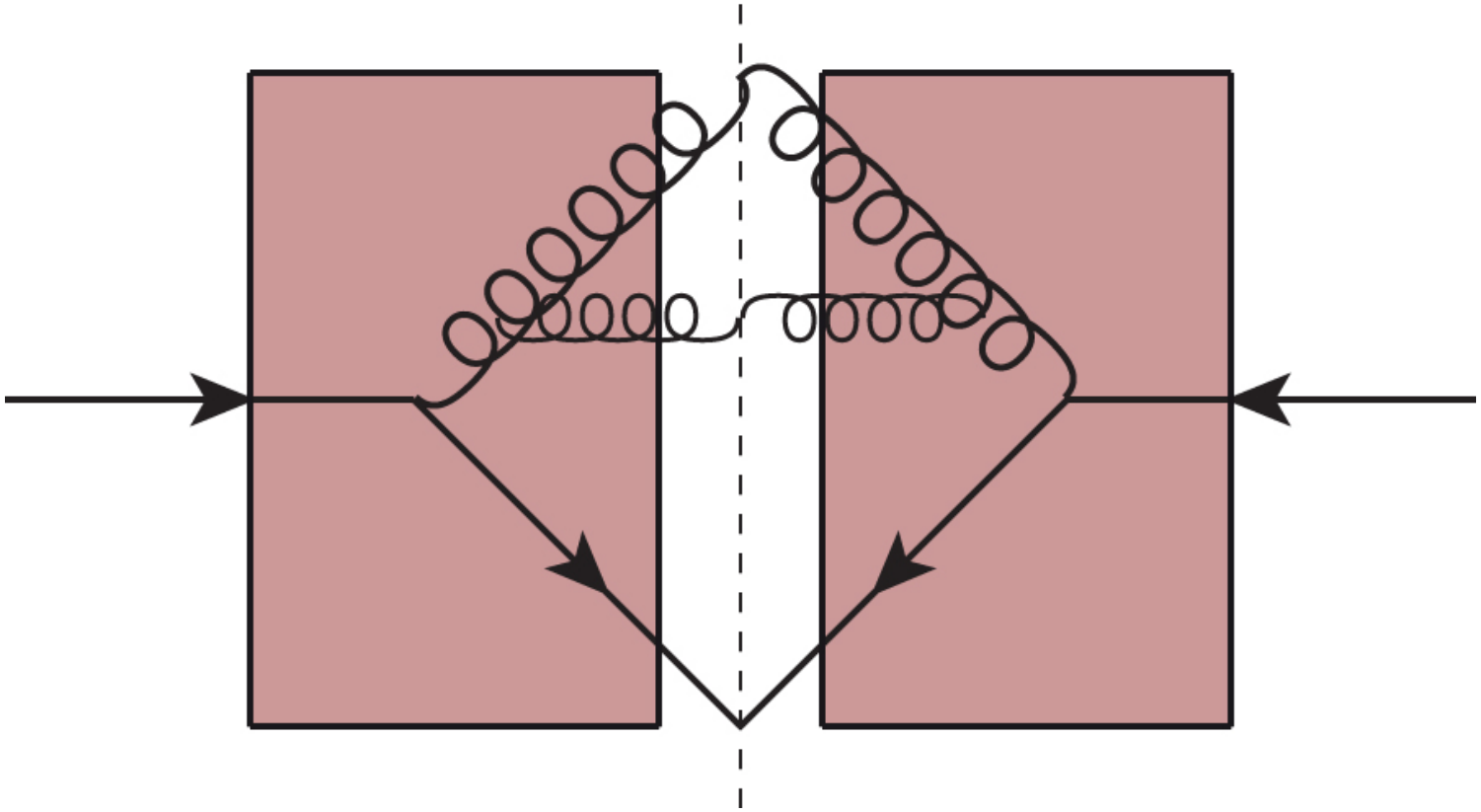
BDMPS $\mathcal{R}_q$

$$C_F \mathcal{R}_q = (k^+)^2 \langle |\mathcal{M}_q|^2 \rangle$$



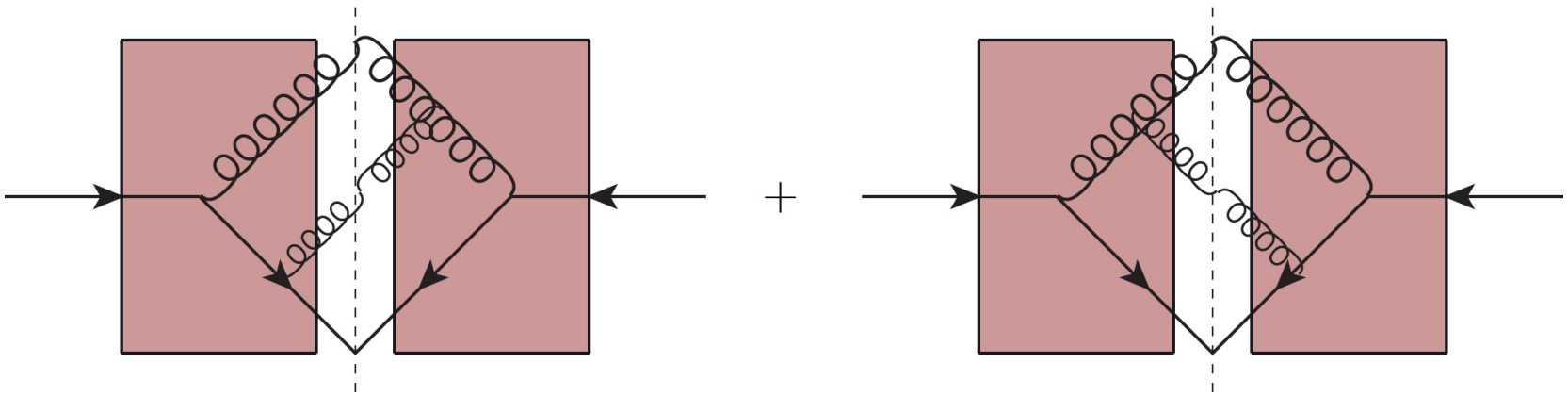
BDMPS $\mathcal{R}_g$

$$C_A \mathcal{R}_g = (k^+)^2 \langle |\mathcal{M}_g|^2 \rangle$$



Interference $\mathcal{J}$

$$\frac{-C_A}{2}\mathcal{J} = (k^+)^2\,\mathrm{Re}\langle \mathcal{M}_q \mathcal{M}_g^\dagger \rangle$$



BDMPS Spectrum of the Massive Quark

- Recovery of the BDMPS spectrum for heavy quarks.

$$\mathcal{R}_q = 2\mathrm{Re}\int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \exp\left[i\frac{k^+}{2}\theta_{DC}^2(y^+ - y'^+)\right]$$
$$\times \int \mathrm{d}^2\mathbf{z}\, \exp\left[-i\mathbf{k}\cdot\mathbf{z} - \frac{1}{2}\int_{y'^+}^\infty \mathrm{d}\xi\, n(\xi)\sigma(\mathbf{z})\right] \boldsymbol{\partial}_y\cdot\boldsymbol{\partial}_z\, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\Big|_{\mathbf{y}=\mathbf{0}}$$

(Armesto, Salgado, Wiedemann; Dokshitzer, Karzeev; Zhang, Wang, Wang; Djordjevic, Gyulassy, ...)

Interference Spectrum

- The interference spectrum for the massive quark-gluon antenna:

$$\boxed{\begin{aligned}\mathcal{J} = \mathrm{Re}\Bigg\{ &\int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \, \left(1 - \Delta_{\mathsf{med}}(y^+, 0)\right) \, e^{i\frac{k^+}{2} y^+ (\theta^2_{DC} + \delta\mathbf{n}^2)} \\ &\times \int \mathrm{d}^2\mathbf{z} \exp\left[-i\bar{\boldsymbol{\kappa}} \cdot \mathbf{z} - \frac{1}{2}\int_{y'^+}^\infty \mathrm{d}\xi \, n(\xi)\sigma(\mathbf{z})\right] \\ &\times (\boldsymbol{\partial}_y - ik^+\delta\mathbf{n}) \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\Big|_{\mathbf{y}=\delta\mathbf{n}y^+} \Bigg\} + \text{sym.}\end{aligned}}$$

- Same result as in the massless case... except for the mass phase!

Discussion

- $\exp\left(i\frac{k^+}{2}\theta_{DC}^2 y^+\right)$ is a quickly oscillating exponencial if the mass is big enough:

$$\theta_{DC}^2 \gg \frac{2}{k^+ L}$$

- Different mass phases for massive quark-gluon antenna and BDMPS/massive quark-antiquark antenna.
  - qg antenna:

$$\exp\left(i\frac{k^+}{2}\theta_{DC}^2 y^+\right)$$

  - BDMPS/$q\bar{q}$ antenna:

$$\exp\left(i\frac{k^+}{2}\theta_{DC}^2 (y^+ - y'^+)\right)$$

Discussion

- The effect of the mass will suppress the interferences, thus losing coherence more easily than in the massless antenna.
- Interference in the massive qg antenna will dissappear faster than in the massive $q\bar{q}$ one.
- The loss of coherence implies a larger energy loss.
- Phenomenological relevance to be investigated.

Summary

- Energy loss of heavy quarks is a remaining puzzle.
- Dead cone effect makes the energy loss smaller.
- Role of interferences in in-medium jets has been investigated in recent years.
- Here we compute the interferences for a heavy quark-gluon antenna.
- Coherence more easily lost for heavy quarks implies enhanced energy loss.