Frequency shifts in gravity resonance spectroscopy

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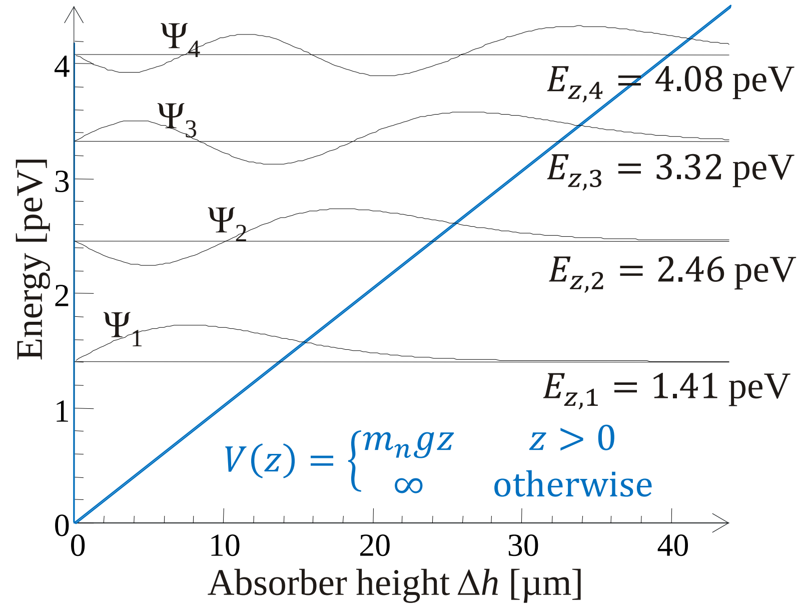
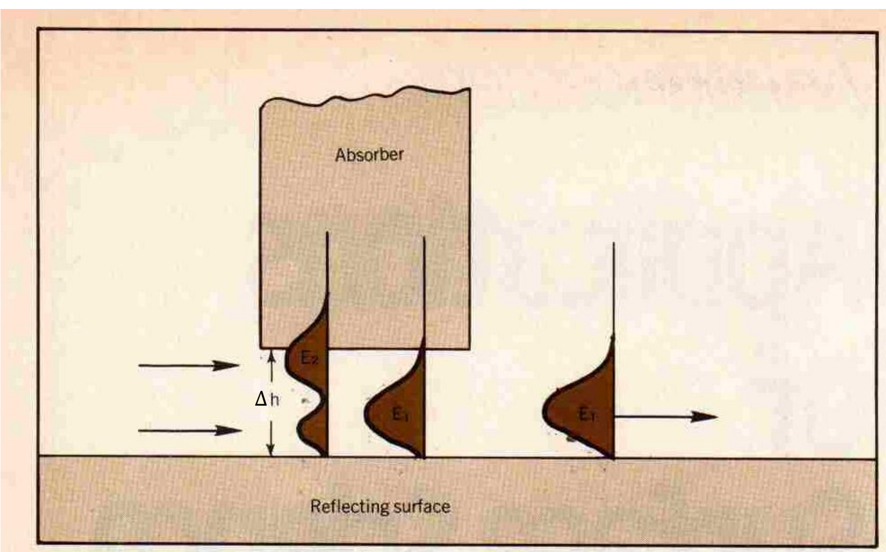
Gravitationally Bound states – The idea

$$-\frac{\hbar^2}{2m_\mathrm{n}}\frac{\partial^2}{\partial z^2}\Psi(z)+V(z)\Psi(z)=E_z\Psi(z)$$

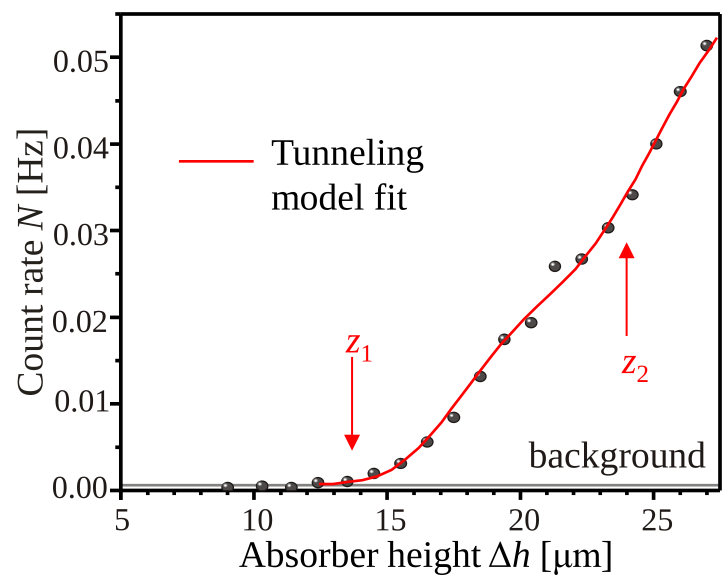
$$\Psi_k(z)\propto \mathrm{Ai}\left(\frac{z-z_k}{l_0}\right);\ z_k=l_0\cdot\lambda_k;\ \ E_{z,k}=m_n g l_0\cdot\lambda_k$$

with $l_0=\sqrt[3]{\hbar^2/2m_n^2 g}=5.87$ µm, and the roots of the Airy function, $\lambda_k=2.34, 4.09, 5.52, 6.79, \ldots$

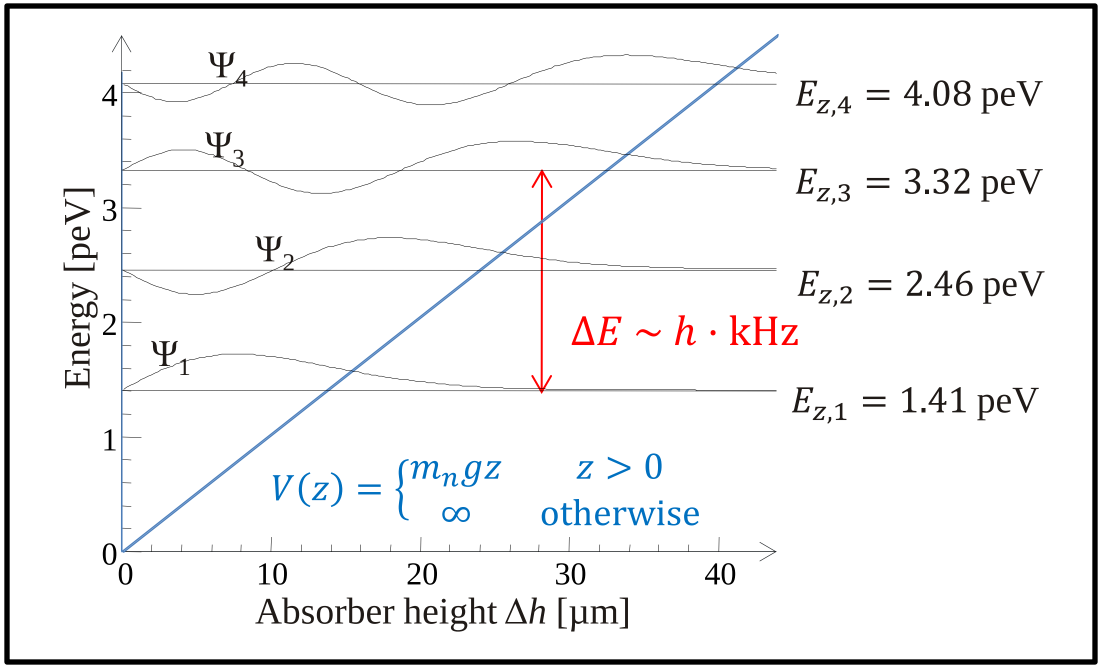




V.I. Lushikov,
Physics Today, June 1977



Development of gravity resonance spectroscopy



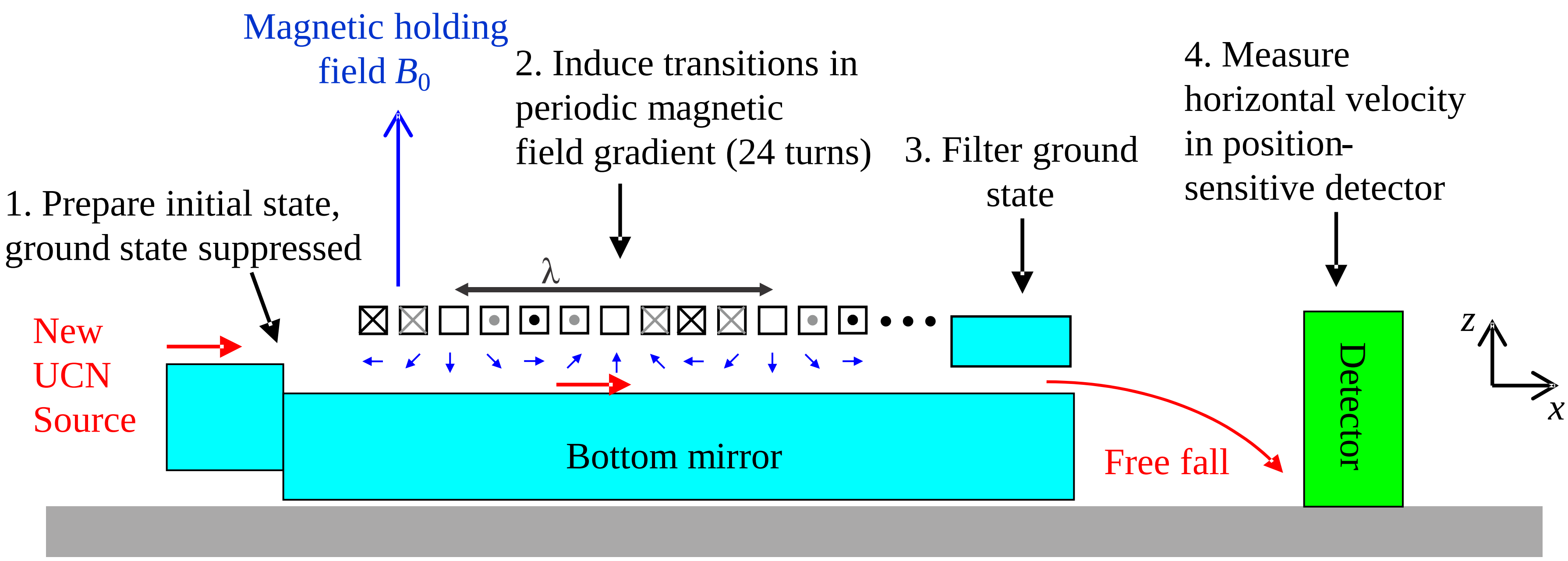
Accuracy of position-type observables: ~ 10%

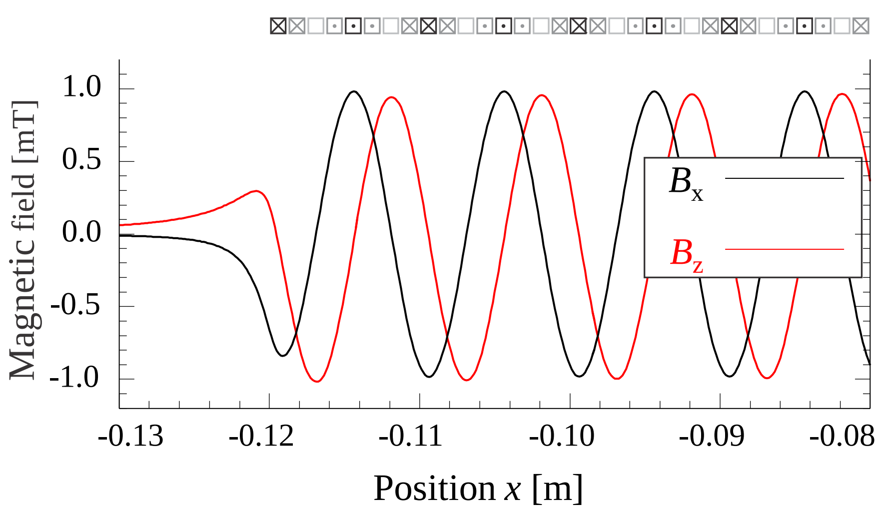
Improvement: do **spectroscopy**

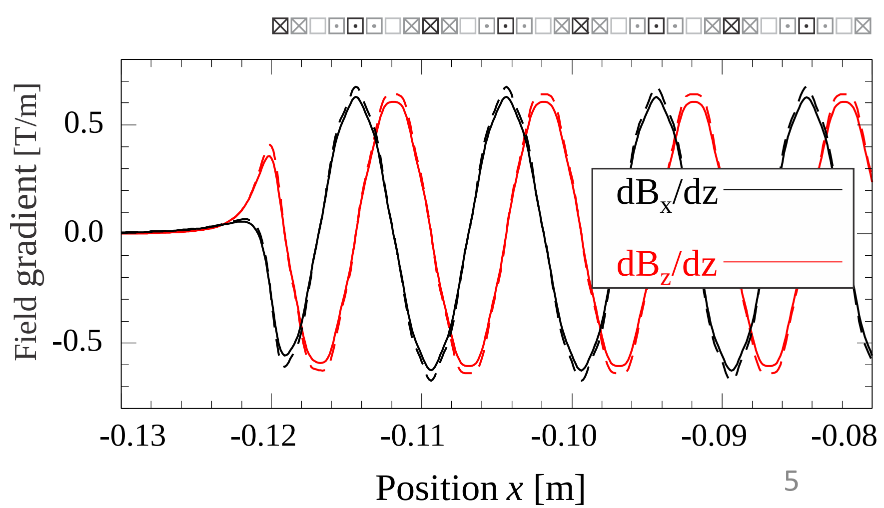
Induce state transitions through:

- **Oscillating magnetic field gradients**
- **Vibrations** (→ QuBounce, H. Abele, T. Jenke et al.)
- Oscillating Masses

GRANIT in Flow-Through mode







Resonance transitions ~ Rabi oscillations

Schroedinger equation:

$$H = \underbrace{\frac{\hbar^2}{2m_\mathrm{n}}\frac{\partial^2}{\partial z^2} + m_\mathrm{n} g z}_{H_0\text{, defines } |N\rangle\text{ , } E_N = \hbar\omega_N} + V(z)\cdot\cos\omega t$$

Ansatz:

$$|\psi(t)\rangle = \sum_{N=1} a_N(t)\mathrm{e}^{-i\omega_N t}|N\rangle \quad \text{with } H_0|N\rangle = \hbar\omega_N|N\rangle$$

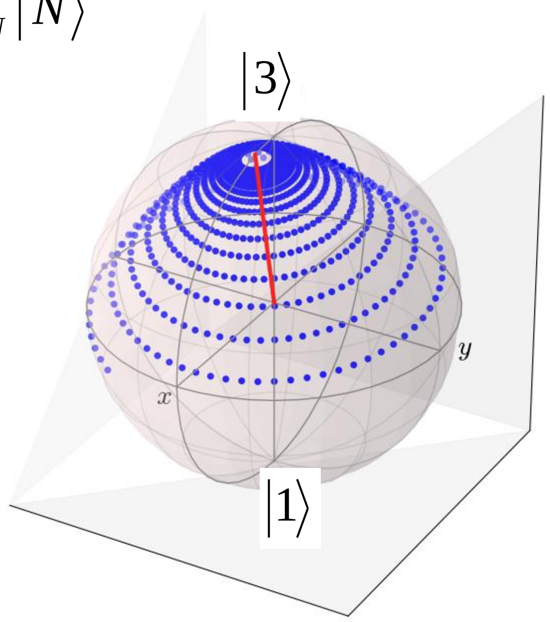
Insert in Schroedinger equation, restrict to states "1" and "3":

$$\hbar\Omega_{13} = \langle 1|V(z)|3\rangle \qquad \omega_{13} = \omega_3 - \omega_1$$

$$i\frac{da_3(t)}{dt} = \Omega_{13}a_1(t)\mathrm{e}^{-i\omega_{13}t}\cos\omega t + \Omega_{33}a_3(t)\cos\omega t$$

$$i\frac{da_1(t)}{dt} = \Omega_{13}a_3(t)\mathrm{e}^{+i\omega_{13}t}\cos\omega t + \Omega_{11}a_1(t)\cos\omega t$$

$$\hbar\Omega_{kk} = \langle k|V(z)|k\rangle$$



Neglect self coupling and fast oscillating terms:

Rabi formula:

$$\boxed{P_{3\to 1}\left(\tau_\mathrm{passage}\right) = \Omega_{13}{}^2\,\frac{\sin^2\left(\sqrt{(\omega-\omega_{13})^2+\Omega_{13}{}^2}\,\frac{\tau_\mathrm{passage}}{2}\right)}{(\omega-\omega_{13})^2+\Omega_{13}{}^2}}$$

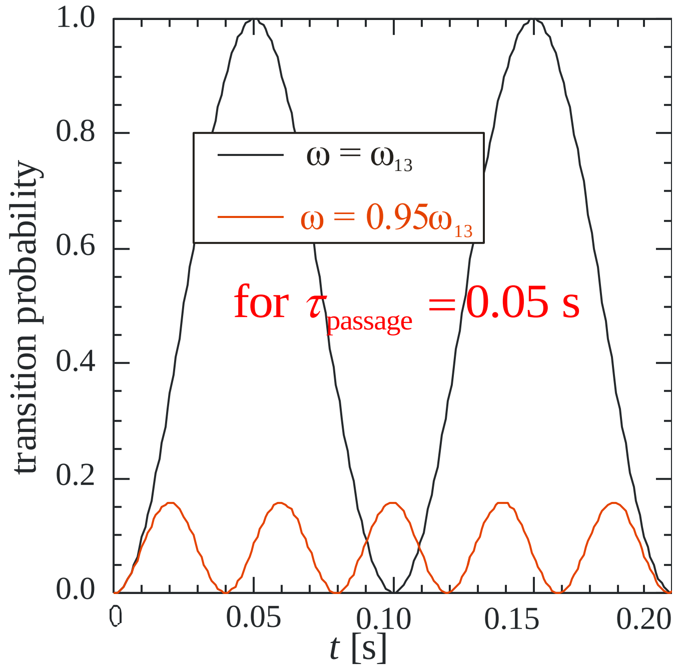
Discussion of Rabi formula

Rabi formula:

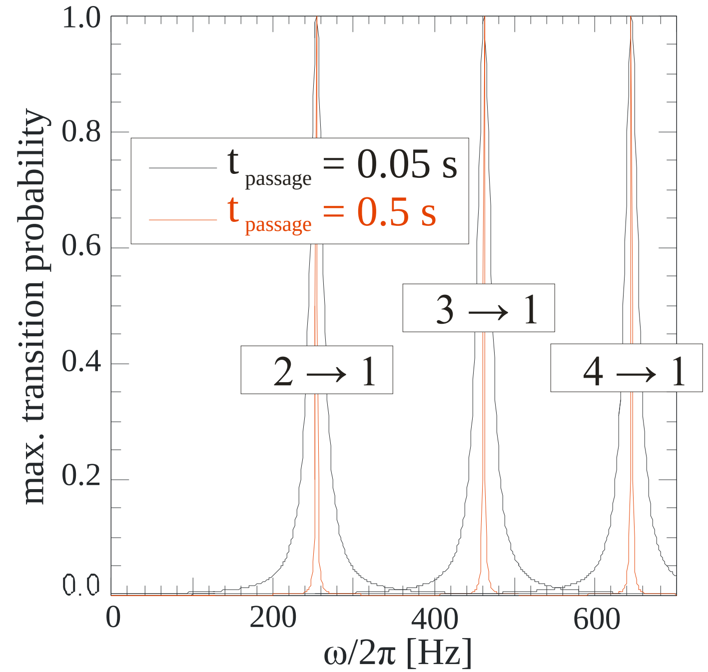
$$P_{3\to1}\left(\tau_{\text{passage}}\right) = \Omega_{13}^{\;2}\,\frac{\sin^2\left(\sqrt{\left(\omega-\omega_{13}\right)^2+\Omega_{13}^{\;2}}\,\frac{\tau_{\text{passage}}}{2}\right)}{\left(\omega-\omega_{13}\right)^2+\Omega_{13}^{\;2}}$$

Transition probability at maximum for $\omega = \omega_{13}$ and $\Omega_{13} \cdot \tau_{\text{passage}} = \pi$

"Rabi oscillations"

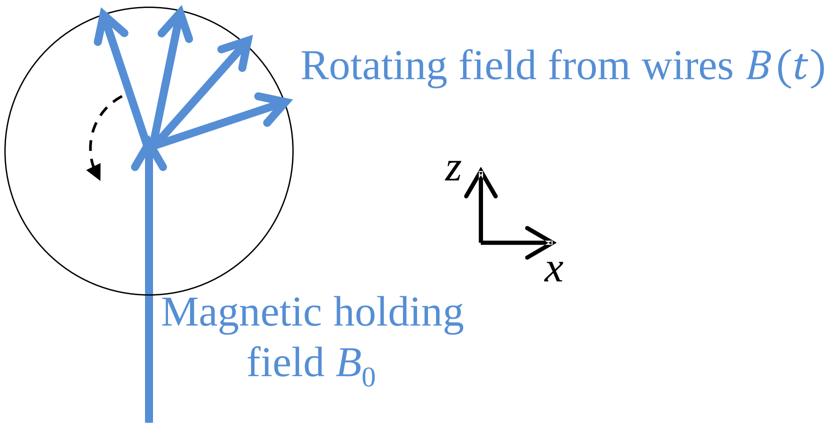


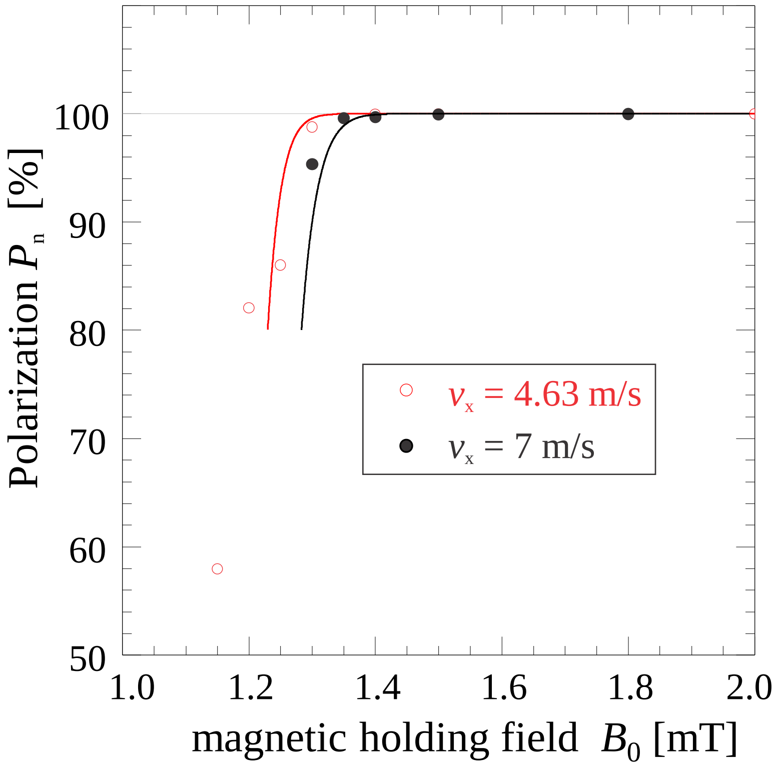
Resonance behavior



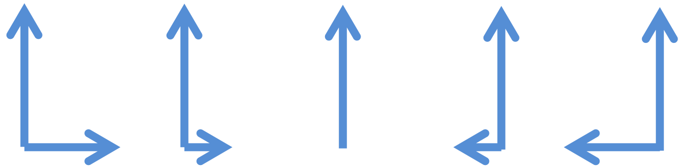
Adiabaticity of Spin Movement

Rotating field + Holding field:
Neutron spin in changing magnetic field



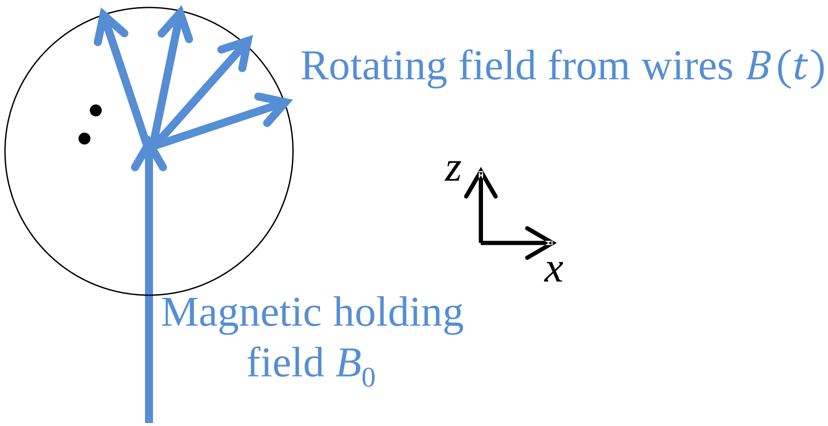


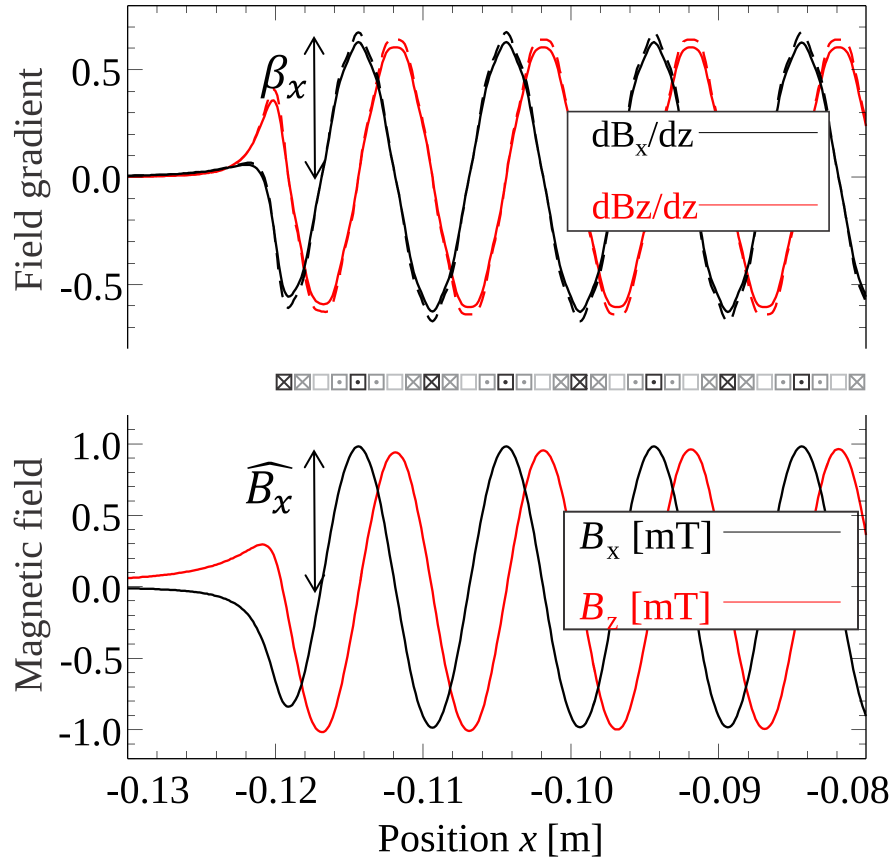
Model: $\boldsymbol{B_z}$ constant, $\boldsymbol{B_x} \propto \boldsymbol{t}$



E. Majorana, Il Nuovo Cimento 9, 43 (1932)

Stern-Gerlach Shift





Rotating field + Holding field:
Neutron spin in non-zero vertical magnetic field gradient.

$$\Longrightarrow \frac{\partial \vec{\mu_n} \cdot \vec{B}}{\partial z} z = \pm |\vec{\mu_n}| \frac{\beta_x \widehat{B_x}}{2B_0} z + \text{higher orders}$$

+oscillating terms

$$H = \frac{\hbar^2}{2m_n} \frac{\partial^2}{\partial z^2} + m_n g z \pm \left| \vec{\mu}_n \right| \frac{\beta_x B_x}{2} z + \ldots + \text{oscillating terms}$$

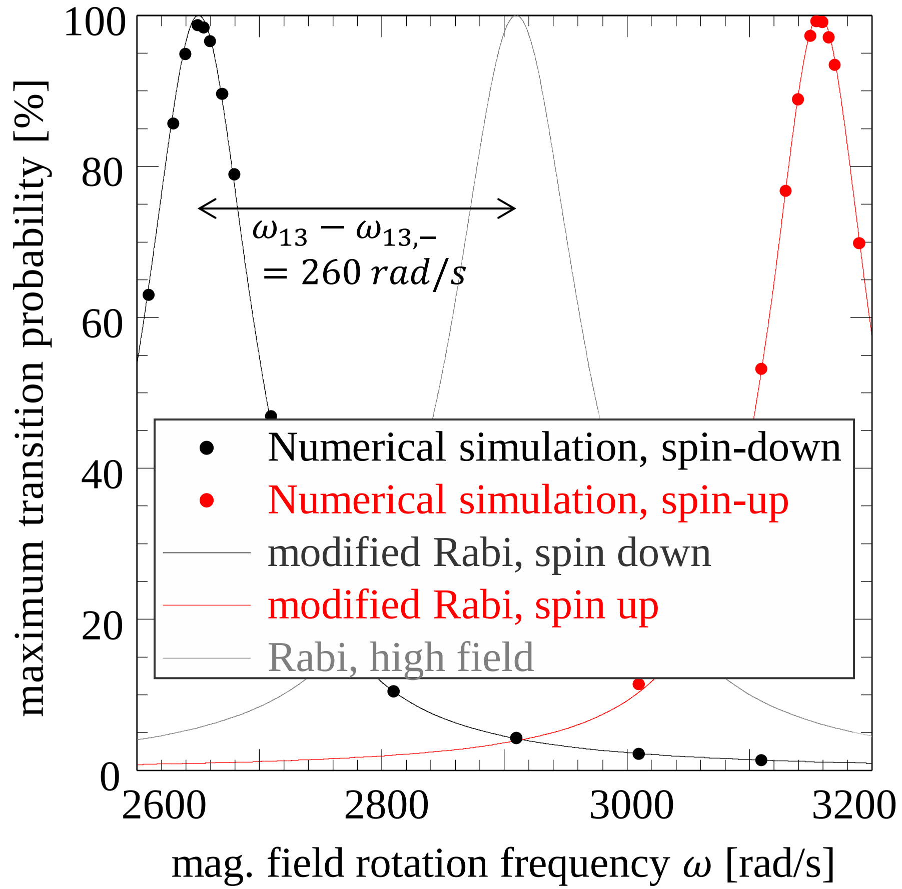
Solutions are similar to before, but wave function and energy levels are shifted through:

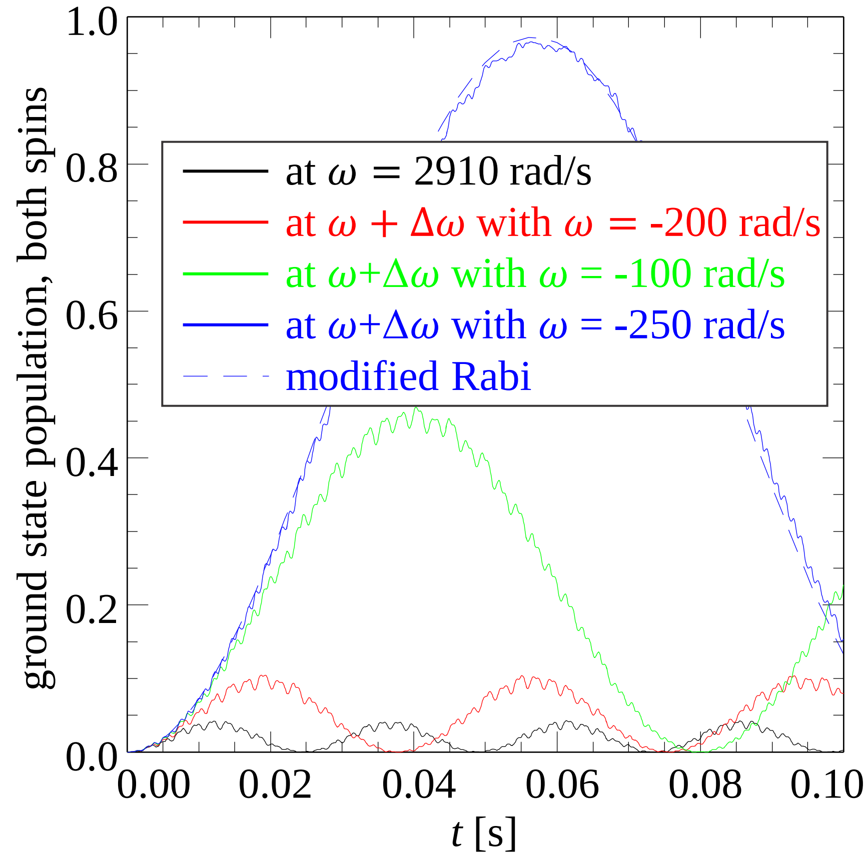
$$g \rightarrow g_{\pm} = g \pm \frac{|\vec{\mu_n}|}{m_n} \left(\frac{\beta_x \widehat{B_x}}{2B_0} + \text{higher orders} \right)$$

Stern-Gerlach shift

$$\omega_{13,\pm} = \omega_{13}\left(1 + \frac{|\overrightarrow{\mu_n}|}{m_n g}\left(\frac{\beta_x \widehat{B_x}}{3B_0} + \text{higher orders}\right)\right)$$

$$\Omega_{13,\pm} = \Omega_{13}(1 + \text{higher orders})$$





Data points: Brute force simulation (24 coupled differential equations)
(in principle, sufficient to model the experiment, but "understanding" helps.

Self-coupling terms

What means: "Neglect self coupling and fast oscillating terms"? Well known is the Bloch-Siegert shift due to the counter-rotating field, but it is tiny here.

$$i\frac{da_3(t)}{dt} = \Omega_{13} a_1(t) e^{-i\omega_{13}t} \cos\omega t + \Omega_{33} a_3(t) \cos\omega t$$

$$i\frac{da_1(t)}{dt} = \Omega_{13} a_3(t) e^{+i\omega_{13}t} \cos\omega t + \underbrace{\Omega_{11} a_1(t) \cos\omega t}$$

Self-coupling; the analogy in a simple spin system would be to provide an oscillating magnetic field along the holding field direction.

Transformation:

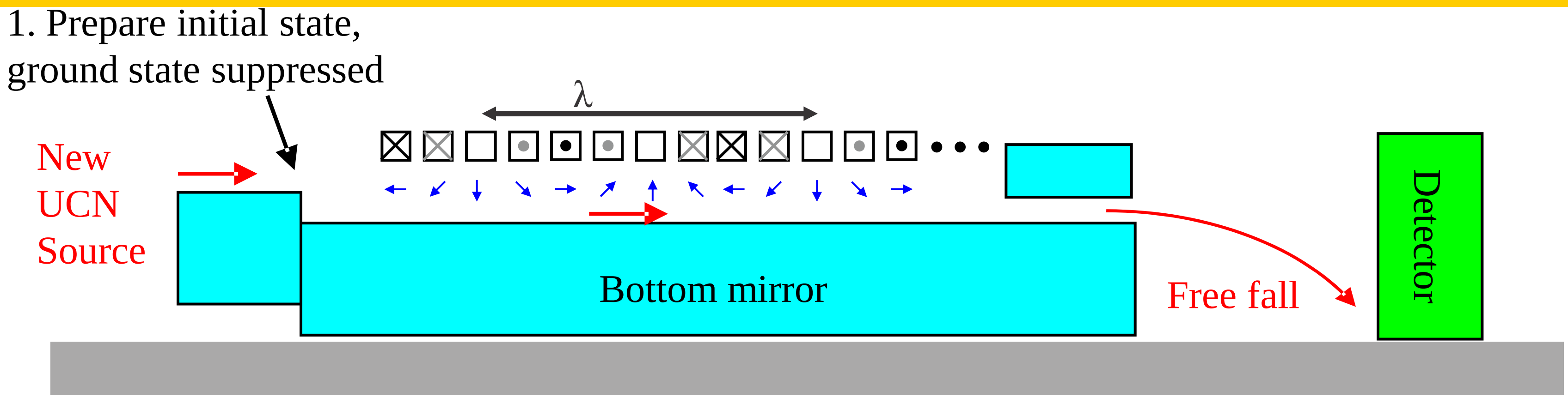
$$a_1 \to a_1 \exp(i\Omega_1 \sin\omega t/\omega)$$
$$a_3 \to a_3 \exp(i\Omega_3 \sin\omega t/\omega)$$

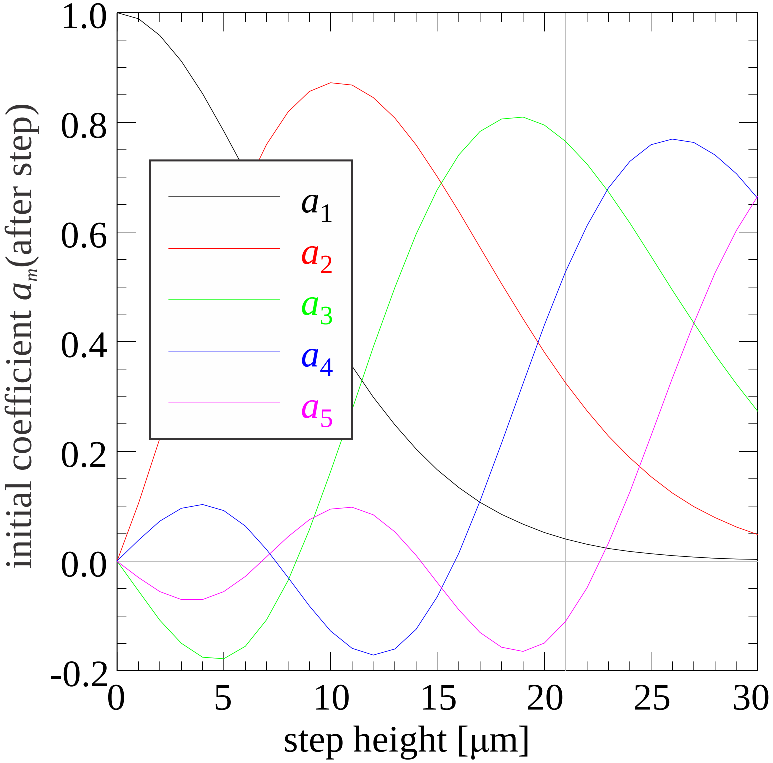
$$i\frac{da_3(t)}{dt} = \Omega_{13} \exp\left(i(\Omega_3 - \Omega_1)\sin\omega t/\omega t\right) a_1(t) e^{-i\omega_{13}t} \cos\omega t$$

$$i\frac{da_1(t)}{dt} = \Omega_{13} \underbrace{\exp\left(i(\Omega_1 - \Omega_3)\sin\omega t/\omega t\right)} a_3(t) e^{+i\omega_{13}t} \cos\omega t$$

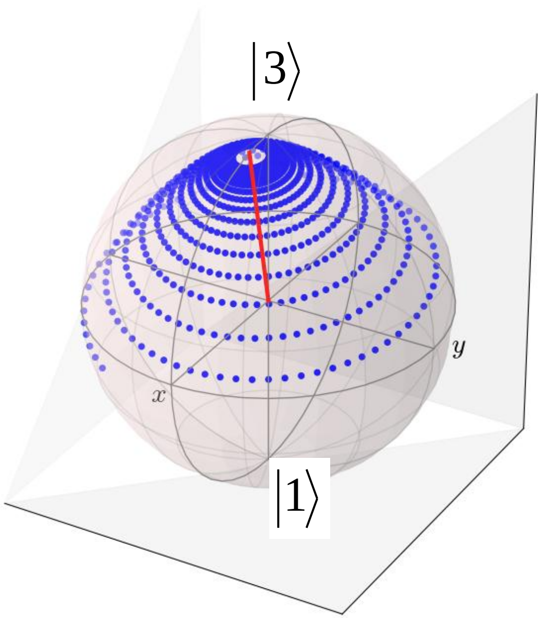
Average value: $1 - \left(\frac{\Omega_3 - \Omega_1}{2\omega}\right)^2$ This correction cannot be observed

Interference shift





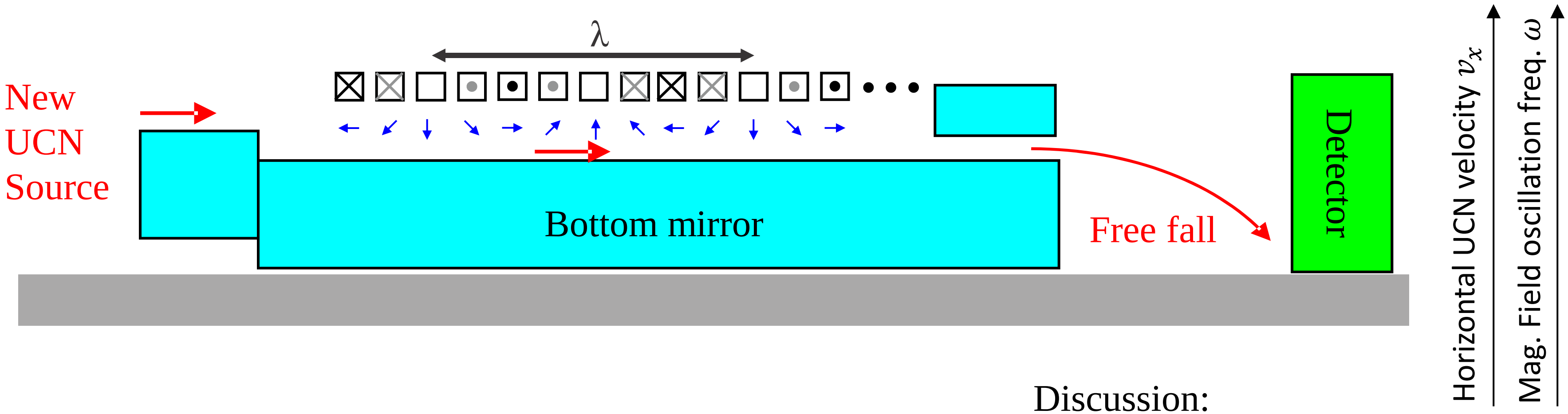
$$\frac{\Delta\omega_{13,\pm}}{\omega_{13,\pm}} \propto \frac{a_1(0)\Omega_{13,\pm}}{a_3(0)\omega_{13,\pm}} \cos(\arg(a_1(0) - a_3(0)))$$

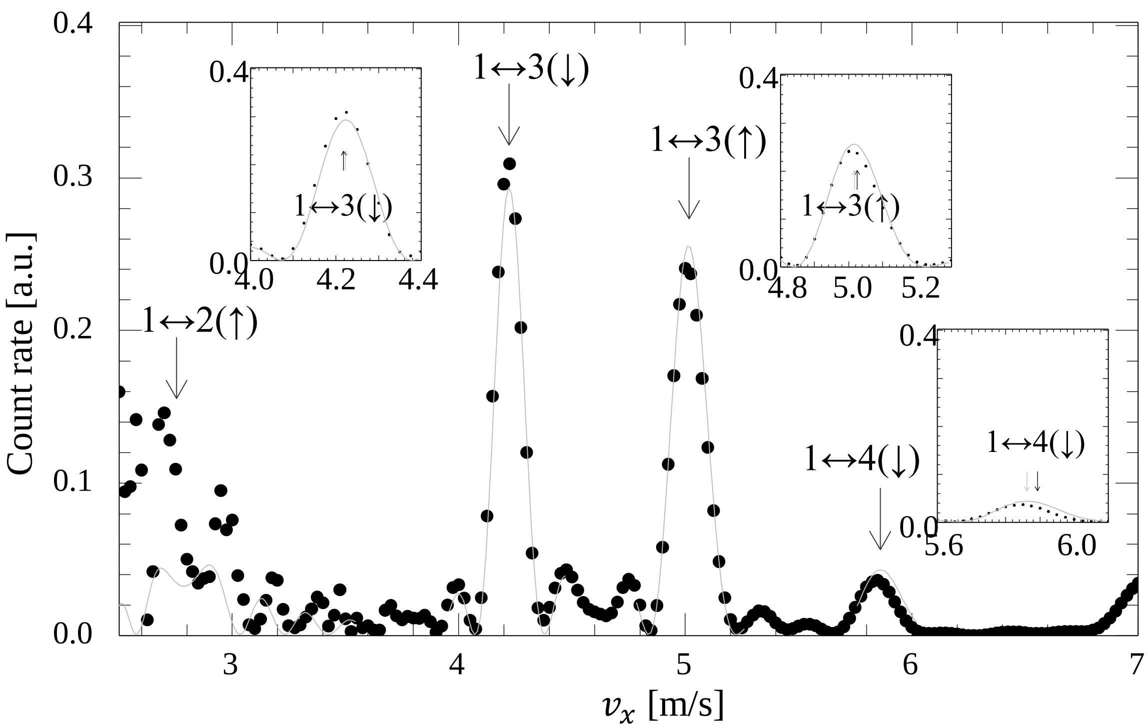


Discussion:

1. Shift is given for full inversion, will be much bigger if transition probability is not large!
2. Phase difference is hard to simulate
3. Interference shift cancels if magnetic field phase is varied.

Expected signal



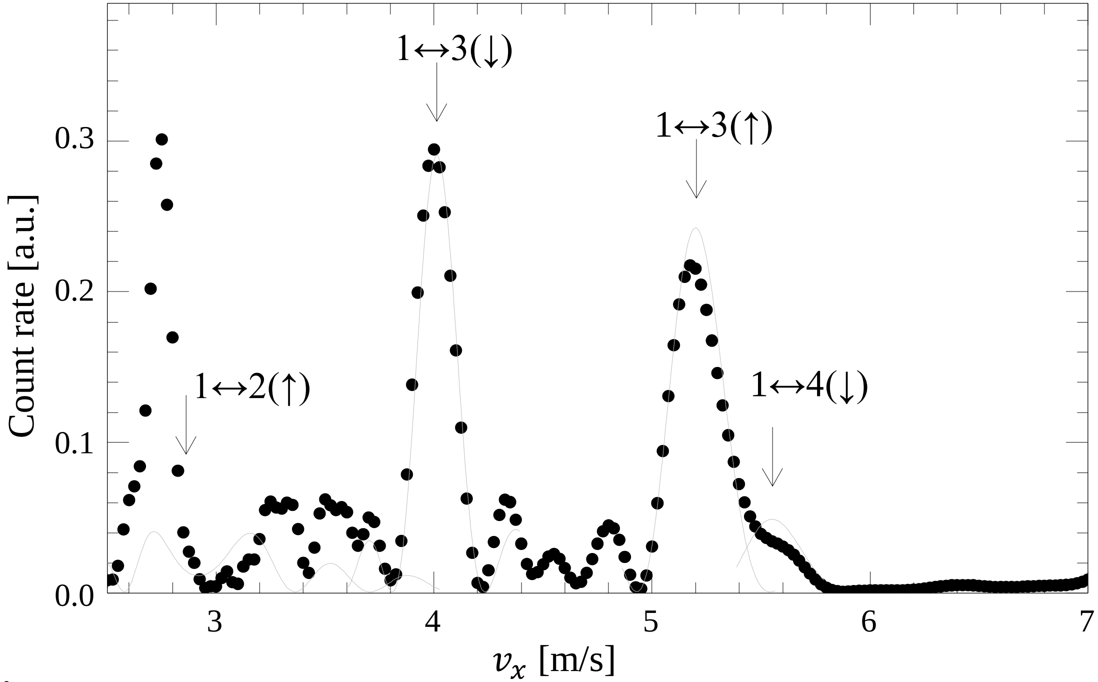


Discussion:
(Show Stern-Gerlach and Interference shift)

(1) For given velocity component, neutrons are polarized.
(2) Average peak position is best measure of transition frequency.
(3) 1↔2 transition not well described by two state theory
(4) 1↔4 transition only visible if large UCN velocities are available (or of periodicity of wire system is changed)
(5) For precise peak determination, structure in background needs to be determined.

Expected signal: Present setup

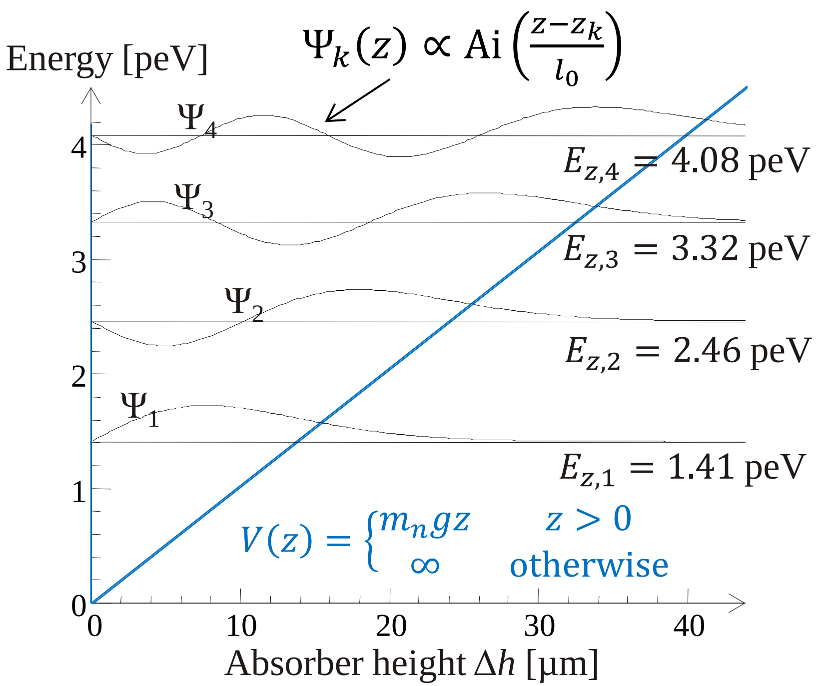
Modification for initial setup: Less wires (128 instead of 192).
Want to increase rotating magnetic field strength by 50% (that is, Ω by 50%) to reach full inversion for $1\leftrightarrow3(\downarrow)$.
Consequence:

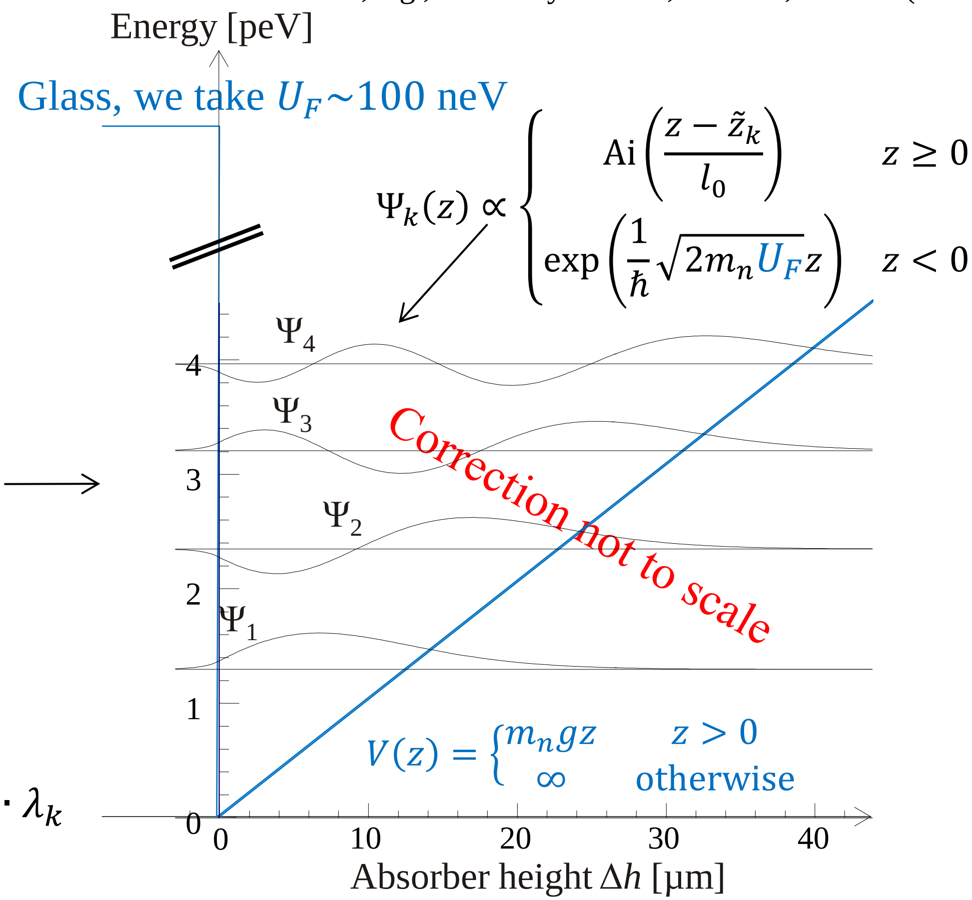


Discussion:
- Stern-Gerlach shift increased by 50%.
- Peak width larger
- We should reduce field a little bit, to avoid overlap between $1\leftrightarrow3(\uparrow)$ and $1\leftrightarrow4(\downarrow)$
- Peaks not reproduced perfectly $\Rightarrow$ another shift?

Fermi-potential of glass

See, e.g., A.E. Meyerovitch, PRA 73, 063616 (2006)





$\Psi_k(z) \propto \mathrm{Ai}\left(\frac{z-z_k}{l_0}\right)$; $z_k = l_0 \cdot \lambda_k$; $E_{z,k} = m_n g l_0 \cdot \lambda_k$

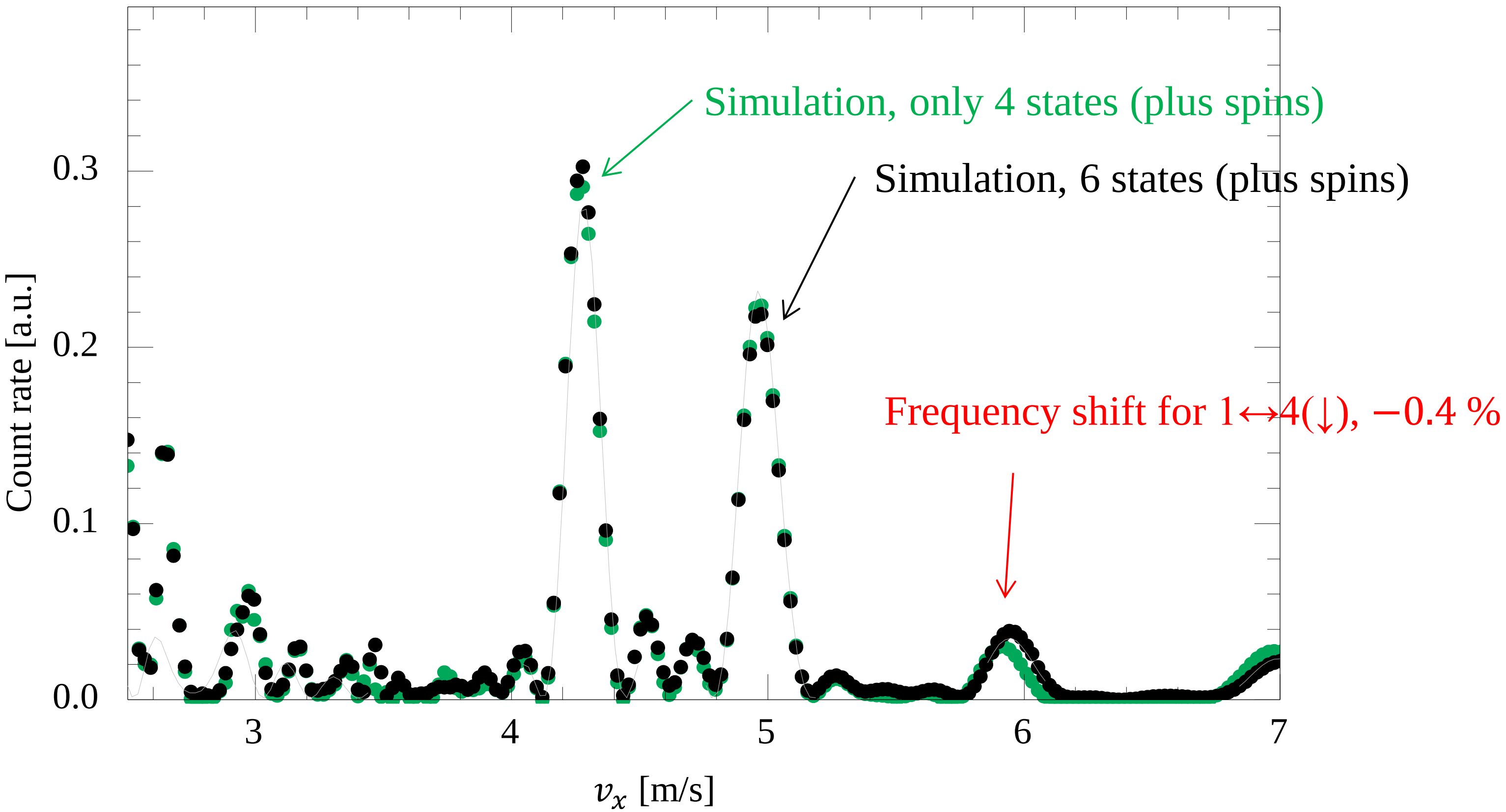
with $\lambda_k = 2.34, 4.09, 5.52, 6.79, \ldots$

$$\longrightarrow \quad \tilde{z}_k = l_0 \cdot \tilde{\lambda}_k;\ E_{z,k} = m_n g l_0 \cdot \tilde{\lambda}_k;\ \tilde{\lambda}_k = \lambda_k - 1\Big/\sqrt{\frac{U_F}{m_n g l_0} - \lambda_k}$$

$\longrightarrow$ Energies and positions get corrections of about 0.1%

$\longrightarrow$ Energy differences experience only a tiny correction (10^{-8}).

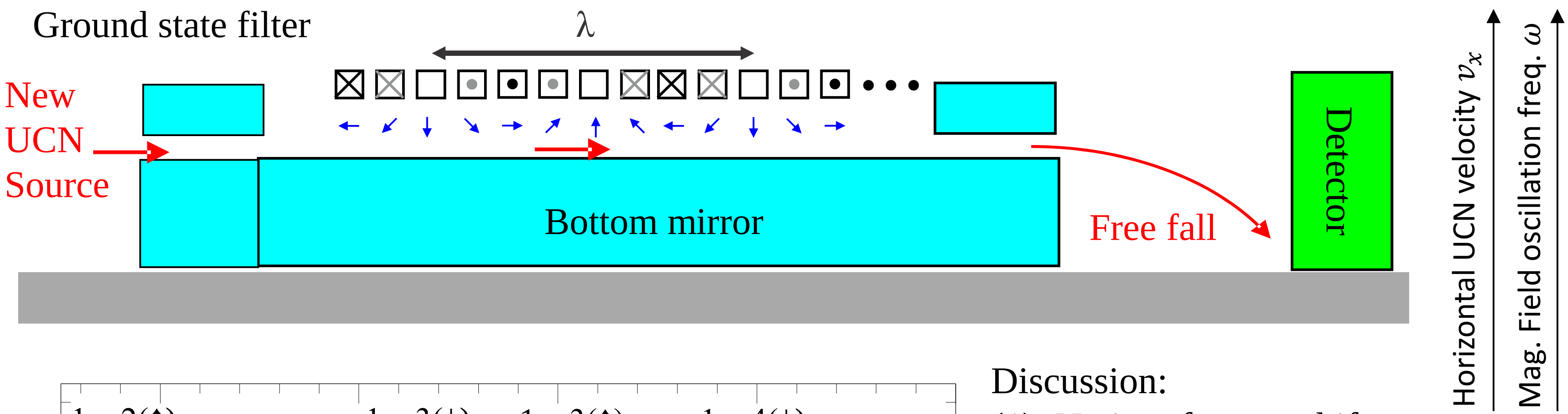
Spectator state shift

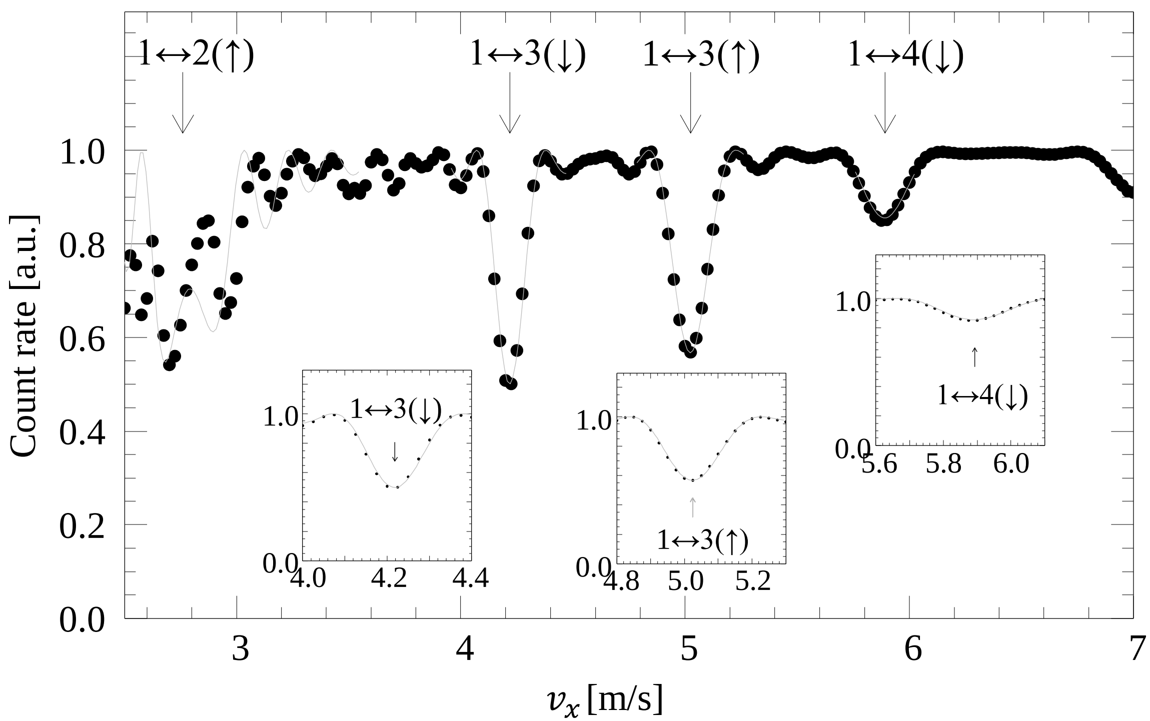


Discussion:

- Frequency shift due to spectator states (states other than the two of the transitions)
- Effect is general property of n-niveau system for n>2
- Can describe shift of 1↔4
- Cannot describe why 1↔3is better (State 2 and 4 make effect that cancels)
- Need to be resolved!

Possible variation: Transmission measurement





Discussion:

(1) No interference shift

(2) Not foreseen in present GRANIT setup

(3) Average peak position is best measure of transition frequency.

(4) Still, 1↔2 transition not well described by this theory

(5) 1↔4 transition only visible if large UCN velocities are available (or of periodicity of wire system is changed)

(6) For precise peak determination, structure in background needs to be determined.

Summary

- About 10%-characterization of gravitationally bound quantum states of UCN achieved previously
- GRANIT will attempt substantially higher precision already in flow through-mode (<< 1%)
- This talk reports on our prediction of several frequency shifts, the most important being the Stern-Gerlach-Shift that is specific to this setup, and is of the order of 10%
- Other shifts: Interference shift, spectator state-shift need to be taken into account.