Non-dispersive phase shifters

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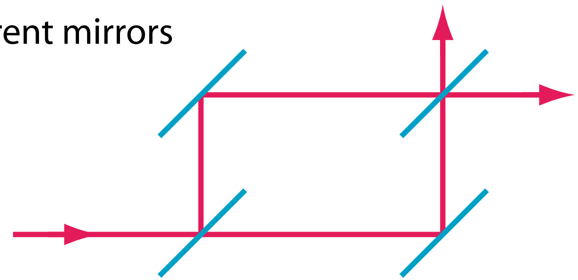
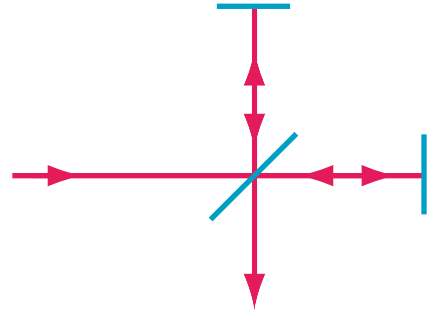
Typical Interferometer Types

Semi-transparent mirrors

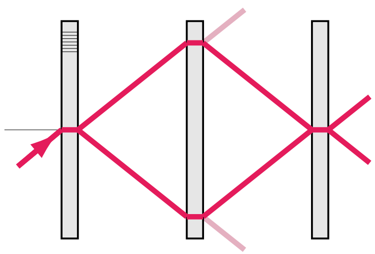


Mach-Zehnder



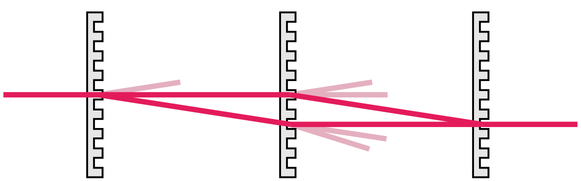
Michelson

Bragg crystals



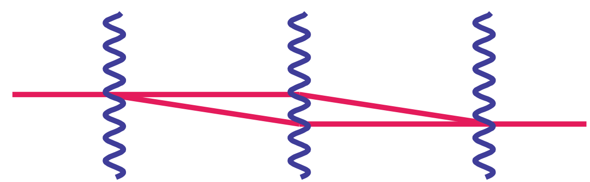
Rauch & Werner (Th N), Bonse (X ray), Fally (Cold N)

Diffraction gratings



Zeilinger (VCN)

Laser fields

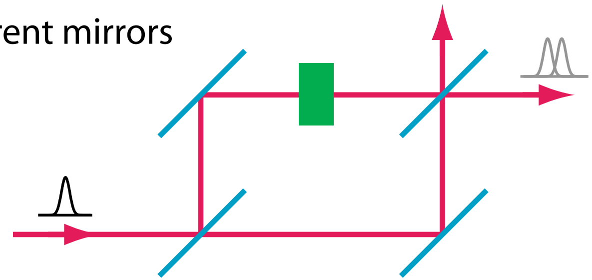


Atominterferometry
Arndt (Molecules)

only small angle splitting

Typical Interferometer Types and Phase Shifters

Semi-transparent mirrors



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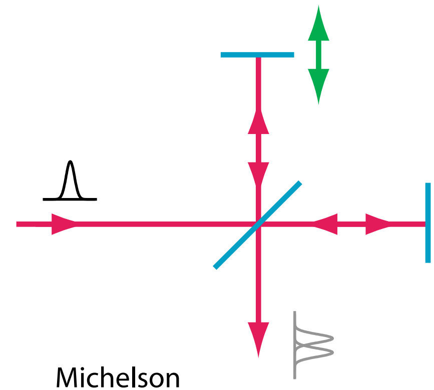
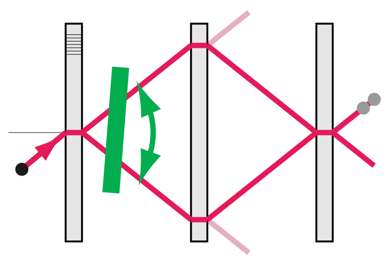


Michelson

Bragg crystals



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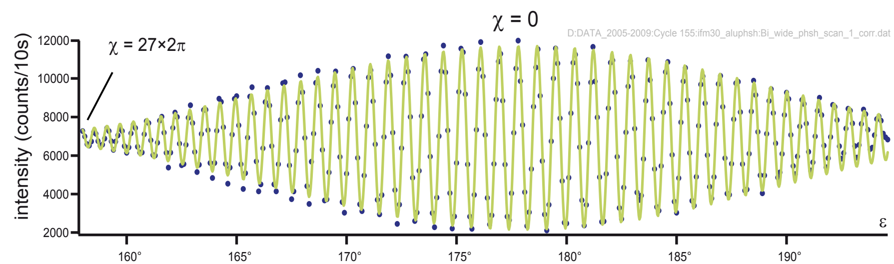
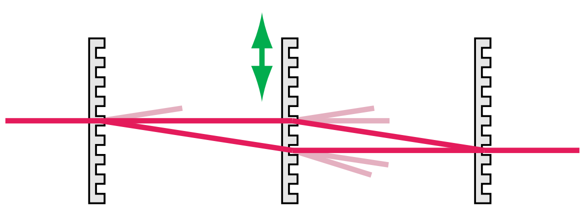


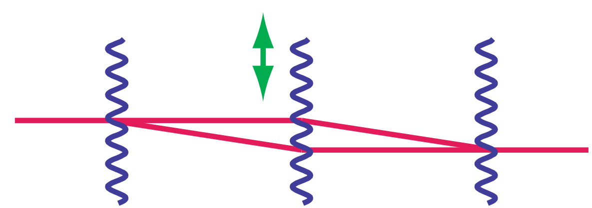
Diffraction gratings



Zeilinger (VCN)

Laser fields

non-dispersive phase shift

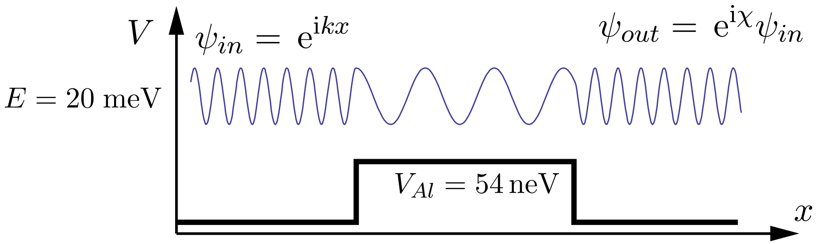


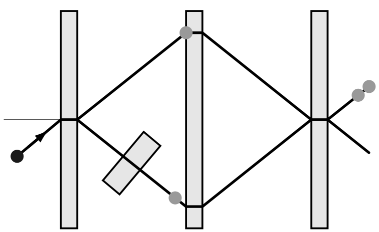
only small angle splitting

Atominterferometry
Arndt (Molecules)

Phase shift and spatial displacement

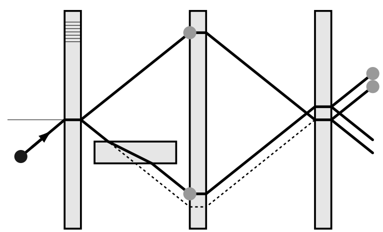
Phase shifter = region of changed potential V





$\delta k/k = 0.02$

$l_c = 7\,\mathrm{nm}$



$\delta k_\perp/k_\perp = 5 \cdot 10^{-6}$

$l_{c\perp} = 80\,\mu\mathrm{m}$

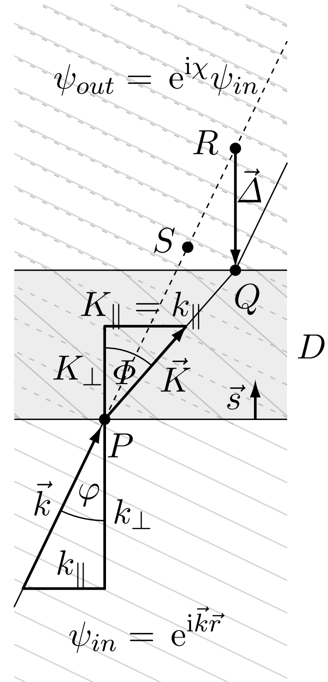
1mm Aluminium:

$\chi = 6.6 \cdot 2\pi$

$\Delta = 1.3\,\mathrm{nm}$

$\chi_\perp = 12.6 \cdot 2\pi$

$\Delta_\perp = 9\,\mathrm{nm}$



Phase shift

$$\chi = D(K_\perp - k_\perp)$$

Spacial displacement

$$\vec{\Delta} = \vec{s}\,D\left(\frac{K_\perp - k_\perp}{K_\perp}\right)$$

$$\chi = \vec{K}\,\vec{\Delta}$$

if wave packet separation $\vec{\Delta}$ > coherence length

$\Rightarrow$ no interference visible

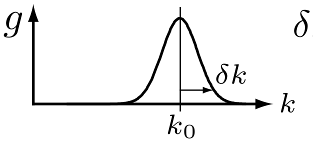
Is χ and $\vec{\Delta}$ always coupled?

Dispersion of wave packets

free wave packet

$$\psi(x,t) = \int g(k-k_0)\, e^{ikx} e^{-i\omega t} dk$$

$$= e^{ik_0 x} \int g(\delta k)\, e^{i\delta k x} e^{-i\omega t} d\delta k$$

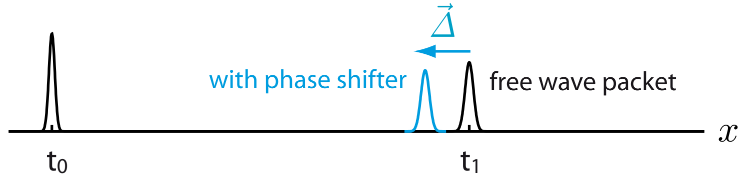


$$\delta k = k - k_0$$

$$g(\delta k) = \frac{1}{\sqrt{2\pi}\sigma_k}\, e^{-\frac{\delta k^2}{2\sigma_k^2}}$$

$$\omega = \frac{E}{\hbar} = \frac{\hbar^2 (k_0 + \delta k)^2}{2m}$$

Dispersion of wave packets



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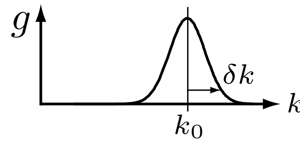

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with phase shifter

$$\chi = D(K-k) = Dk\left(\sqrt{1-\frac{2mV}{\hbar^2 k^2}} - 1\right)$$

expand around k_0:

$$= \chi(k_0) + \chi'(k_0)\delta k + \frac{1}{2}\chi''(k_0)\ \delta k^2 + \ldots$$

$$= \chi_0 - \frac{\chi_0}{K_0}\delta k + \chi_0 \frac{K_0 + k_0}{2K_0^3}\delta k^2 + \ldots \qquad \chi_0 = D(K_0 - k_0)$$

$$\psi(x,t) = e^{ik_0 x} \int g(\delta k)\, e^{i\chi}\, e^{i\delta k x} e^{-i\omega t} d\delta k$$

$$= e^{i(k_0 x + \chi_0)} \int g(\delta k)\ e^{i\delta k \left(x - \frac{\chi_0}{K_0}\right)}\ e^{-i\omega t}\ e^{i\mathcal{O}(\delta k^2)}\ d\delta k$$

zero-order term
$\Rightarrow$ phase shift
χ_0

first-order term
$\Rightarrow$ spatial displacement
$\Delta = \chi_0 / K_0$

higher order terms
$\Rightarrow$ shape of wave packet

General result for wave packets, not limited to Gaussians.

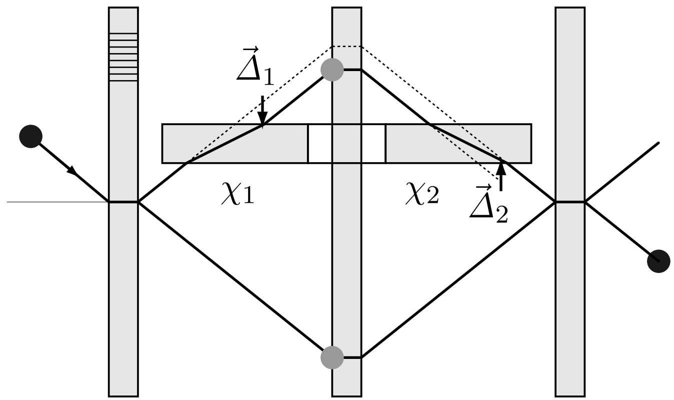
Examples of non-dispersive phases

Topological phases and geometric phases

- Aharonov-Bohm effects, no forces, only potentials
- 4π spin symmetry
- other spin gymnastics (Berry phase, Pancharatnam phase, ...)

They are all completely wave legth independent!

New concept: Dual phase shifter parallel to lattice planes



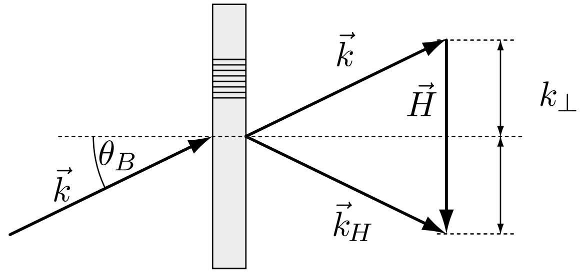
Lemmel & Wagh
Phys Rev A 2010

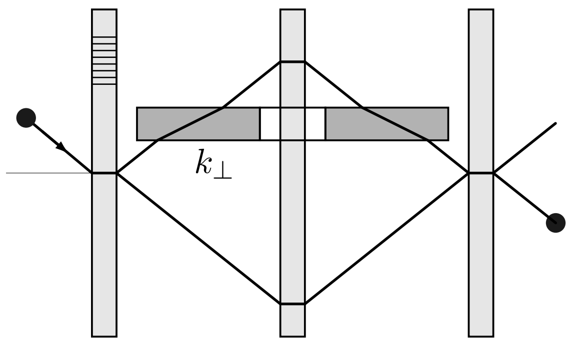
$$
\begin{array}{ccccc}
\vec{\Delta}_1 & + & \vec{\Delta}_2 & = & \vec{\Delta}_{tot} = 0 \\
\chi_1 & + & \chi_2 & = & \chi_{tot} = 2\chi
\end{array}
$$

Where the first order cancels

„Triple Laue“ or „LLL interferometer“:
beam splitter, mirrors and analyzer are
Bragg crystals in symmetric Laue geometry

Bragg‘s law: $$k\ \sin\theta_B = k_\perp \ = \frac{H}{2} = \frac{\pi}{d}$$





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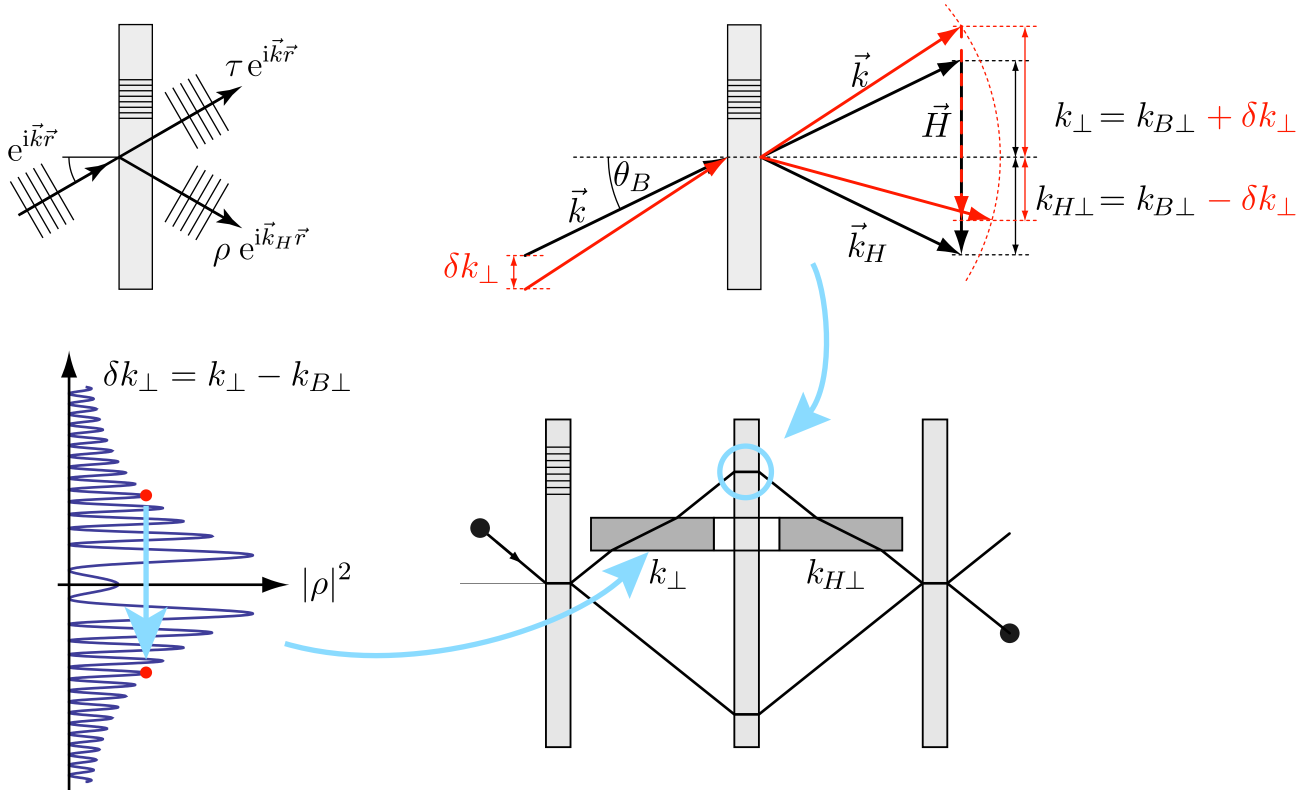
Dynamical diffraction:



Mirror crystal turns the fast components of the first sample transit into the slow ones of the second transit and v.v.

$$\chi_{tot} = \chi(\delta k_\perp) + \chi(-\delta k_\perp)$$

all odd dispersion orders cancel
$\Rightarrow$ zero order sums up, $\chi_{tot} = 2\chi$
$\Rightarrow$ first order cancels, no spatial displacement

Where the first order cancels

„Triple Laue“ or „LLL interferometer“: beam splitter, mirrors and analyzer are Bragg crystals in symmetric Laue geometry

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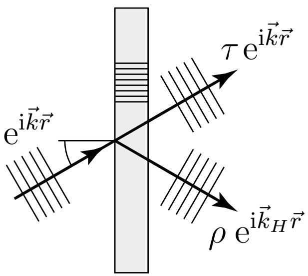

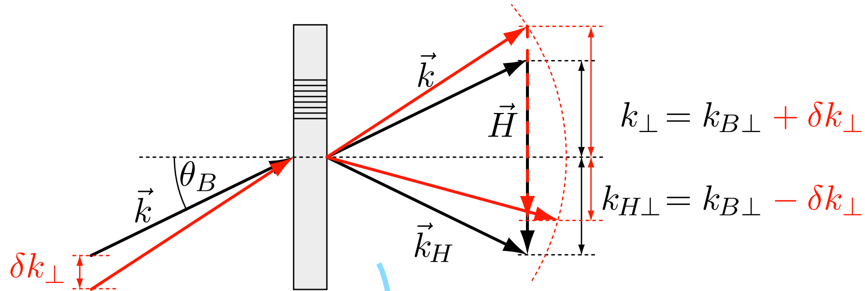

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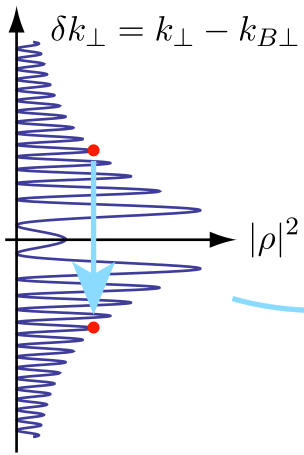

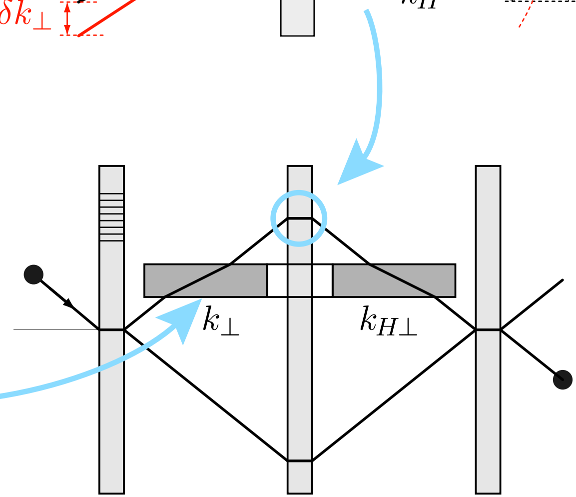

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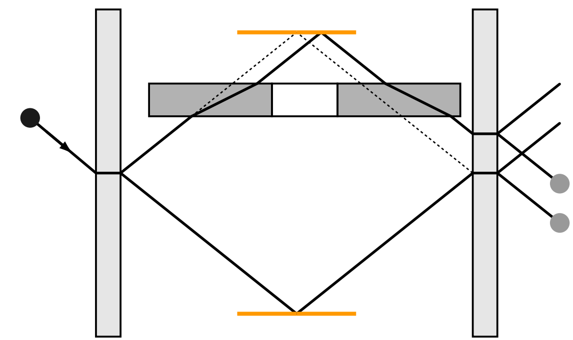
Comparison to ordinary mirror:



$$\chi_{tot} = \chi(\delta k_\perp) + \chi(\delta k_\perp)$$

$\Rightarrow$ double phase shift

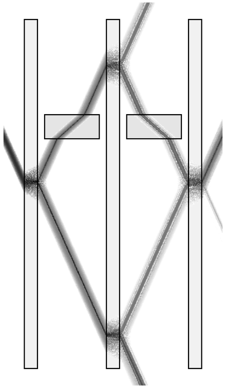
$\Rightarrow$ double spatial displacement

Is it true? $\chi_{tot} = 2\chi$

- **Schrödinger equation**

  plane wave solution analytically for the whole system,

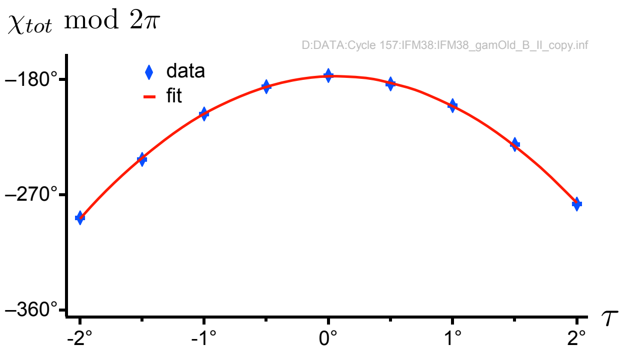
  $\Rightarrow \quad \chi_{tot} = 2\chi$



- **Experiment**

  sample tilt τ changes effective thickness

$$\chi_{tot}(\tau) = \chi_{tot}(0) / \cos\tau$$



curvature reveals complete χ_{tot}, not modulo 2π

- measured: $\chi_{tot} = (466 \pm 27) \cdot 2\pi$
- expected: $\chi_{tot} = 456 \cdot 2\pi$

$$\Rightarrow \quad \chi_{tot} = 2\chi$$

- **Path integral**

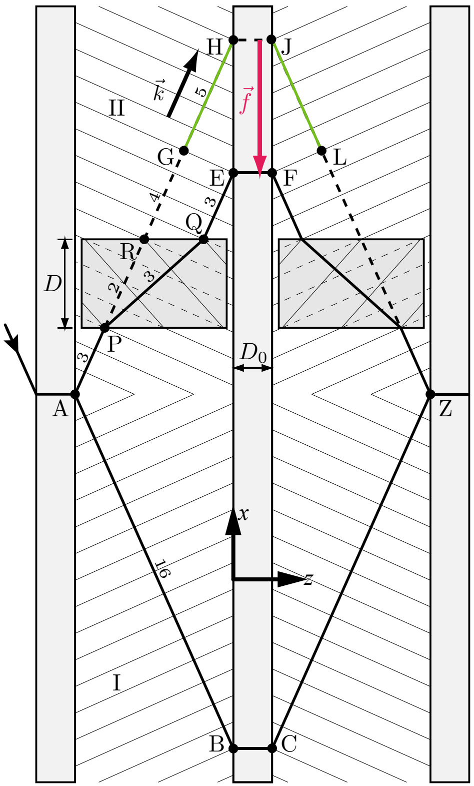
  integrate Lagrangian $\mathcal{L}$ along interferometer loop

$$\chi_{tot} = \frac{1}{\hbar} \oint \mathcal{L}\, dt = \oint \vec{k}(s)\, d\vec{s}$$

$$= 2\chi + \underbrace{\chi_{shortcut} + \chi_{sliding}}_{\text{cancel}}$$

$$\chi_{sliding} = -2\,\vec{k}\vec{f}$$

$$\chi_{shortcut} = k\, s_{shortcut} = 2\,\vec{k}\vec{f}$$



- **Path integral: perturbative**
  - integrate perturbation $\delta\mathcal{L}$ of Lagrangian by phase shifter
  - along undisturbed path

$$\chi_{tot} = \frac{1}{\hbar} \oint \delta\mathcal{L}\, dt = \oint \delta\vec{k}(s)\, d\vec{s} = 2\,(K_\perp - k_\perp)\, D = 2\chi$$

UCN interferometer?

$V_{Al} = 54\,\mathrm{neV}$

1 mm Aluminium phase shifter

ThN — silicon crystal

e.g. Lemmel, Rauch ILL S18

$\lambda = 2\,\text{Å}$	$d = 2\,\text{Å}$	$\delta k/k = 0.02$	$\chi = 6.6 \cdot 2\pi$
$E = 20\,\mathrm{meV}$	$\theta_B = 30°$	$l_c = 7\,\mathrm{nm}$	$\Delta = 1.3\,\mathrm{nm}$
$v = 2000\,\mathrm{m/s}$	$\Delta_P = 70\,\mu\mathrm{m}$	$\delta k_\perp / k_\perp = 5 \cdot 10^{-6}$	$\chi_\perp = 12.6 \cdot 2\pi$
	$D = 3\,\mathrm{mm}$	$l_{c\perp} = 80\,\mu\mathrm{m}$	$\Delta_\perp = 9\,\mathrm{nm}$

CN — polymer "crystal"

Pruner, Klepp, Fally ILL D22 (2006)

$\lambda = 2\,\mathrm{nm}$	$d = 380\,\mathrm{nm}$	$\delta k/k = 0.1$	$\chi = 66 \cdot 2\pi$
$E = 200\,\mu\mathrm{eV}$	$\theta_B = 0.15°$	$l_c = 10\,\mathrm{nm}$	$\Delta = 130\,\mathrm{nm}$
$v = 200\,\mathrm{m/s}$	$\Delta_P = 0.7\,\mathrm{mm}$	$\delta k_\perp / k_\perp = 0.12$	
	$D = 2.7\,\mathrm{mm}$	$l_{c\perp} = 6\,\mu\mathrm{m}$	

VCN — phase grating

Zouw, Zeilinger ILL PF2 (2000)

$\lambda = 10\,\mathrm{nm}$	$d = 2\,\mu\mathrm{m}$	$\delta k/k = 0.2$	$\chi = 330 \cdot 2\pi$
$E = 8\,\mu\mathrm{eV}$	$\theta_{\mathrm{dif}} = 0.26°$	$l_c = 22\,\mathrm{nm}$	$\Delta = 3\,\mu\mathrm{m}$
$v = 40\,\mathrm{m/s}$			

VCN — polymer "crystal" — Alu ph.sh.

hypothetical

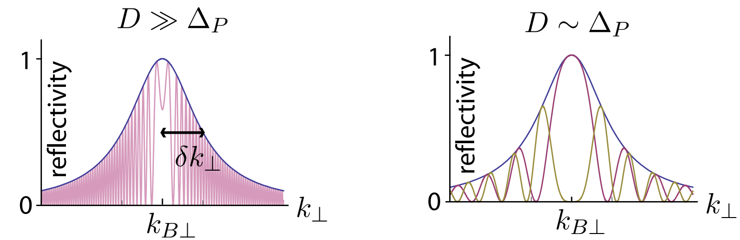
$\lambda = 30\,\mathrm{nm}$	$d = 30\,\mathrm{nm}$		$\chi = 1000 \cdot 2\pi$
$E = 900\,\mathrm{neV}$	$\theta_B = 30°$		$\Delta = 31\,\mu\mathrm{m}$
$v = 13\,\mathrm{m/s}$	$\Delta_P = 67\,\mu\mathrm{m}$	$\delta k_\perp / k_\perp = 8 \cdot 10^{-4}$	$\chi_\perp = 2000 \cdot 2\pi$
	$D = 0.5\,\mathrm{mm}$	$l_{c\perp} = 80\,\mu\mathrm{m}$	$\Delta_\perp = 145\,\mu\mathrm{m}$

UCN

?

$\lambda = 66\,\mathrm{nm}$	any crystal possible?		control phase by magnetic fields?
$E = 190\,\mathrm{neV}$			
$v = 6\,\mathrm{m/s}$			

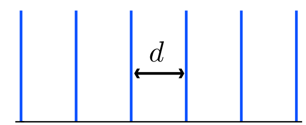
Crystal thickness



$\Delta_P = \text{Pendellösung length}$

Crystal potential

real crystal: point-like scattering centers

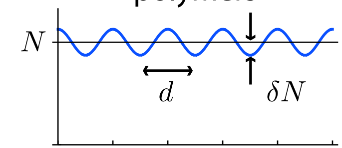


dynamical diffraction theory

$\delta k_\perp = d\,2Nb_c$

$\Delta_P = k_\parallel/(2Nb_c)$

density modulated polymers



rigorous coupled wave theory

$\delta k_\perp = d\,\delta N b_c$

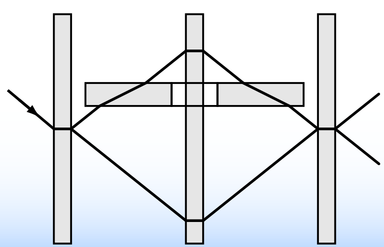
$\Delta_P = k_\parallel/(\delta N b_c)$

$b_c = \text{coherent scattering length}$

$N = \text{atom number density}$

Phase shifter

material



magnetic field