Analysis and design optimization of a rough neutron mirror

M. Escobar*, F. Lamy,**
A. E. Meyerovich*, and V. V. Nesvizhevsky**

* *Department of Physics, University of Rhode Island, USA*
** *Institute Laue-Langevin, Grenoble, France*

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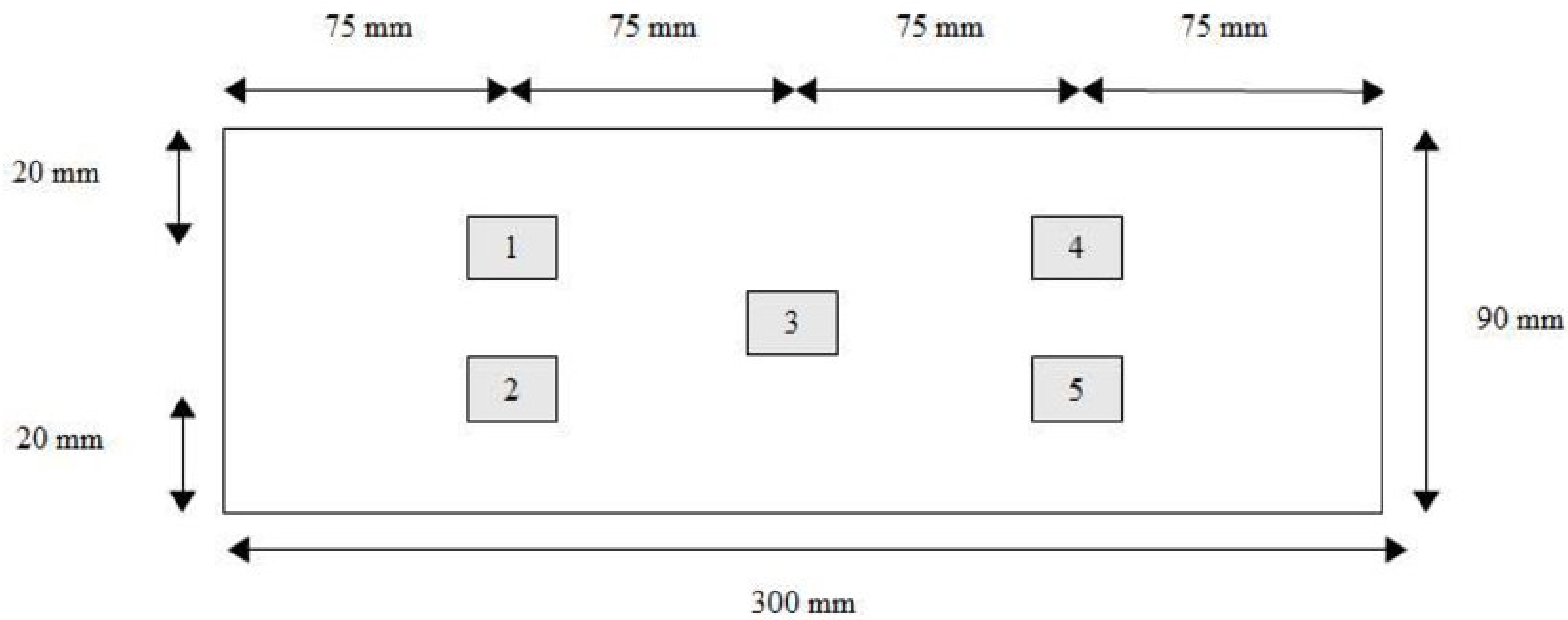


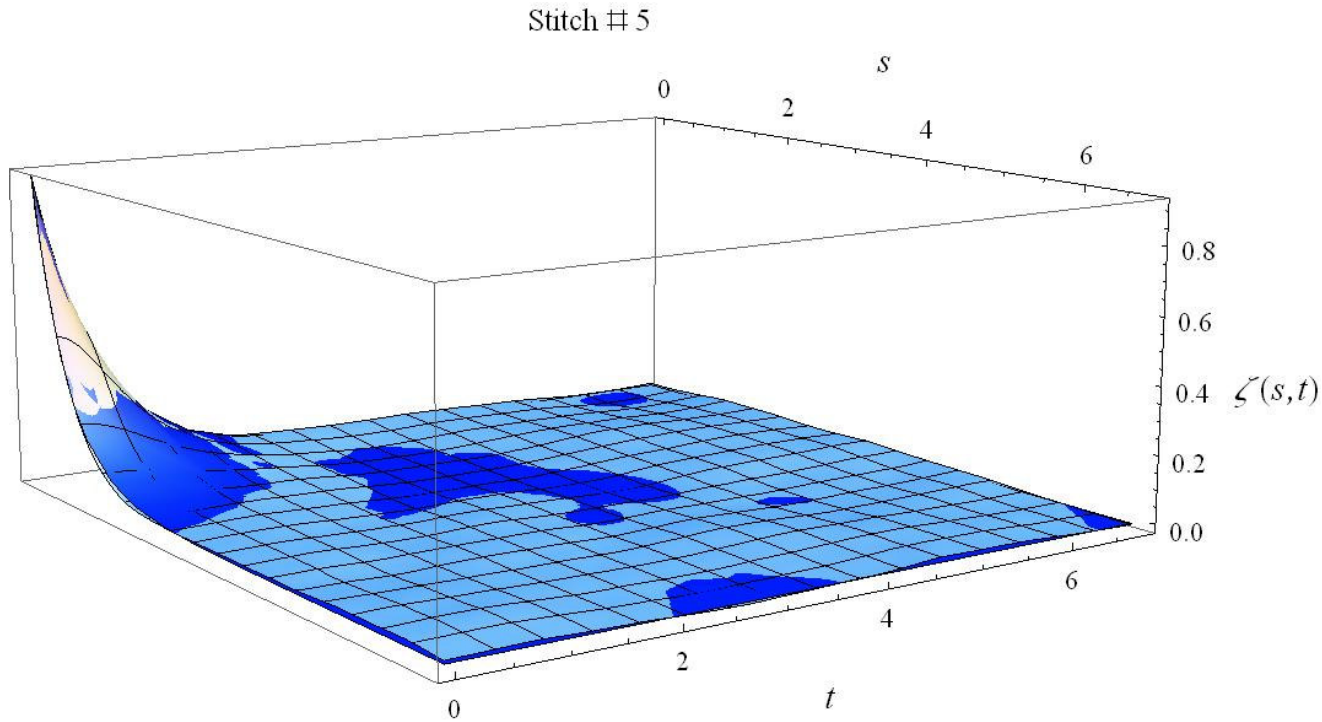
The size of each stitch is $0.504 \times 0.504\ mm^2$.

The data set for each stitch contains $2{,}557 \times 2{,}557 \approx 6.5 \times 10^6$ points.

The distance between individual data points is 0.19 µm.

Up to 2% of profile date are “bad”.

Roughness correlation function



Lengths in units of $\ell = \hbar^{2/3}\left(2m^2 g\right)^{-1/3} \simeq 5.87\ \mu m$ - *size of the lowest level*

Roughness correlation function

#	η	*E: r_s, r_t*, $\sigma\times 10^2$	*PL: r_s, r_t, u*, $\sigma\times 10^2$
1	1.03	0.62, 0.63, 1.42	0.45, 0.45, 0.9997, 1.56
2	1.14	0.70, 0.73, 2.69	0.44, 0.46, 0.864, 2.04
3	0.97	0.64, 0.74, 2.71	0.28, 0.31, 0.658, 1.37
4	0.99	0.64, 0.66, 2.35	0.32, 0.34, 0.73, 1.26
5	0.96	0.57, 0.60, 0.83	0.46, 0.48, 1.10, 1.20
Av	1.02	0.64, 0.67, 1.84	0.38, 0.40, 0.842, 1.33

E: fit with anisotropic exponential function

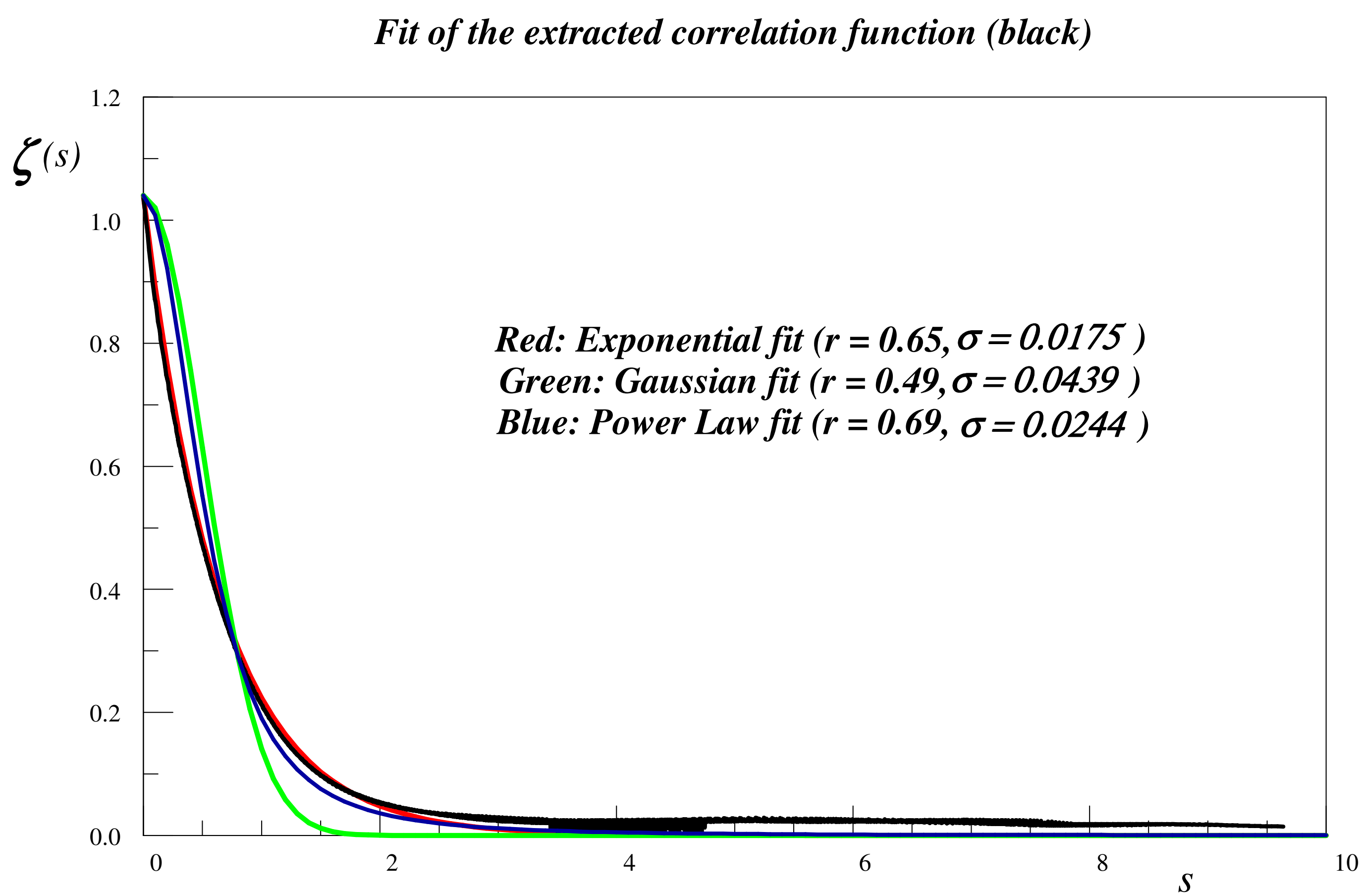
PL: fit with anisotropic power law function

η: amplitude of roughness, $r_{s,t}$: correlation radii in x,y-directions

Anisotropy is about 4%, the difference between stitches about 10%

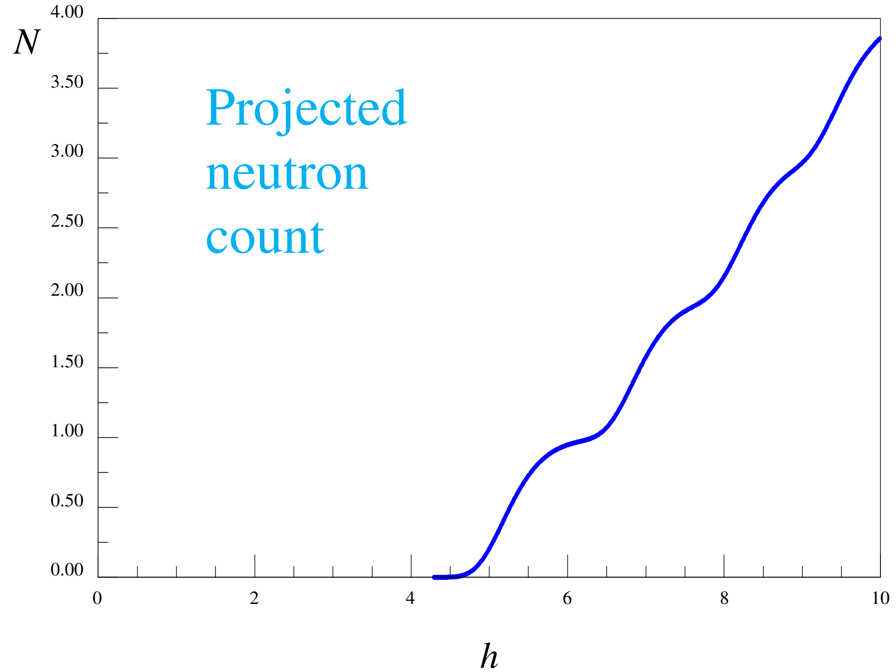
Quality of fits σ does not depend on the fitting function!

Identification of the roughness correlation function – actual mirror



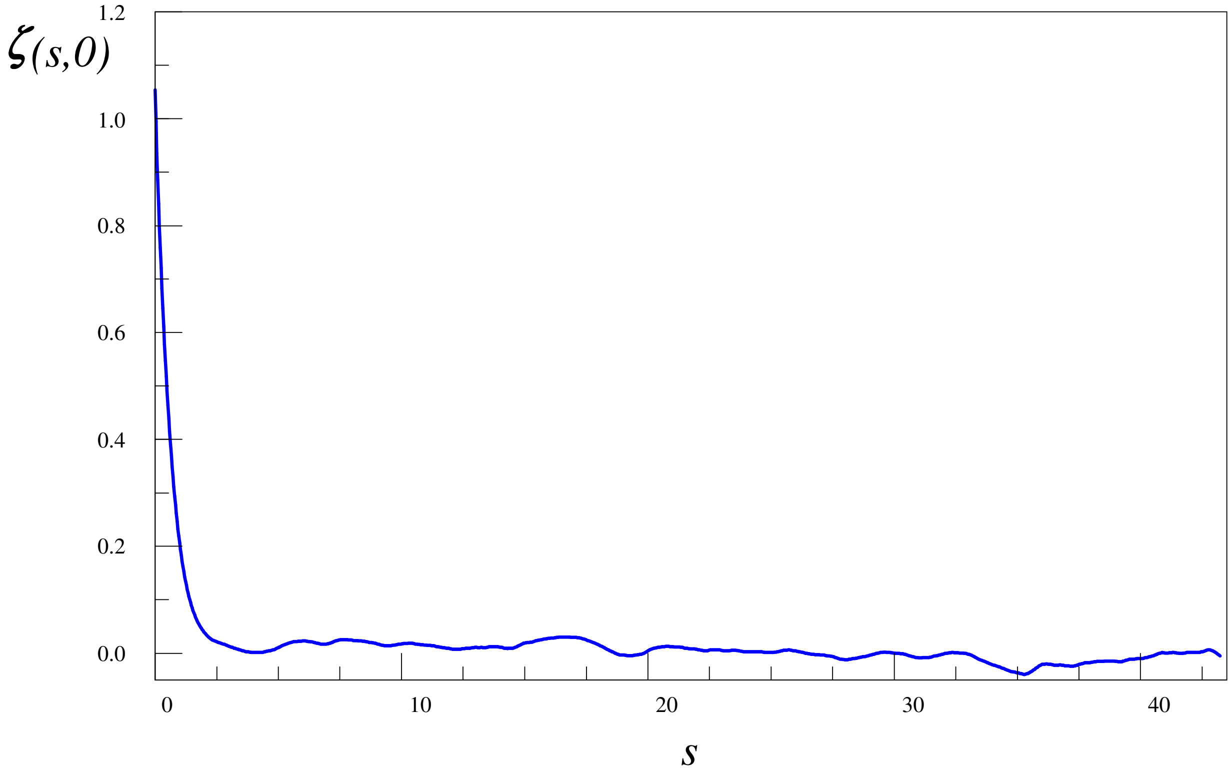
Roughness correlation function

Our *very reluctant* choice is the isotropic exponential correlation function $\zeta(s) = \eta^2 exp(-s/r)$ with $r = 0.65$, $\eta = 1.02$



Misgivings: η is too large which is not good for both theory and experiment

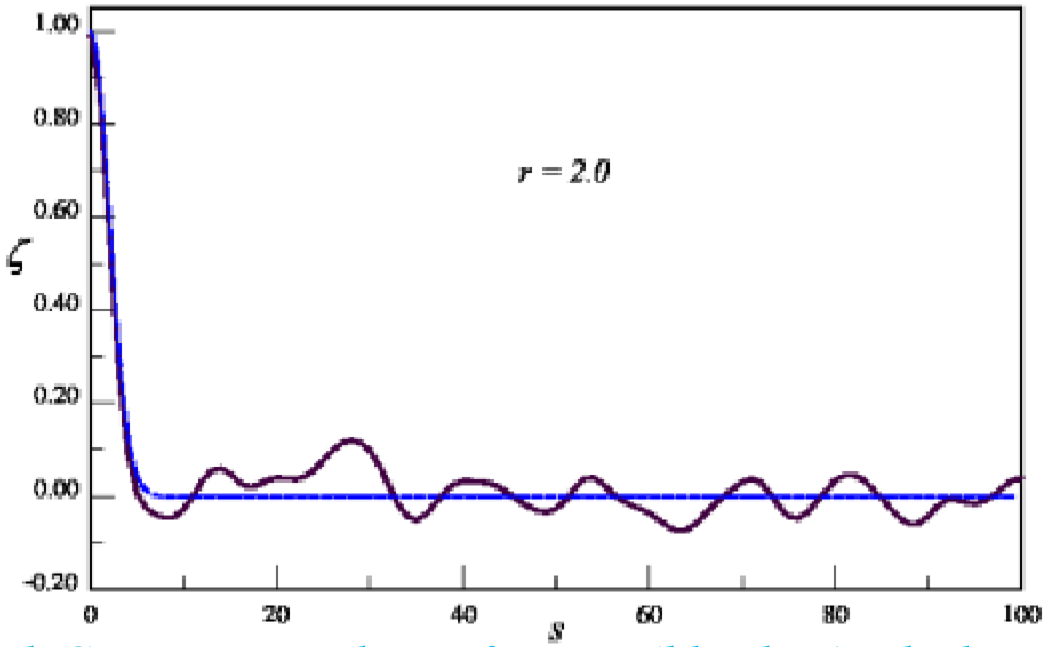
Roughness correlation function



An example of a long fluctuation-driven tale which explains difficulties with fitting. The fluctuations are due to the finite size of samples. Actual mirror.

Identification of the roughness correlation function

We generated random rough surfaces with known correlation functions, "measured" the profile, and fitted the extracted correlator with different correlation functions. The issue is the same: fat tails



Rough surface with Gaussian correlation function (blue line). Black – extracted correlator . The fitting error σ *comes mostly from the tail!*

Identification of the roughness correlation function

We generated surfaces with a known (Gaussian) correlation function and fitted the extracted correlator with different correlation functions.

#	G: r, $\sigma\times10^4$	E: r, $\sigma\times10^4$	PL: r, $\sigma\times10^4$	N: $\sigma\times10^{17}$	Φ_G, Φ_E, Φ_{PL}, Φ_n
1	1.19, 5.24	1.59, 5.81	1.44, 5.81	1.92	23.86, 18.19, 18.81, 21.96
2	1.15, 4.49	1.53, 4.56	1.36, 4.64	1.83	23.33, 17.84, 18.65, 21.14
3	1.25, 4.37	1.69, 4.40	1.54, 4.47	1.69	23.56, 17.26, 17.85, 20.96

Good quality of fit σ does not confirm appropriateness of a choice of a fitting function and does not reflect the quality of predictions for observables! (Φ for this surface should be 23.48)

Identification of the roughness correlation function – actual mirror

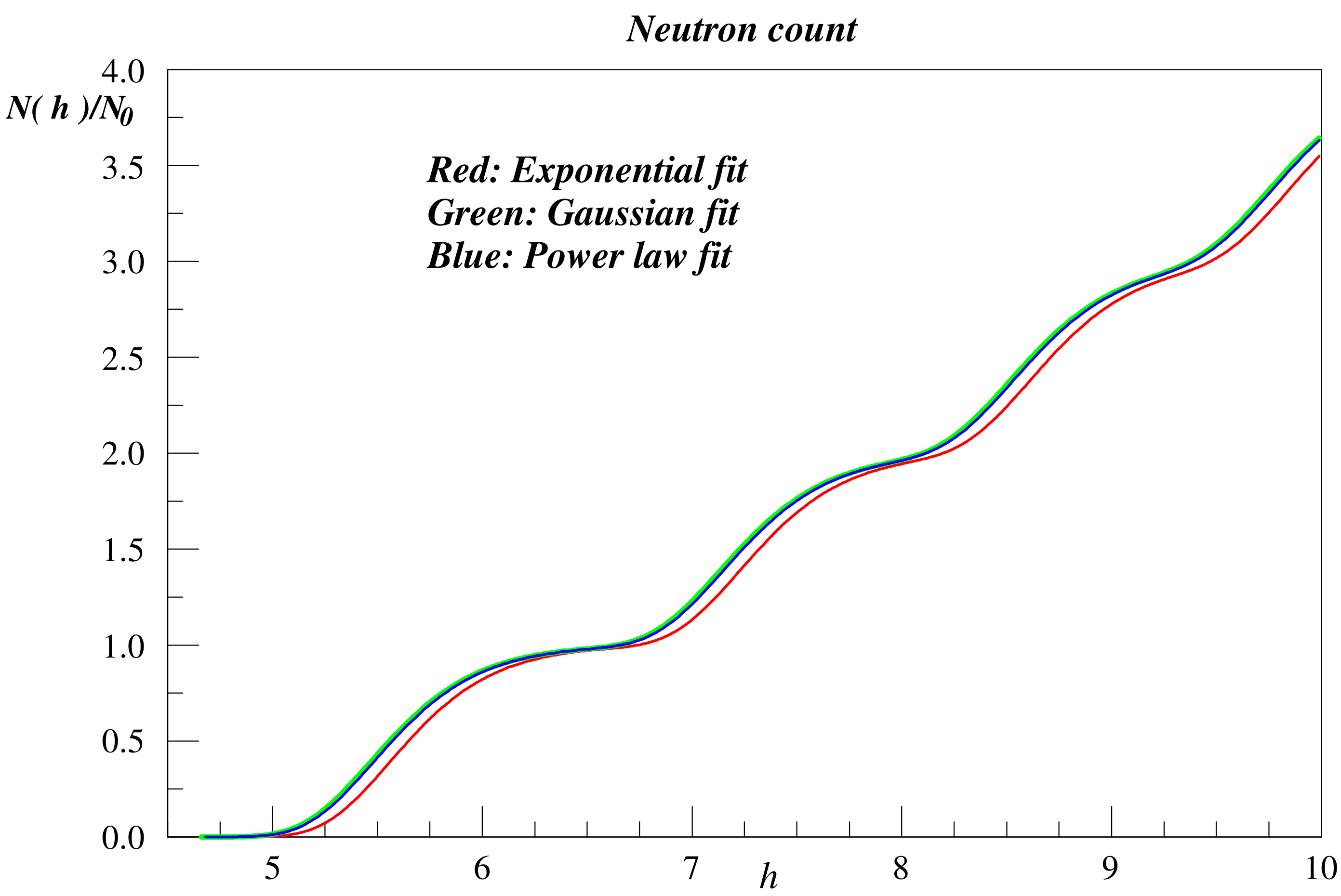
#	η	$r_E, \sigma_E \times 10^2$	$r_G, \sigma_G \times 10^2$	$r_{PL}, \sigma_{PL} \times 10^2$	$(\Phi_E, \Phi_G, \Phi_{PL}) \times 10^{-3}$
1	1.03	0.62, 1.42	0.49, 3.93	0.66, 2.04	5.44, 3.94, 4.18
2	1.14	0.72, 2.44	0.56, 5.75	0.76, 3.15	6.23, 4.52, 4.78
3	0.97	0.68, 2.61	0.50, 4.94	0.70, 3.23	4.63, 3.46, 3.60
4	0.99	0.65, 2.25	0.49, 4.56	0.68, 2.80	4.92, 3.64, 3.81
5	0.96	0.58, 0.68	0.47, 3.0	0.62, 1.48	4.87, 3.50, 3.75
Av.	1.02	0.65, 1.75	0.50, 4.39	0.69, 2.44	5.22, 3.83, 4.01

E – exponential fit $\eta\, exp(-s/r)$

G – Gaussian fit $\eta\, exp(-s^2/2r^2)$

PL – power law fit $\eta/(1+ s^2/r^2)^{3/2}$

Predicted neutron count with a new mirror



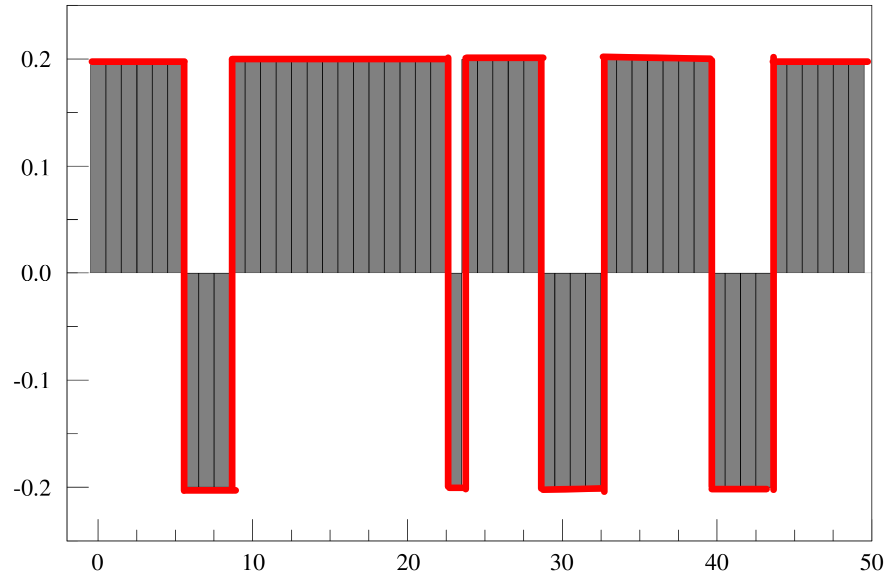
Predicted neutron count: Sources of errors

1. *Amplitude of roughness is too large – the theory looses its accuracy*
2. *Amplitude of roughness is too large – position of the reference plane is uncertain and the gap between the mirrors is not well-defined*
3. *Energy levels have uncertain values and are too broad*
4. *Uncertainty in the roughness parameters and the correlation function*
5. *There is a possibility of roughness-driven localization*

Proposal for a new rough mirror

Computationally generate a random 1D grid using Monte Carlo simulations based on the1D Ising Hamiltonian and transfer this pattern onto a real mirror. Because of a large spatial scale (~ 6 μ), the procedure seems feasible.

Element of an "Ising" mirror



Mirror parameters:
η = 0.2, r = 2

Ising parameters:
$J/T = 0.7$, n = 1,000,
10^6 Ising cycles

Red line: mirror surface, grey bars: Ising "spins" (~ 6 μ wide)

Proposal for a new rough mirror

Advantages:

- Known (almost exponential) roughness correlation function

$$\zeta(x) = \eta^2[\exp(-\lfloor x \rfloor/r) + (x-\lfloor x \rfloor)(\exp(-\lceil x \rceil/r)-\exp(-\lfloor x \rfloor/r))]$$

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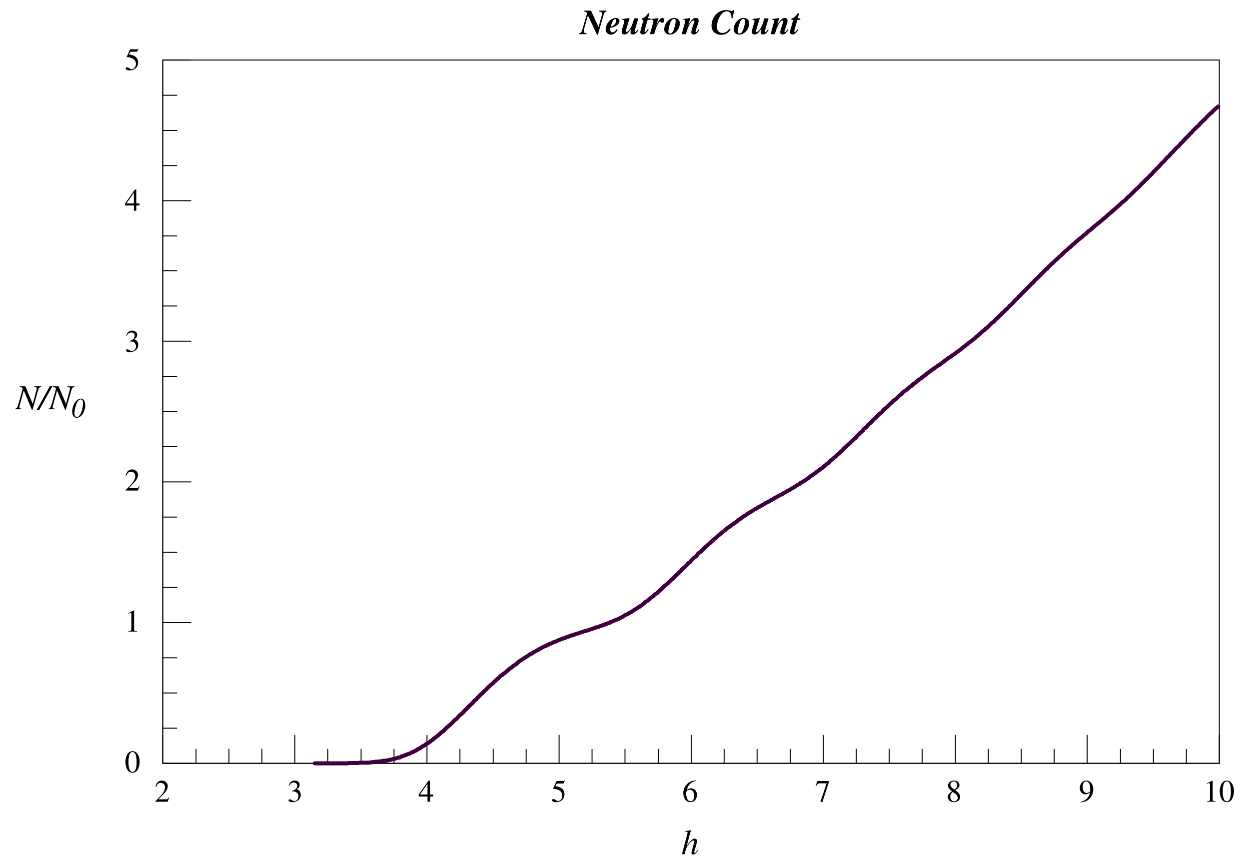
- The environment is more controllable
- No problems with determining the reference plane, and, by extension, the distance between the mirrors
- No losses due to sideway scattering
- Our preference is $r = 2$ and $\eta \sim 0.2 \div 0.4$ resulting in Φ in $43.5 \div 170$ range. This is sufficient to observe a step-wise dependence of neutron count.

Proposal for a new rough mirror



Neutron count for the 1D Ising-inspired mirror with the roughness amplitude $\eta = 0.4$ and the correlation radius $r = 2$

Conclusions

- *We report precise measurements of the surface profile of the new rough neutron mirror*

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- *The extracted correlation function is reasonably close to the isotropic exponential function with $\eta = 1.02$ and $r = 0.65$*
- *The experimental setup seems to be sufficient to reveal the quantum steps in neutron count*
- *There are theoretical and experimental issues that can hinder the exact quantitative interpretation of the data*
- *We propose a new mirror design with roughness based on the Monte Carlo simulations for the* 1D *Ising model which provides a more controlled environment*