Radiation spectrum of a massive quark-gluon antenna

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Abstract.
We compute the radiation spectrum of antennas containing a heavy quark, focusing on the case of a massive quark-gluon antenna. We apply the classical Yang-Mills formalism (CYM), and treat the interactions of the jet with the medium in the multiple soft scattering approximation. This work completes the studies of antenna radiation inside a medium. The main conclusion is that decorrelation occurs faster in the case in which at least one of the emitters is a heavy quark, and is faster in the massive quark-gluon case.

1. Introduction

The hot and dense state of matter created in heavy ion collisions is known to modify the evolution of jets. This has been experimentally studied first at the RHIC at BNL [1] and then at the LHC at CERN [2, 3, 4]. The most successful model for this jet quenching effect is the one based on the enhancement of the gluon radiation spectrum induced by the medium [5, 6, 7, 8, 9, 10, 11, 12].

In this model, radiation off massive quarks is expected to be suppressed in regions of phase space relevant for phenomenological applications due to the dead-cone effect. On the other hand, RHIC data on the suppression of the non-photonic electrons (expected to be dominated by heavy quark decays) is compatible, taking it at face value, with no mass effect in the radiation [13]. In order to understand this experimental data, it seems that new developments in the theoretical description are needed.

Several improvements have been made in the theoretical side in the recent years[14, 15, 16, 17]. Also, efforts have been made so as to compute interference effects, which are known to play a significant role in the vacuum case [18, 19, 20, 21, 22], leading to the well known fact that subsequent emissions are angular ordered. The emerging picture in the medium case is that interferences become important when the transverse size of the antenna is smaller compared to the characteristic transverse size of the medium, and are irrelevant in the opposite case [23, 24, 25, 27, 28, 29].

Motivated by all the above considerations, we compute in this work the interference effects for antennas containing heavy quarks. We perform the calculation using a semiclassical approach and treat the interactions of the partons with the medium in the multiple soft scattering approximation. The main conclusion of the paper is that coherence is more easily lost when one of the emitters is a heavy quark, and that a massive quark-gluon antenna loses coherence faster than a massive quark-antiquark antenna. The loss of coherence is an effect that in general implies a larger energy loss.

2. Calculation of the amplitude

The amplitude for one gluon emission can be calculated using the reduction formula [25]

$$\mathcal{M}^a(k) = -\sum_\lambda \int\limits_{x^+=+\infty} \mathrm{d}x^- \mathrm{d}^2\mathbf{x}\, \mathrm{e}^{ik\cdot x}\, 2\partial_x^+ \mathbf{A}^a(x)\cdot \boldsymbol{\epsilon}_\lambda(\vec{k}) \tag{1}$$

with $k^\mu = (\omega, \vec{k})$ being the 4-momentum of the emitted gluon and $\mathbf{A}$ the transverse gauge field. The gauge field is obtained from the classical Yang-Mills (CYM) equations

$$[D_\mu, F^{\mu\nu}] = J^\nu \tag{2}$$

where $D_\mu \equiv \partial_\mu - igA_\mu$ and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$, and with the current J^μ being covariantly conserved, i.e.,

$$[D_\mu, J^\mu] = 0 \tag{3}$$

The initial state of a component of the antenna with momentum $p^\mu = (E, \vec{p})$ and charge color vector Q^a is represented by this vacuum current:

$$J^{\mu,a}_{(0)}(x) = g\frac{p^\mu}{E}\delta^{(3)}\left(\vec{x} - \frac{\vec{p}}{E}t\right)\theta(t)Q^a \tag{4}$$

The medium will affect this vacuum current by inducing a color rotation:

$$J^\mu(x) = U_p(x^+, 0)J^\mu_{(0)}(x) \tag{5}$$

described by a Wilson line:

$$U_p(x^+, 0; \mathbf{r}) \equiv \mathcal{P}\exp\left\{\int_0^{x^+} \mathrm{d}\xi\, T\cdot A^-_{\mathrm{med}}(\xi, \mathbf{p}\xi/p^+)\right\} \tag{6}$$

Leaving only terms that are linear on the medium induced field and performing the calculation in the light-cone gauge ($A^+ = 0$, with light-cone coordinates defined as $A^\pm = (A^0 \pm A^3)/\sqrt{2}$ and $\mathbf{A} = (A^1, A^2)$), we get the following expression for the amplitude:

$$\begin{aligned}\mathcal{M}^a(\vec{k}) = \sum_\lambda \frac{g}{k^+} \int\limits_{x^+=+\infty} \mathrm{d}^2\mathbf{x}\, e^{ik^-x^+} e^{-i\mathbf{k}\cdot\mathbf{x}} \int_0^{+\infty} \mathrm{d}y^+\, e^{ik^+\frac{p^-}{p^+}y^+} \\ \times\, \boldsymbol{\epsilon}_\lambda(k)\cdot(i\boldsymbol{\partial}_y + k^+\mathbf{n})\,\mathcal{G}^{ab}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+)\Big|_{\mathbf{y}=\mathbf{n}y^+} U_p^{bc}(y^+, 0)Q^c\end{aligned} \tag{7}$$

where we have explicited the color structure, defined the dimensionless vector $\mathbf{n} = \mathbf{p}/p^+$ and $\mathcal{G}$ is a Green's function that takes into account both the color rotation of the gluon and its Brownian motion in the transverse plane due to interactions with the medium field. These features of the Green's function $\mathcal{G}$ can be easily seen thanks to its expression as a path integral in the transverse plane:

$$\mathcal{G}(x^+, \mathbf{x}; y^+, \mathbf{y}|k^+) = \int\limits_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(x^+)=\mathbf{x}} \mathcal{D}\mathbf{r}\exp\left\{\frac{ik^+}{2}\int_{y^+}^{x^+} \mathrm{d}\xi\, \dot{\mathbf{r}}^2(\xi)\right\} U(x^+, y^+; \mathbf{r}) \tag{8}$$

Reached this point, we wonder what could be the effect due to the presence of a mass M. To see that, we recall that the only difference between a massive quark and a massless one comes from the dispersion relation:

$$2p^+p^- - \mathbf{p}^2 = M^2 \tag{9}$$

So we can understand the existence of a mass as a modification to the relation of p^- with respect of the other components of the momentum. Since the only place where this p^- component appears (provided we neglect any dependence of the medium field on the x^- variable) in the previous expressions is in an exponential, the presence of mass brings us a new phase.

$$\exp\left(ik^+\frac{p^-}{p^+}y^+\right) = \exp\left(i\frac{k^+}{2}\theta_{DC}^2 y^+\right)\exp\left(i\frac{k^+}{2}\mathbf{n}^2 y^+\right) \tag{10}$$

where θ_{DC} is the so called *dead-cone angle*, defined as

$$\theta_{DC} \equiv \frac{M}{p^+} \tag{11}$$

From this point on, we denote with a bar quantities (such as momentum) related to the gluon leg of the antenna, while those related to the quark leg are left without bar; e.g., p^μ represents the quark momentum and $\bar{p}^\mu$ represents the gluon momentum.

Writing down the two possible amplitudes, i.e.,

- when the gluon is radiated off a heavy quark:

$$\begin{aligned}\mathcal{M}_q^a(\vec{k}) = \sum_\lambda \frac{g}{k^+} \int\limits_{x^+=+\infty} \mathrm{d}^2\mathbf{x}\, e^{ik^-x^+} e^{-i\mathbf{k}\cdot\mathbf{x}} \int_0^{+\infty} \mathrm{d}y^+ \exp\left[i\frac{k^+}{2}(\theta_{DC}^2+\mathbf{n}^2)y^+\right] \\ \times\, \boldsymbol{\epsilon}_\lambda(k)\cdot(i\boldsymbol{\partial}_y + k^+\mathbf{n})\,\mathcal{G}^{ab}(x^+,\mathbf{x};y^+,\mathbf{y}|k^+)\Big|_{\mathbf{y}=\mathbf{n}y^+} U_p^{bc}(y^+,0)Q_q^c\end{aligned} \tag{12}$$

- when the gluon is radiated off a gluon:

$$\begin{aligned}\mathcal{M}_g^a(\vec{k}) = \sum_\lambda \frac{g}{k^+} \int\limits_{x^+=+\infty} \mathrm{d}^2\mathbf{x}\, e^{ik^-x^+} e^{-i\mathbf{k}\cdot\mathbf{x}} \int_0^{+\infty} \mathrm{d}y^+ \exp\left[i\frac{k^+}{2}\bar{\mathbf{n}}^2 y^+\right] \\ \times\, \boldsymbol{\epsilon}_\lambda(k)\cdot(i\boldsymbol{\partial}_y + k^+\bar{\mathbf{n}})\,\mathcal{G}^{ab}(x^+,\mathbf{x};y^+,\mathbf{y}|k^+)\Big|_{\mathbf{y}=\bar{\mathbf{n}}y^+} U_{\bar{p}}^{bc}(y^+,0)Q_g^c\end{aligned} \tag{13}$$

we can explicitly see the difference due to the presence of mass: a complex phase in (12) that (13) lacks.

3. Calculation of the spectrum

With the amplitudes (12) and (13) we can compute the radiation spectrum of the antenna, which reads:

$$dN = \frac{\alpha_s}{(2\pi)^2}\left[C_F\mathcal{R}_q + C_A\mathcal{R}_g - C_A\mathcal{J}\right]\frac{d^3k}{(k^+)^3} \tag{14}$$

where we have defined the independent radiation off the heavy quark $\mathcal{R}_q$

$$C_F\mathcal{R}_q = (k^+)^2\langle|\mathcal{M}_q|^2\rangle \tag{15}$$

the independent radiation off the gluon $\mathcal{R}_g$

$$C_A\mathcal{R}_g = (k^+)^2\langle|\mathcal{M}_g|^2\rangle \tag{16}$$

and the interference spectrum between both emitters $\mathcal{J}$

$$-C_A\mathcal{J} = (k^+)^2\,\mathrm{Re}\langle\mathcal{M}_q\mathcal{M}_g^\dagger\rangle \tag{17}$$

3.1. Independent Radiation Spectrum.
The spectrum of independent radiation off the heavy quark $\mathcal{R}_q$ is directly evaluated from (15) taking the limit $\mathbf{n} \to 0$ and it reads

$$\mathcal{R}_q = 2\mathrm{Re} \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \exp\left[i\frac{k^+}{2}\theta_{DC}^2 (y^+ - y'^+) \right] \times \\ \times \int \mathrm{d}^2\mathbf{z} \exp\left[-i\mathbf{k}\cdot\mathbf{z} - \frac{1}{2}\int_{y'^+}^\infty \mathrm{d}\xi\, n(\xi)\sigma(\mathbf{z}) \right] \boldsymbol{\partial}_y \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\big|_{\mathbf{y}=\mathbf{0}} \quad (18)$$

This expression serves as a check of the formalism, since it is the same found in [26].

For the independent radiation off the gluon we can evaluate (16) or, given the similarities between (12) and (13), just take the limit $\theta_{DC} \to 0$ in (18), which gives us the following result:

$$\mathcal{R}_g = 2\mathrm{Re} \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \int \mathrm{d}^2\mathbf{z} \exp\left[-i\mathbf{k}\cdot\mathbf{z} - \frac{1}{2}\int_{y'^+}^\infty \mathrm{d}\xi\, n(\xi)\sigma(\mathbf{z}) \right] \times \\ \times \boldsymbol{\partial}_y \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\big|_{\mathbf{y}=\mathbf{0}} \quad (19)$$

3.2. Interference Spectrum.
Taking the medium averages and following the same procedure as in [25], we found that the interference spectrum $\mathcal{J}$ yields

$$\mathcal{J} = \mathrm{Re}\Bigg\{ \int_0^\infty \mathrm{d}y'^+ \int_0^{y'^+} \mathrm{d}y^+ \, \big(1 - \Delta(y^+, 0)\big) \exp\left[i\frac{k^+}{2}(\theta_{DC}^2 + \delta\mathbf{n}^2) y^+ \right] \times \\ \times \int \mathrm{d}^2\mathbf{z} \exp\left[-i\bar{\boldsymbol{\kappa}}\cdot\mathbf{z} - \frac{1}{2}\int_{y'^+}^\infty \mathrm{d}\xi\, n(\xi)\sigma(\mathbf{z}) \right] \times \\ \times (\boldsymbol{\partial}_y - ik^+\delta\mathbf{n}) \cdot \boldsymbol{\partial}_z \, \mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y})\big|_{\mathbf{y}=\delta\mathbf{n}y^+} \Bigg\} + \mathrm{sym.} \quad (20)$$

with $\bar{\boldsymbol{\kappa}} = \mathbf{k} - k^+\bar{\mathbf{n}}$ (and $\boldsymbol{\kappa} = \mathbf{k} - k^+\mathbf{n}$) and $\mathcal{K}$ being the path integral

$$\mathcal{K}(y'^+, \mathbf{z}; y^+, \mathbf{y}|k^+) = \int_{\mathbf{r}(y^+)=\mathbf{y}}^{\mathbf{r}(y'^+)=\mathbf{z}} \mathcal{D}\mathbf{r} \exp\left\{ \int_{y^+}^{y'^+} \mathrm{d}\xi \, \left(i\frac{k^+}{2}\dot{\mathbf{r}}^2(\xi) - \frac{1}{2}n(\xi)\sigma(\mathbf{r}) \right) \right\} \quad (21)$$

that takes into account the gluon multiple scattering with the medium and its Brownian motion in the transverse plane from $\mathbf{r}(y^+) = \mathbf{y}$ to $\mathbf{r}(y'^+) = \mathbf{z}$. The symmetric part is obtained interchanging $q \leftrightarrow g$, i.e., putting a bar in quantities without it and viceversa.

The only place where mass appears in the interference spectrum (20) is in the phase $\exp(ik^+\theta_{DC}^2 y^+/2)$. If the dead-cone angle is large, satisfying

$$\theta_{DC} \gg \sqrt{\frac{2}{k^+L}} \quad (22)$$

where L is the medium length, the mass phase will oscillate quickly, annihilating the interference. Putting it in space coordinates, this decoherence condition reads

$$\theta_{DC} \gg \frac{1}{\sqrt{\omega L}} \quad (23)$$

This loss of coherence implies a larger energy loss.

3.3. Comparison with a massive quark-antiquark antenna

We can also compute the interference spectrum for a massive quark-antiquark antenna. In this case, the mass phase is:

$$\exp\left[i\frac{k^+}{2}(\theta_{DC}^2 y^+ - \bar{\theta}_{DC}^2 y'^+)\right]$$

while in the massive quark-gluon case it was:

$$\exp\left(i\frac{k^+}{2}\theta_{DC}^2 y^+\right)$$

In other words, the mass phase for a heavy quark-antiquark antenna goes with the gluon time formation, while for a heavy quark-gluon antenna it goes with the length of the medium. Thus, the interference between emitters will be destroyed faster in a heavy qg antenna.

4. Summary

In this work the interference effects for antennas containing massive quarks are calculated in a semiclassical framework. We treat the interactions of the partons with the medium in the multiple soft scattering approximation. Our main conclusion is that the presence of a mass scale in the problem favors a faster decorrelation of the emitters of the antenna, an effect which leads to an enhancement of the spectrum and is in competition with the known suppression of the spectrum of a single massive emitter. We also compare the cases of a massive quark-gluon and a massive quark-antiquark antennae.

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