
Neutron optical Measurements studying Quantum Mechanics

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GRANIT-2014 Workshop



04.03.2014

FWF

- **Part I: Violation Error-Disturbance Uncertainty Relation**
 - Theory: Ozawa's universally valid Error-Disturbance Uncertainty Relation (UVUR)
 - Violation of Heisenberg's Error-Disturbance Uncertainty Relation & Validity of Ozawa's UVUR in Neutron Spin Measurements

- **Part I: Violation Error-Disturbance Uncertainty Relation**
 - Theory: Ozawa's universally valid Error-Disturbance Uncertainty Relation (UVUR)
 - Violation of Heisenberg's Error-Disturbance Uncertainty Relation & Validity of Ozawa's UVUR in Neutron Spin Measurements
- **Part II: Hunting for the *Quantum Cheshire Cat***
 - New Quantum Paradox & Neutron Interferometric Observation
- **Summary**

Heisenberg's Uncertainty Principle

Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik.

Von **W. Heisenberg** in Kopenhagen.

Mit 2 Abbildungen. (Eingegangen am 23. März 1927.)



läuterung der Relation $p q - q p = \frac{h}{2\pi i}$. Sei q_1 die Genauigkeit, mit der der Wert q bekannt ist (q_1 ist etwa der mittlere Fehler von q), also hier die Wellenlänge des Lichtes, p_1 die Genauigkeit, mit der der Wert p bestimmbar ist, also hier die unstetige Änderung von p beim Comptoneffekt, so stehen nach elementaren Formeln des Comptoneffekts p_1 und q_1 in der Beziehung

$$p_1 q_1 \sim h. \quad (1)$$

p_1 : discontinuous change of momentum (*disturbance*)

q_1 : *mean error of position measurement*

- In 1927 Heisenberg proposed a reciprocal relation for measurement *mean error* and *disturbance* by the famous γ -ray microscope thought experiment (*Compton effect*)

W. Heisenberg, *Z. Phys*, **43**, 172 (1927).

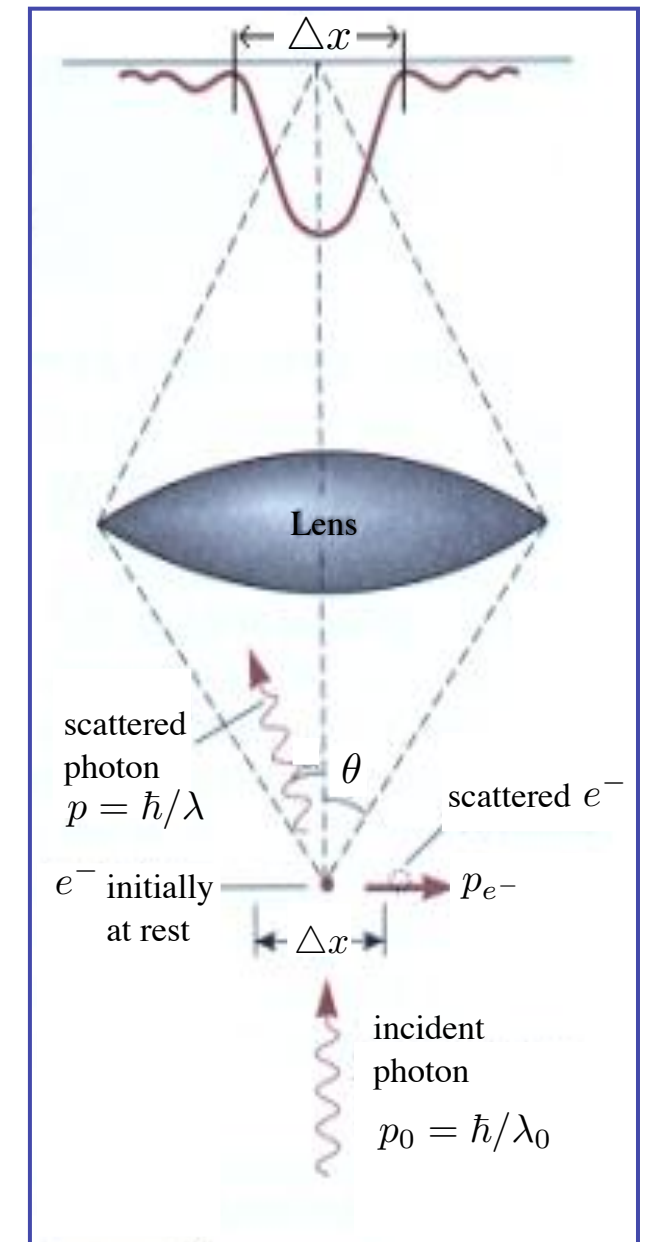
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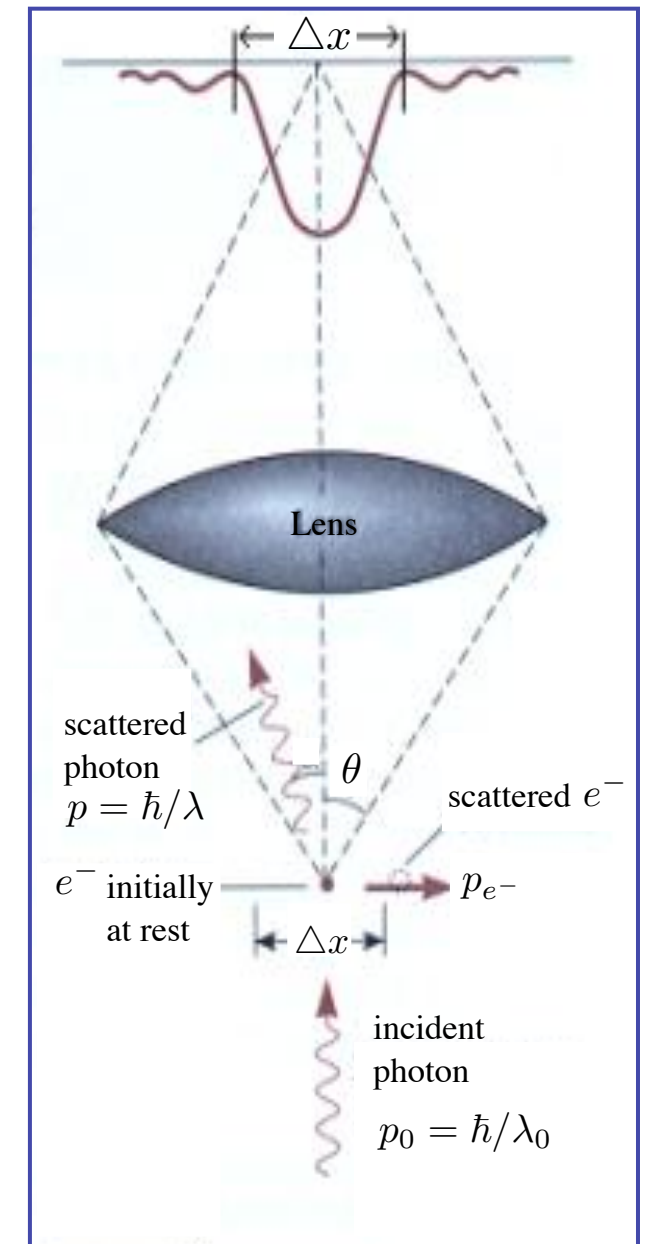
q_1 : mean error of position measurement ($q_1 = \lambda / \sin \theta$)

- more modern treatment for **observables** Q, P

$$\epsilon(Q)\eta(P) \geq \frac{\hbar}{2}$$

ϵ : **error** of first measurement (Q)

η : **disturbance** on a second measurement (P)



W. Heisenberg, *Z. Phys*, **43**, 172 (1927).

Kennard's Relation: From Mean Error to Standard Deviations

- Kennard reformulated Heisenberg's relation as an inequality for the **standard deviations** of position and momentum (**statistical distributions** of not a joint but a **single** measurement either Q or P)

$$\sigma(Q)\sigma(P) \geq \frac{\hbar}{2}$$

standard deviations:

$$\sigma(X) = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$$

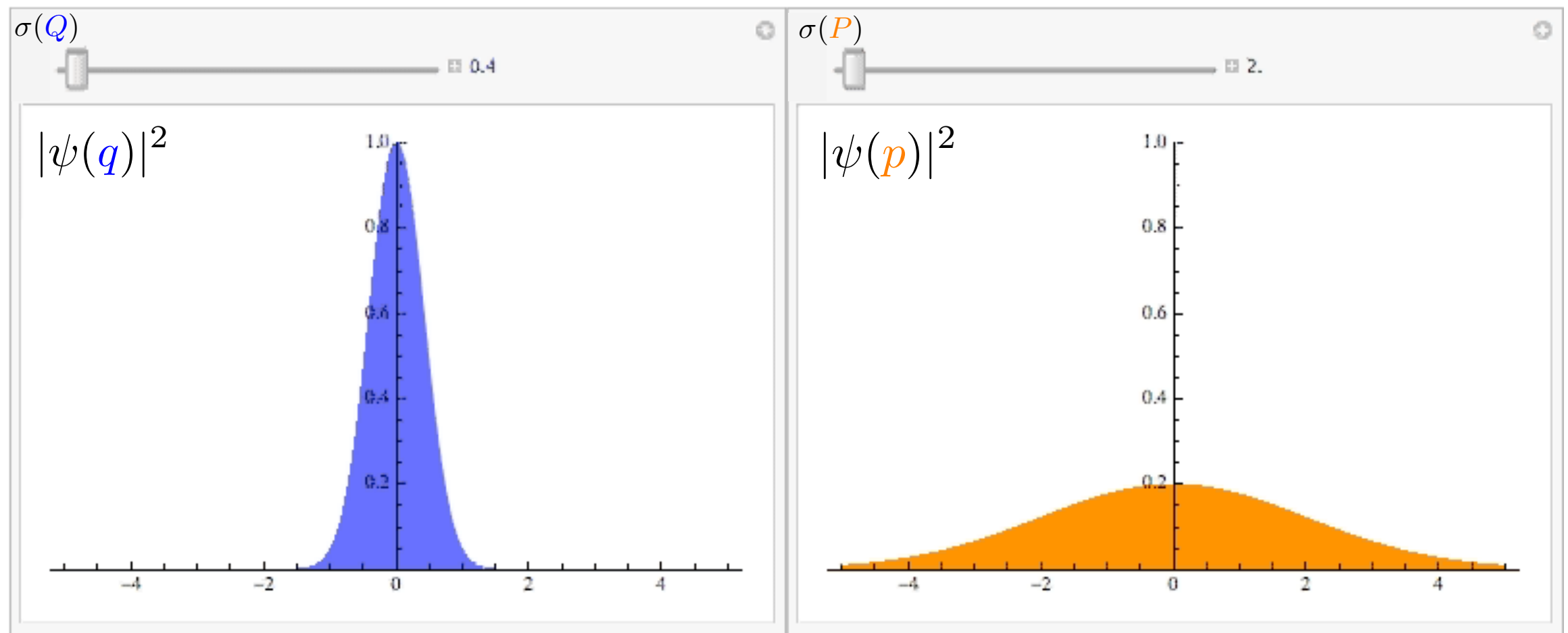
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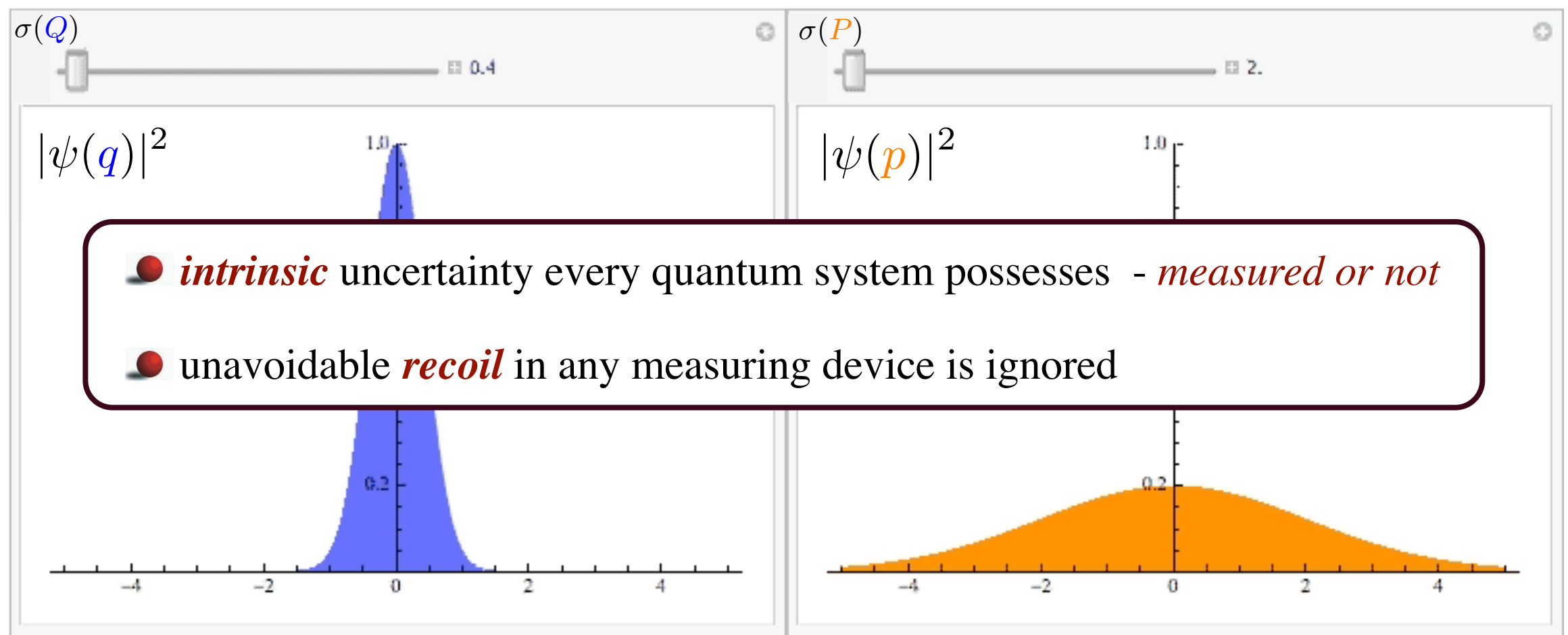
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Robertson's Relation: From Q, P to arbitrary Observables

- Robertson generalized Kennard's relation between standard deviations to arbitrary pairs of observables A and B

$$\sigma(A)\sigma(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

where $[A, B] = AB - BA$ is the commutator between A and B

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- corresponding generalized Heisenberg's Error Disturbance Uncertainty Relation (**EDUR**) ?

$$\epsilon(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

- validity of Heisenberg's generalized EDUR ? certain measurements do **NOT** obey the EDUR - proven to be formally incorrect

Ozawa's Universally Valid Uncertainty Relation (**UVUR**)

PHYSICAL REVIEW A **67**, 042105 (2003)

Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement



Masanao Ozawa

Graduate School of Information Sciences, Tôhoku University, Aoba-ku, Sendai, 980-8579, Japan

(Received 9 October 2002; published 11 April 2003)

The Heisenberg uncertainty principle states that the product of the noise in a position measurement and the momentum disturbance caused by that measurement should be no less than the limit set by Planck's constant $\hbar/2$ as demonstrated by Heisenberg's thought experiment using a γ -ray microscope. Here it is shown that this common assumption is not universally true: a universally valid trade-off relation between the noise and the disturbance has an additional correlation term, which is redundant when the intervention brought by the

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

- rigorous theoretical treatments of quantum measurements:
- **first term:** error of the first measurement, disturbance on the second measurement
- **second and third terms:** crosstalks between spreads of wavefunctions and error/disturbance

Ozawa's Universally Valid Uncertainty Relation (**UVUR**)

composite quantum system: **object** + measurement **probe**

$$\mathcal{H}^{\text{obj}} \otimes \mathcal{H}^{\text{pro}} : U(t)$$

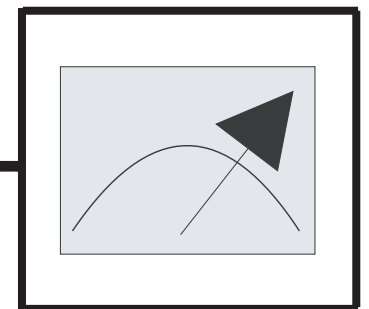
object system: S

$\mathcal{H}^{\text{obj}} : |\psi\rangle, A, B$
(initial state, observables)

probe system: P

$\mathcal{H}^{\text{pro}} : |\xi\rangle, M$
(initial state, meter observable)

A(x)



$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

M. Ozawa, *Phys. Rev. A* **67**, 042105 (2003).

Ozawa's Universally Valid Uncertainty Relation (**UVUR**)

● Measurement defined via *Indirect Measurement Model*

● **Error** is defined as the root-mean-square (rms) of **noise operator**:

$$\epsilon(A) = \left(\langle \xi | \otimes \langle \psi | \underbrace{U^\dagger (I \otimes M) U - A \otimes I}_{N(A) \text{ noise operator}} | \psi \rangle \otimes | \xi \rangle \right)^{1/2}$$

$|\xi\rangle$: initial state of the apparatus in Hilbert space $\mathcal{H}^{\text{app}}$

$|\psi\rangle$ initial state of the object be measured in $\mathcal{H}^{\text{obj}}$

M : meter observable acting on the apparatus's Hilbert space $\mathcal{H}^{\text{app}}$

U : unitary evolution of the composite object-apparatus in Hilbert space $\mathcal{H}^{\text{obj}} \otimes \mathcal{H}^{\text{app}}$

from the time $t = 0$ to $t = \Delta t$

M. Ozawa, *Phys. Rev. A* **67**, 042105 (2003).

Ozawa's Universally Valid Uncertainty Relation (**UVUR**)

● Measurement defined via *Indirect Measurement Model*

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$$\epsilon(A) = \left(\langle \xi | \otimes \langle \psi | \underbrace{U^\dagger (I \otimes M) U - A \otimes I}_{\substack{N(A) \\ \text{noise operator}}} | \psi \rangle \otimes | \xi \rangle \right)^{1/2}$$

state-dependent !

● **Disturbance** is defined in the same manner:

$$\eta(B) = \left(\langle \xi | \otimes \langle \psi | \underbrace{U^\dagger (B \otimes I) U - B \otimes I}_{\substack{D(B) \\ \text{disturbance operator}}} | \psi \rangle \otimes | \xi \rangle \right)^{1/2}$$

$|\xi\rangle$: initial state of the apparatus in Hilbert space $\mathcal{H}^{\text{app}}$

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M. Ozawa, *Phys. Rev. A* **67**, 042105 (2003).

Error and Disturbance for Projection Operators

● Error:

$$\bullet N(A) = U^\dagger (I \otimes M) U - A \otimes I \quad \bullet M = \sum_{\lambda} \lambda |\lambda\rangle \langle \lambda|$$

measurement **operators**: $O_{\lambda} = \langle \lambda | U | \xi \rangle$ acting on obj.-system $\mathcal{H}^{\text{obj}}$ (U on $\mathcal{H}^{\text{obj}} \otimes \mathcal{H}^{\text{pro}}$)

$$\epsilon(A)^2 = \|(O_A - A)|\psi\rangle\|^2$$

output operator: $O_A = \sum_{\lambda} \lambda O_{\lambda}$

$$\|...\| = \|X|\psi\rangle\| = \langle \psi | X^\dagger X | \psi \rangle^{\frac{1}{2}}$$

M. Ozawa, *Ann. Phys.* **311**, 350 (2004)

Error and Disturbance for Projection Operators

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$$\epsilon(A)^2 = ||(O_A - A)|\psi\rangle||^2$$

output operator: $O_A = \sum_{\lambda} \lambda O_{\lambda}$

● Disturbance:

$$\bullet D(B) = U^\dagger (B \otimes I) U - B \otimes I$$

$$\eta(B)^2 = \sum_{\lambda} ||[O_{\lambda}, B]|\psi\rangle||^2$$

both considered experimentally inaccessible !!!

$$||\dots|| = ||X|\psi\rangle|| = \langle \psi | X^\dagger X | \psi \rangle^{\frac{1}{2}}$$

M. Ozawa, *Ann. Phys.* **311**, 350 (2004), R. F. Werner, *Quantum Inf. Comput.* **4**, 546 (2004)

Error and Disturbance for Projection Operators

● Error:

$$\epsilon(A)^2 = \|(O_A - A)|\psi\rangle\|^2$$

output operator: $O_A = \sum_{\lambda} \lambda O_{\lambda}$

operator identity: $O_A A + A O_A = (A + I) O_A (A + I) - A O_A A - O_A$

● Disturbance:

$$\eta(B)^2 = \sum_{\lambda} \|[O_{\lambda}, B]|\psi\rangle\|^2$$

$$\|...\| = \|X|\psi\rangle\| = \langle\psi|X^{\dagger}X|\psi\rangle^{\frac{1}{2}}$$

Error and Disturbance for Projection Operators

● Error:

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output operator: $O_A = \sum_{\lambda} \lambda O_{\lambda}$

different expression for measurement (5 expectation values):

$$\epsilon(A)^2 = \langle\psi|A^2|\psi\rangle + \langle\psi|O_A^2|\psi\rangle + \langle\psi|O_A|\psi\rangle + \langle\psi|AO_AA|\psi\rangle - \langle\psi|(A+I)O_A(A+I)|\psi\rangle$$

with $O_A^2 = \sum_{\lambda} \lambda^2 O_{\lambda}^{\dagger} O_{\lambda}$

● Disturbance:

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with $O_A^2 = \sum_{\lambda} \lambda^2 O_{\lambda}^{\dagger} O_{\lambda}$

● Disturbance:

$$\eta(B)^2 = \sum_{\lambda} \|[O_{\lambda}, B]|\psi\rangle\|^2$$

$$\eta(B)^2 = \langle\psi|B^2|\psi\rangle + \langle\psi|X_B^2|\psi\rangle + \langle\psi|X_B|\psi\rangle + \underbrace{\langle\psi|BX_B B|\psi\rangle}_{\langle\psi'''|X_B|\psi'''\rangle} - \underbrace{\langle\psi|(B+I)X_B(B+I)|\psi\rangle}_{\langle\psi''''|X_B|\psi''''\rangle}$$

with $X_B^2 = \sum_{\lambda} O_{\lambda}^{\dagger} B^2 O_{\lambda}$, and modified output operator: $X_B = \sum_{\lambda} O_{\lambda}^{\dagger} B O_{\lambda}$

remark: another strategy using weak-values

M. Ozawa, *Ann. Phys.* **311**, 350 (2004). A. P. Lund and H. Wiseman, *N.J. Phys.* **12**, 093011 (2010).

“Three-State-Method” for Spin 1/2 Measurements

● **Observables A, B (O_A) are Pauli Spin Matrices:** $\sigma_i^2 = I$, $A = A^\dagger$, $B = B^\dagger$

$$\epsilon(A)^2 = \underbrace{\langle \psi | A^2 | \psi \rangle}_1 + \underbrace{\langle \psi | O_A^2 | \psi \rangle}_1 + \langle \psi | O_A | \psi \rangle + \underbrace{\langle \psi | A O_A A | \psi \rangle}_{\langle \psi' | O_A | \psi' \rangle} - \underbrace{\langle \psi | (A + I) O_A (A + I) | \psi \rangle}_{\langle \psi'' | O_A | \psi'' \rangle}$$

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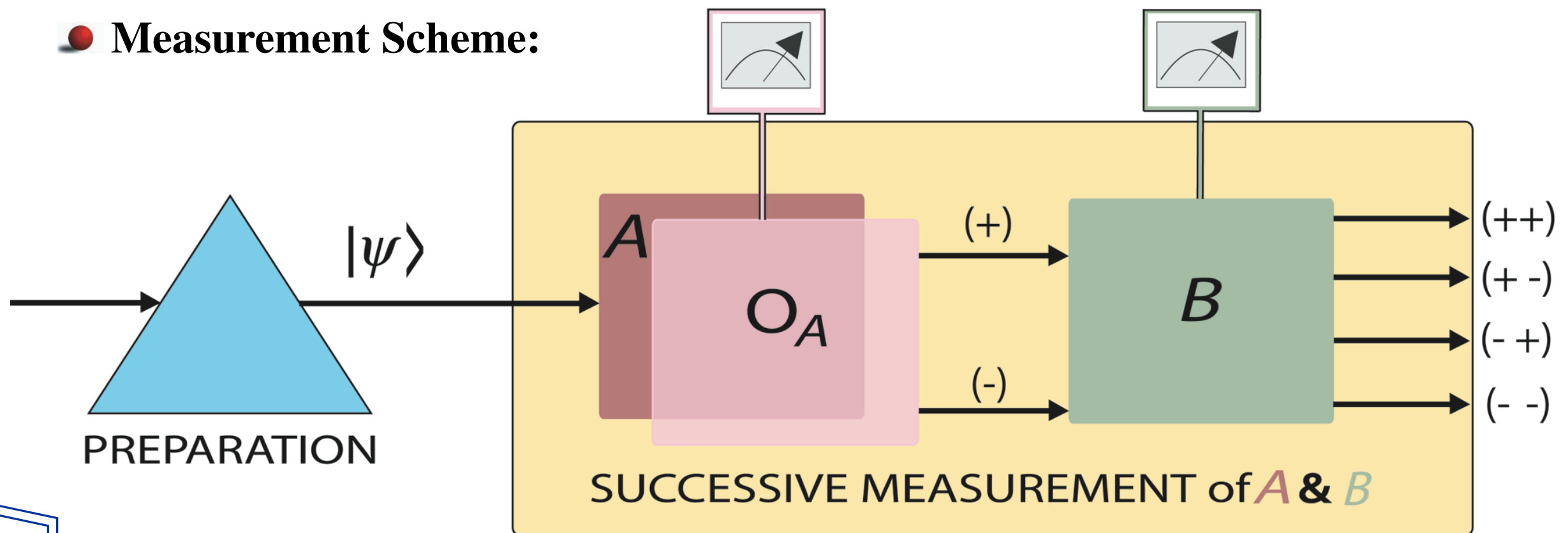
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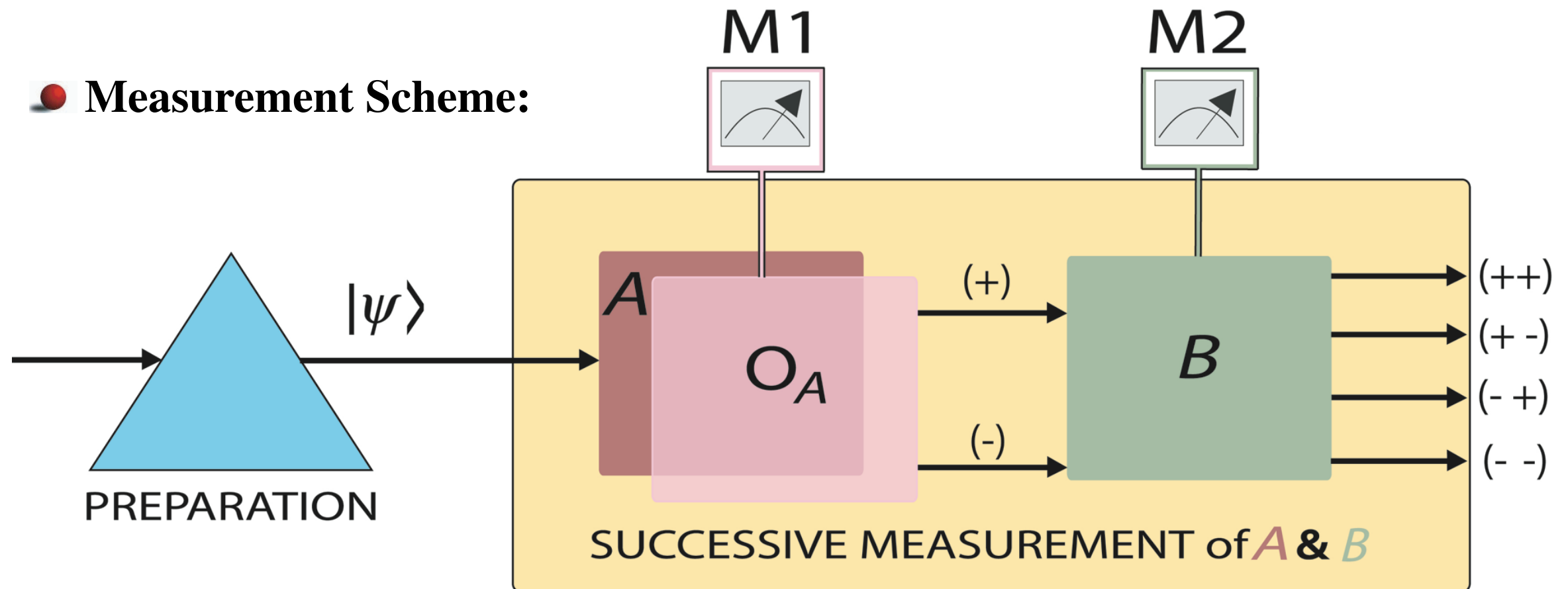
● **Measurement Scheme:**



J. Erhart, S. Sponar, G. Sulyok, G. Badurek, M. Ozawa, Y. Hasegawa, *Nature Phys.* **8**, 185 (2012).

“Three-State-Method” for Spin 1/2 Measurements

Measurement Scheme:



$$\frac{(I_{++} + I_{-+}) - (I_{+-} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi | \overbrace{\sum_{\lambda=+,-} O_{\lambda}^{\dagger} B O_{\lambda}}^{X_B \text{ modified output operator}} | \psi \rangle = \langle \psi | X_B | \psi \rangle \Rightarrow \text{disturbance } \eta(B)$$

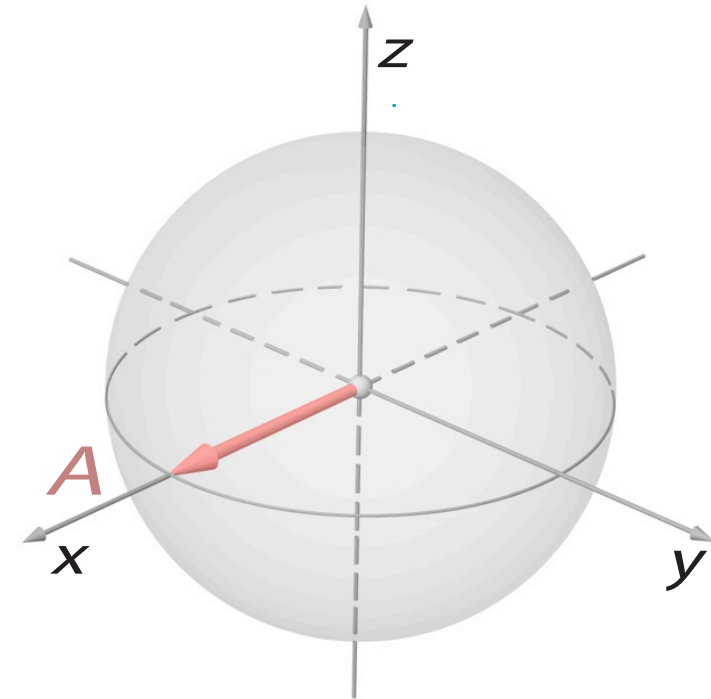
$$\frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} \overset{\text{output operator}}{=} \langle \psi | O_A | \psi \rangle \Rightarrow \text{error } \epsilon(A)$$

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Error and Disturbance for Spin 1/2 Measurements

● set of observables and initial state

$$A = \sigma_x$$

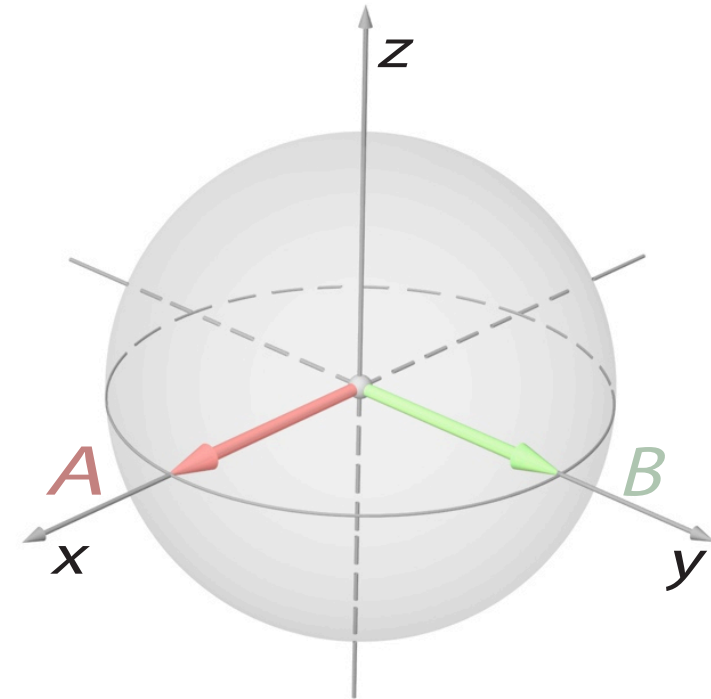


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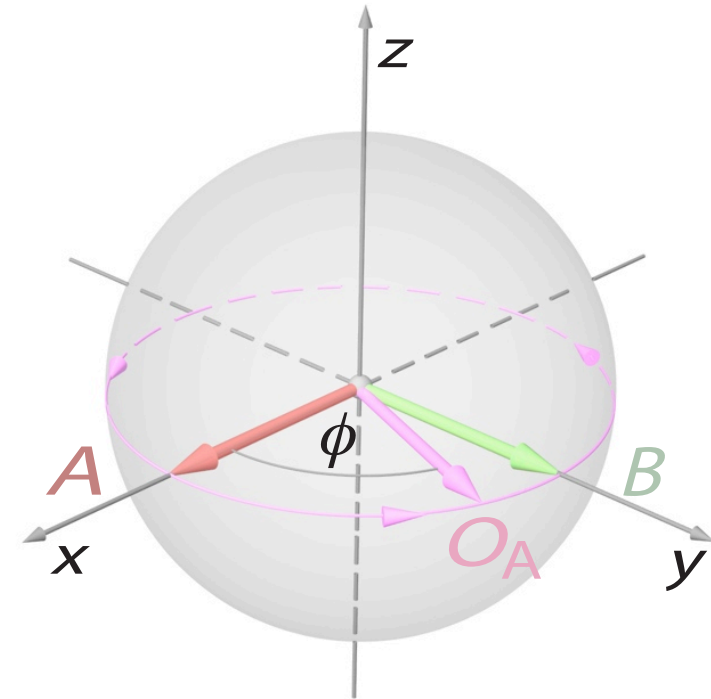
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$$O_A = \sigma_\phi = \sigma_x \sin \phi + \sigma_y \cos \phi$$



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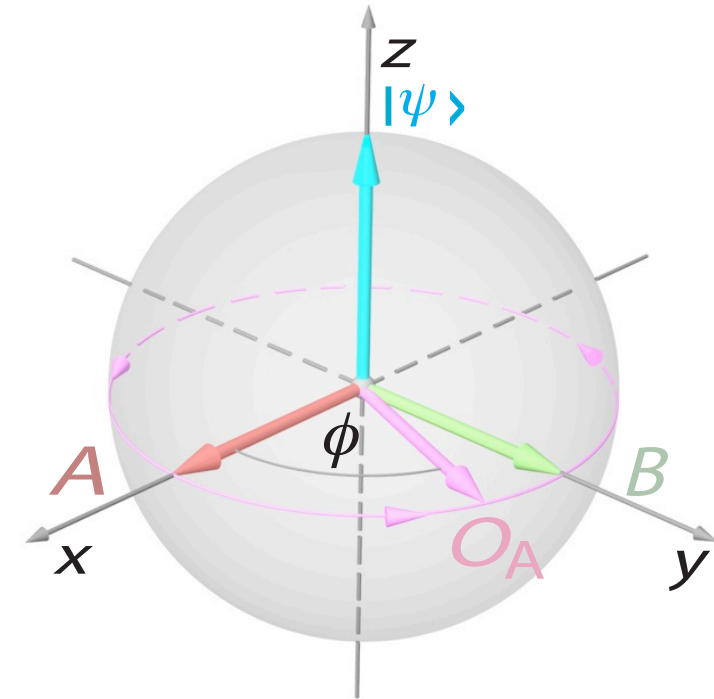
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Error and Disturbance for Spin 1/2 Measurements

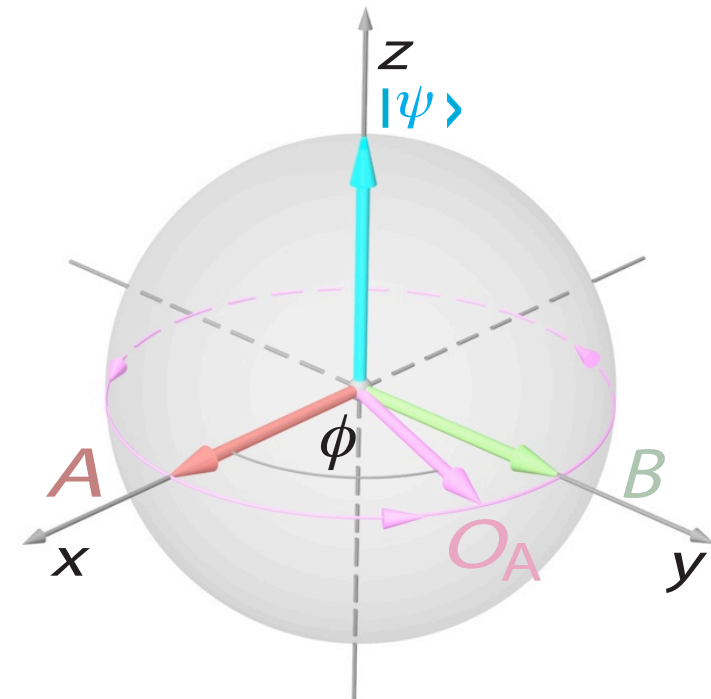
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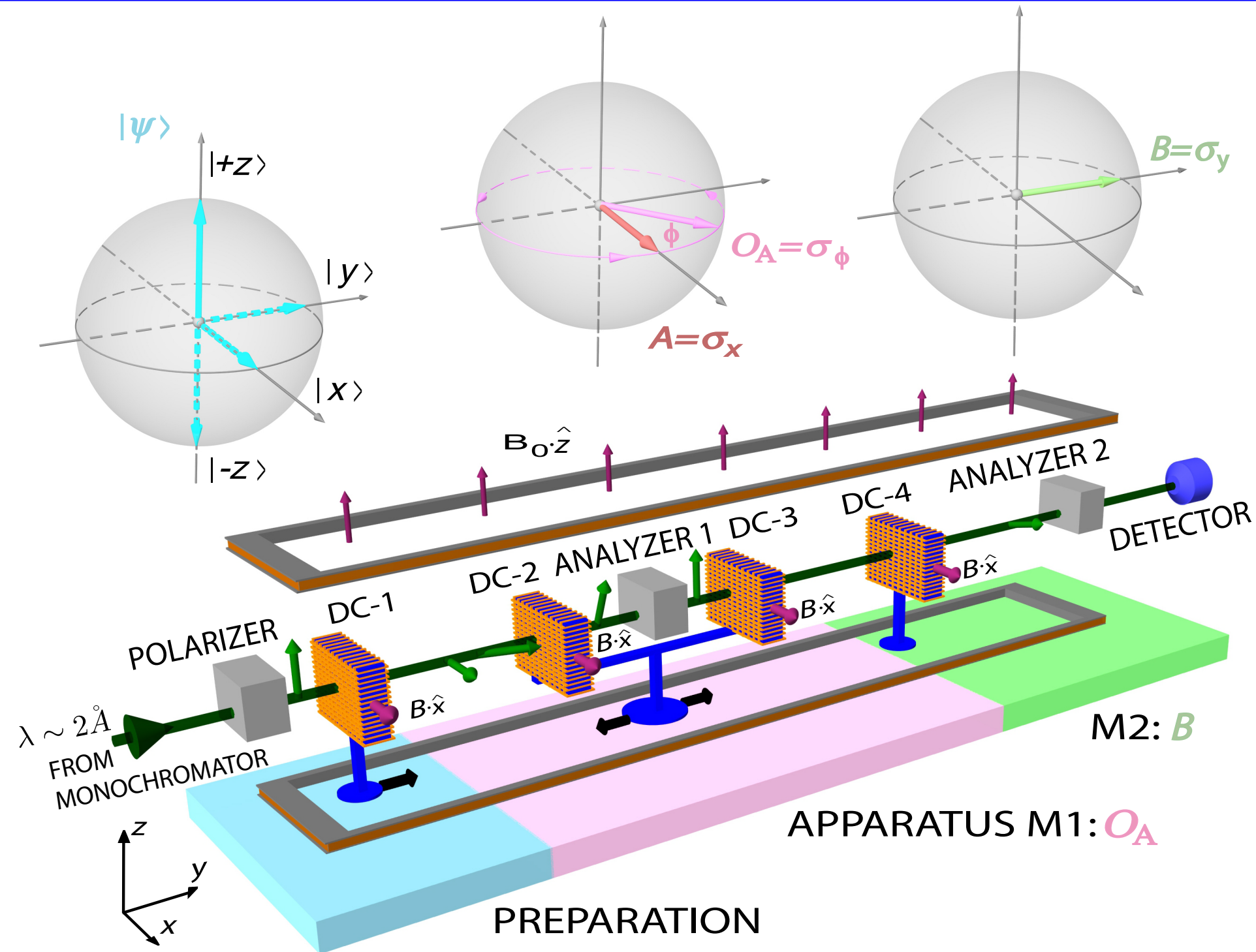
$$|\psi\rangle = | + z \rangle$$



$$\epsilon(A)^2 = \underbrace{\langle +z | \sigma_x^2 | +z \rangle}_1 + \underbrace{\langle +z | \sigma_\phi^2 | +z \rangle}_1 + \underbrace{\langle +z | \sigma_\phi | +z \rangle}_{| - z \rangle} + \underbrace{\langle +z | \sigma_x \sigma_\phi \sigma_x | +z \rangle}_{| x \rangle} - \underbrace{\langle +z | (\sigma_x + I) \sigma_\phi (\sigma_x + I) | +z \rangle}_{| x \rangle}$$

$$\eta(B)^2 = \underbrace{\langle +z | \sigma_y^2 | +z \rangle}_1 + \underbrace{\langle +z | X_B^2 | +z \rangle}_1 + \underbrace{\langle +z | X_B | +z \rangle}_{| - z \rangle} + \underbrace{\langle +z | \sigma_y X_B \sigma_y | +z \rangle}_{| y \rangle} - \underbrace{\langle +z | (\sigma_y + I) X_B (\sigma_y + I) | +z \rangle}_{| y \rangle}$$

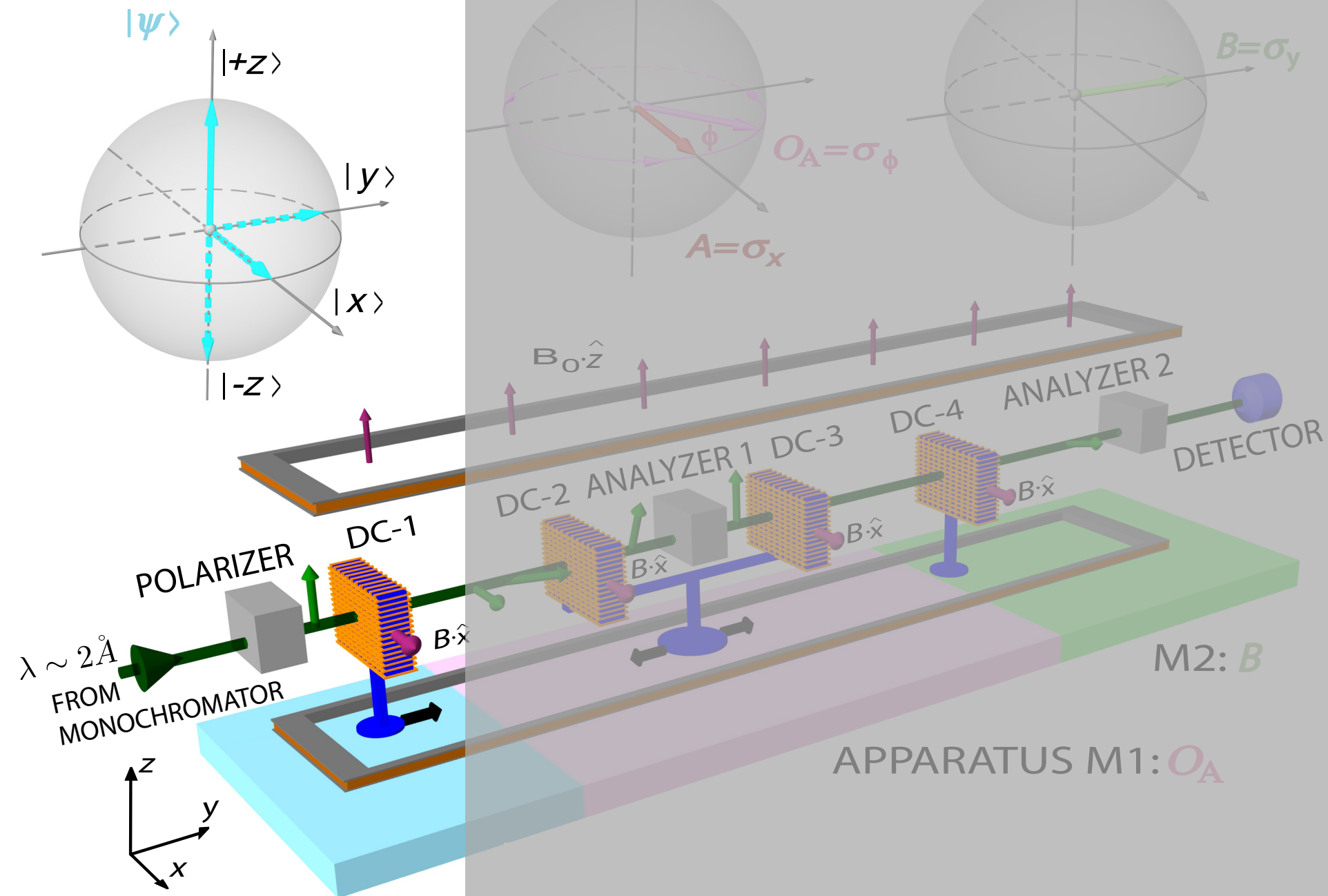
Experimental Setup



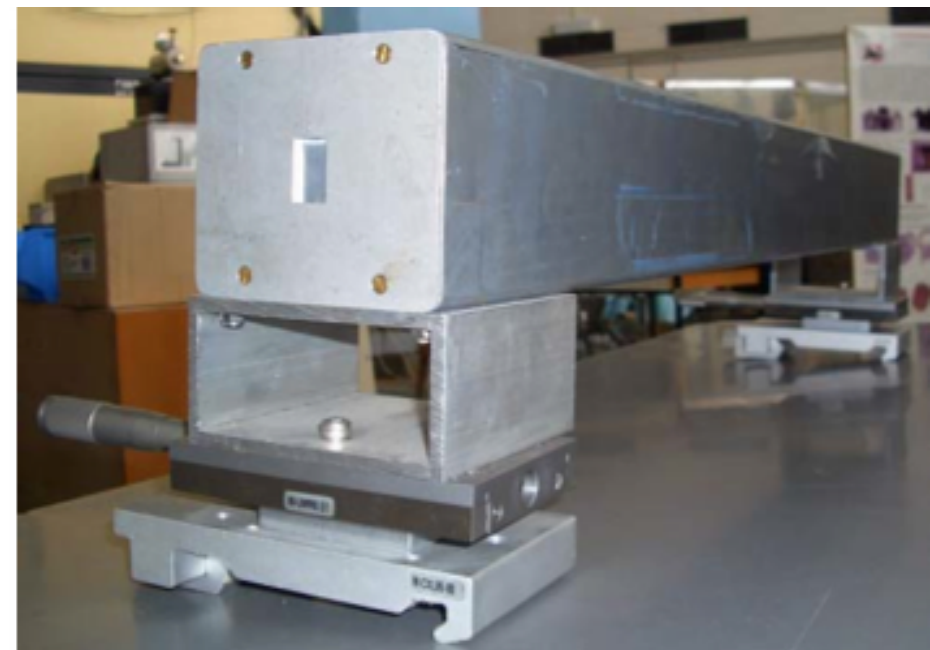
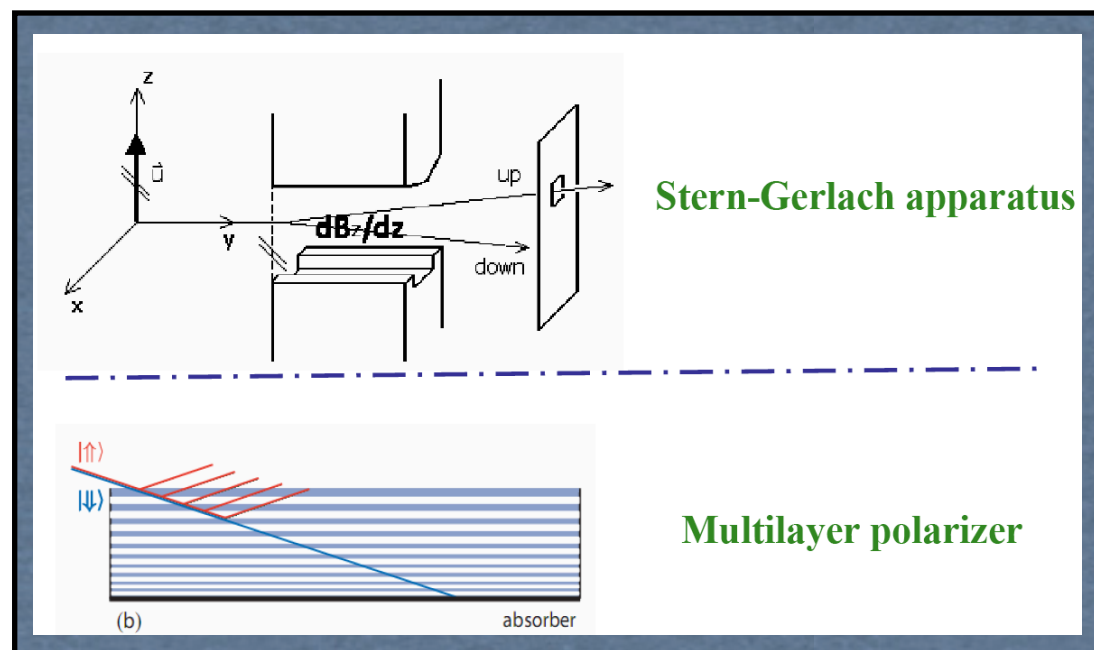
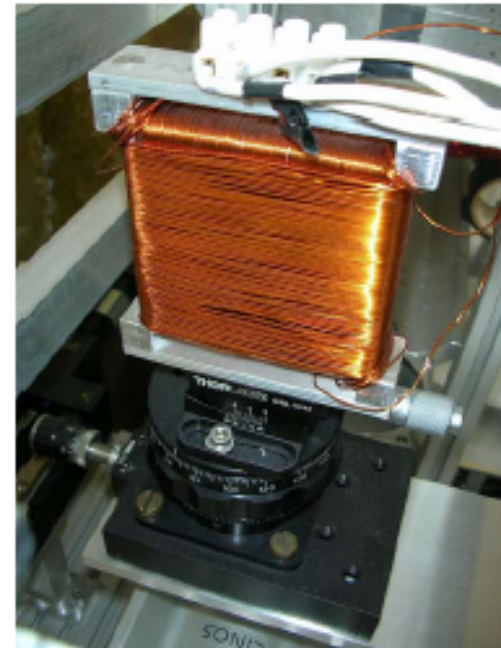
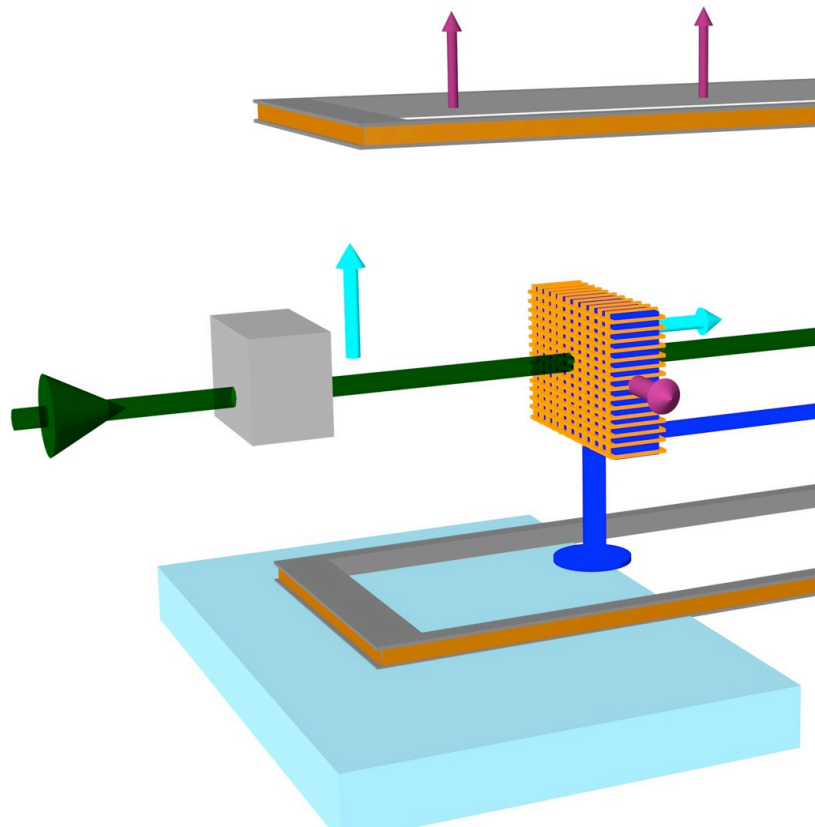
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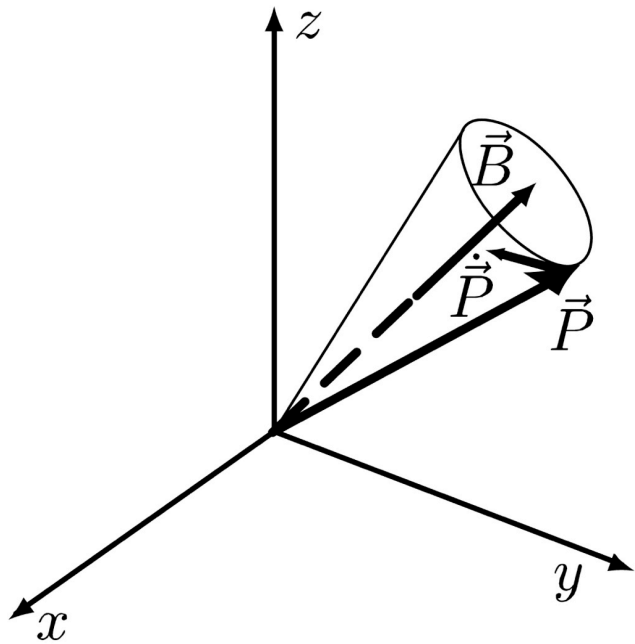
● Preparation:



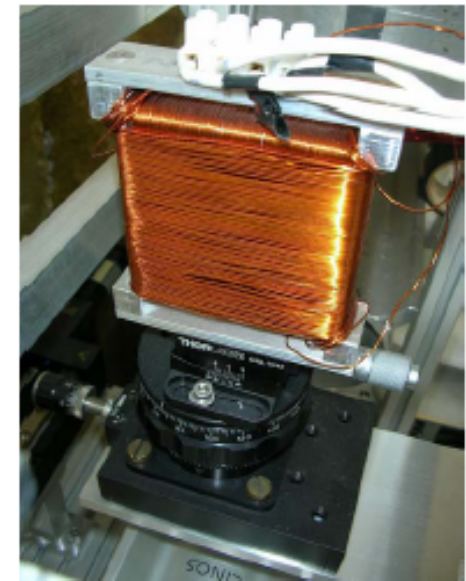
Experimental Setup: State Preparation



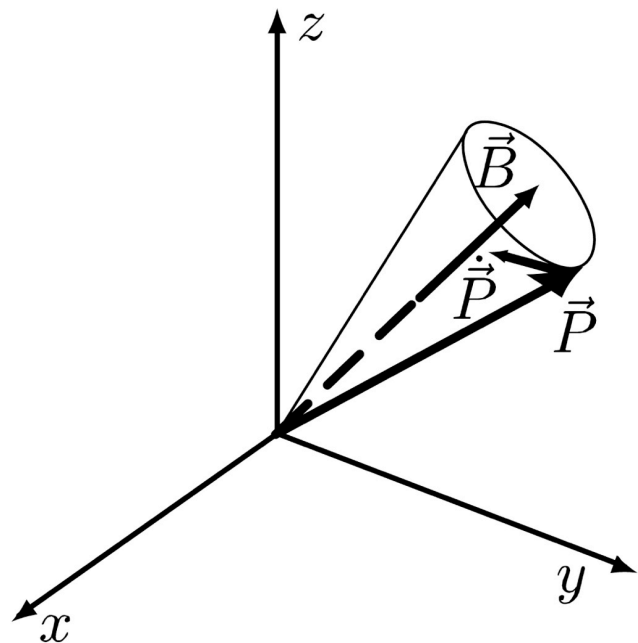
Experimental Setup: State Preparation



$$\dot{\vec{P}}(t) = \vec{P}(t) \times \gamma \vec{B}(t)$$
$$\vec{P}(t) = \langle \psi | \vec{\sigma} | \psi \rangle$$

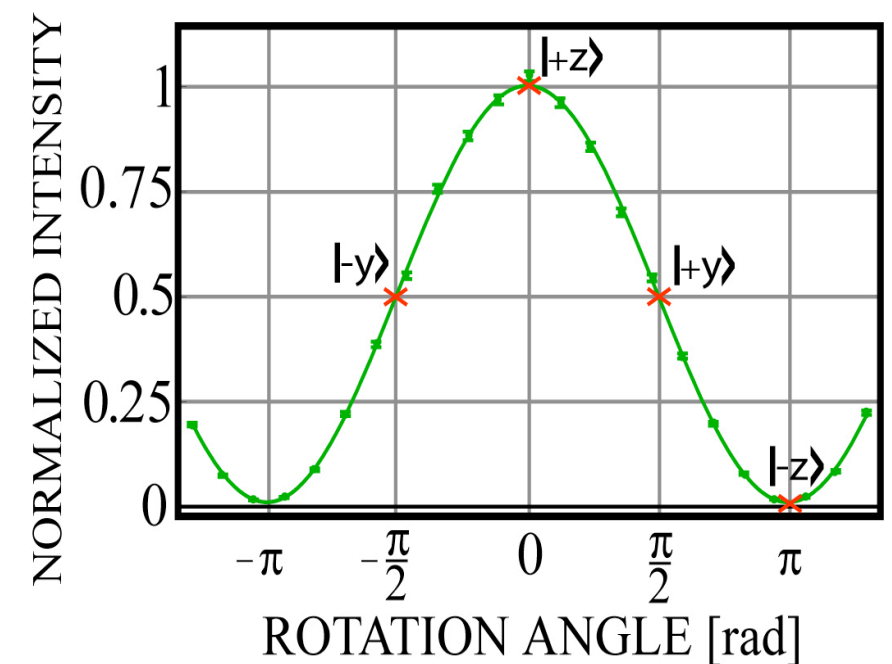
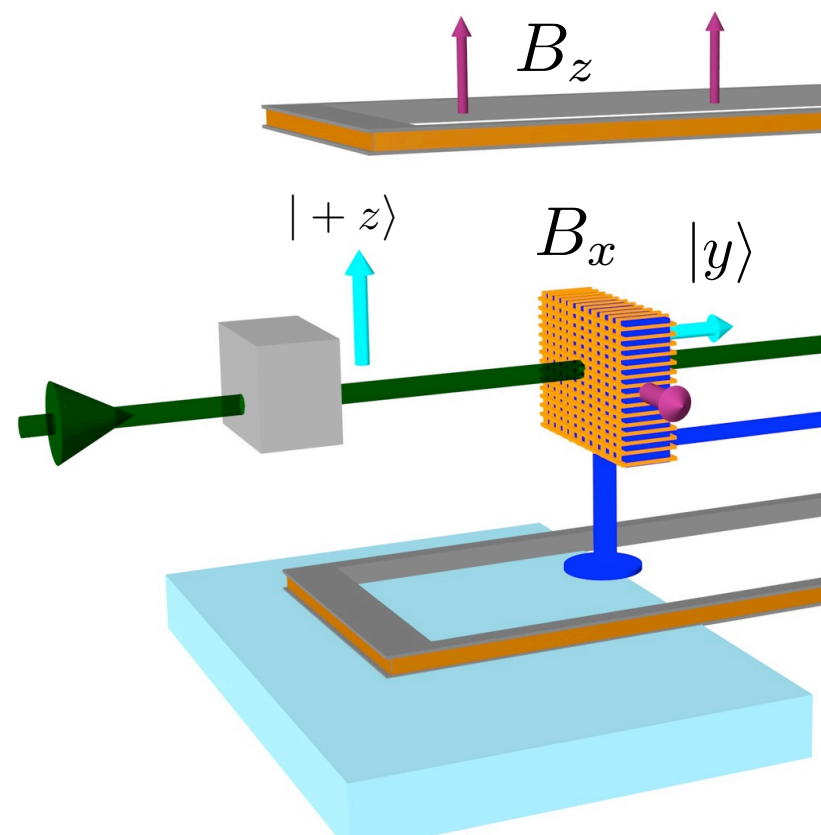
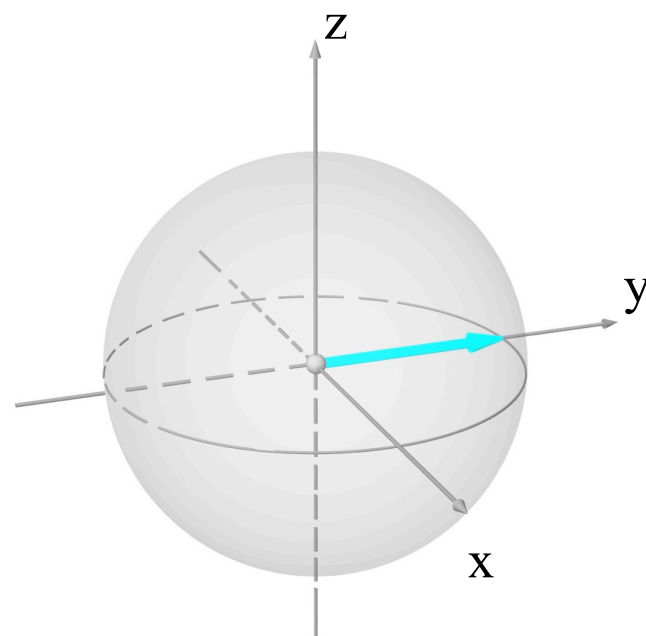
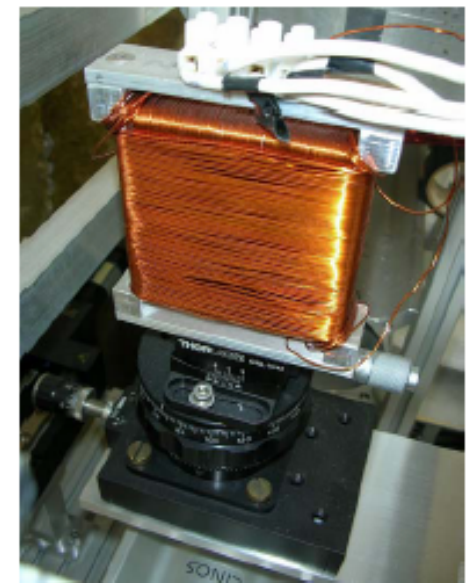


Experimental Setup: State Preparation



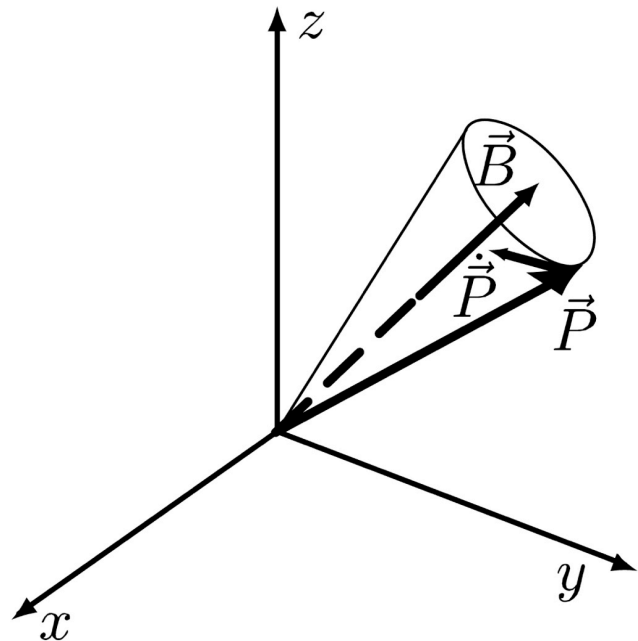
$$\dot{\vec{P}}(t) = \vec{P}(t) \times \gamma \vec{B}(t)$$

$$\vec{P}(t) = \langle \psi | \vec{\sigma} | \psi \rangle$$



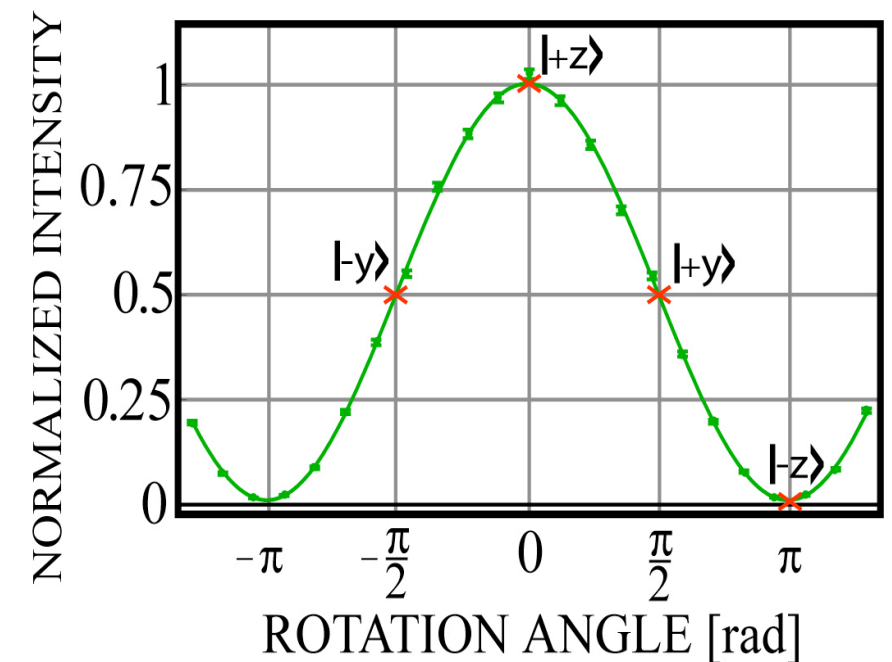
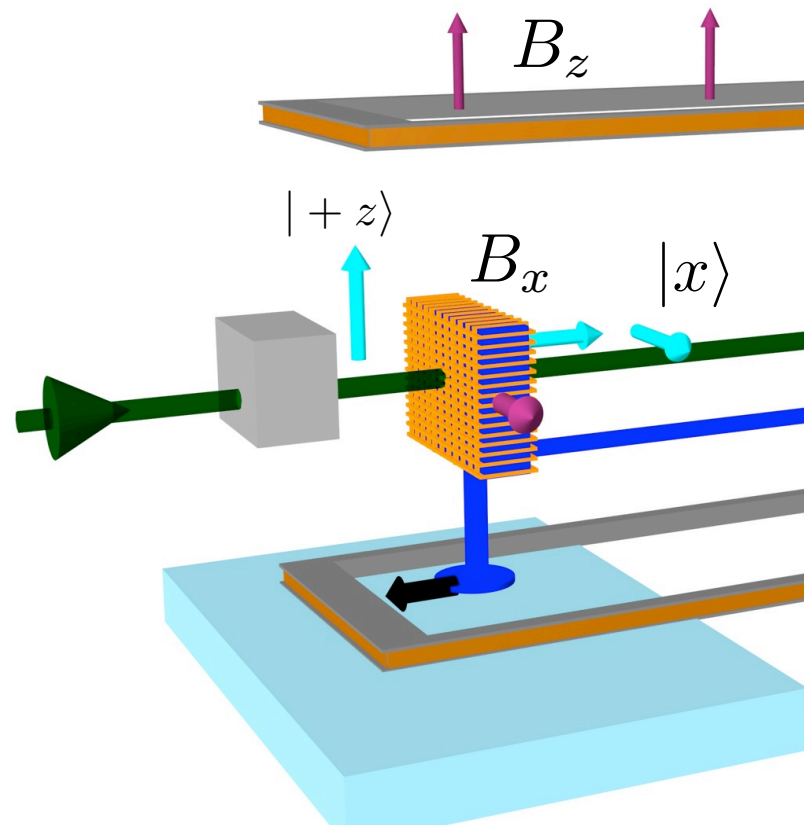
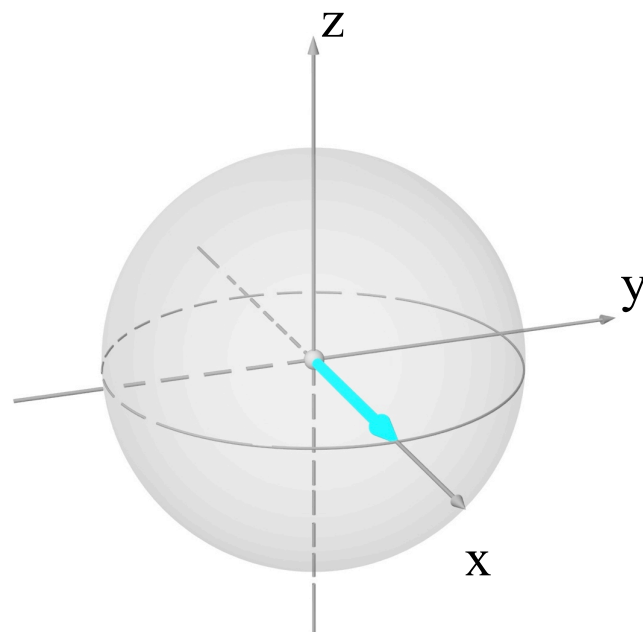
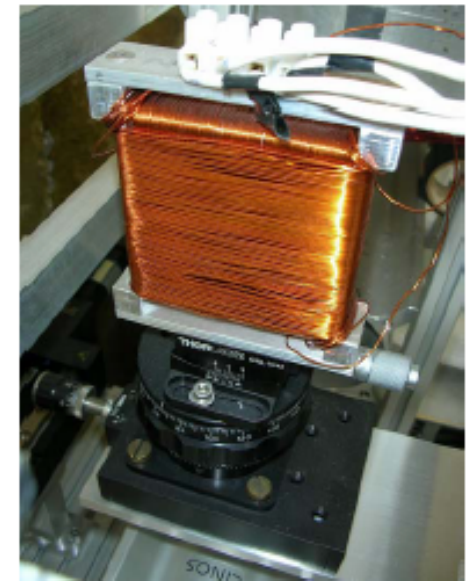
J. Erhart, S. Sponar, G. Sulyok, G. Badurek, M. Ozawa, Y. Hasegawa, *Nature Phys.* **8**, 185 (2012).

Experimental Setup: State Preparation



$$\dot{\vec{P}}(t) = \vec{P}(t) \times \gamma \vec{B}(t)$$

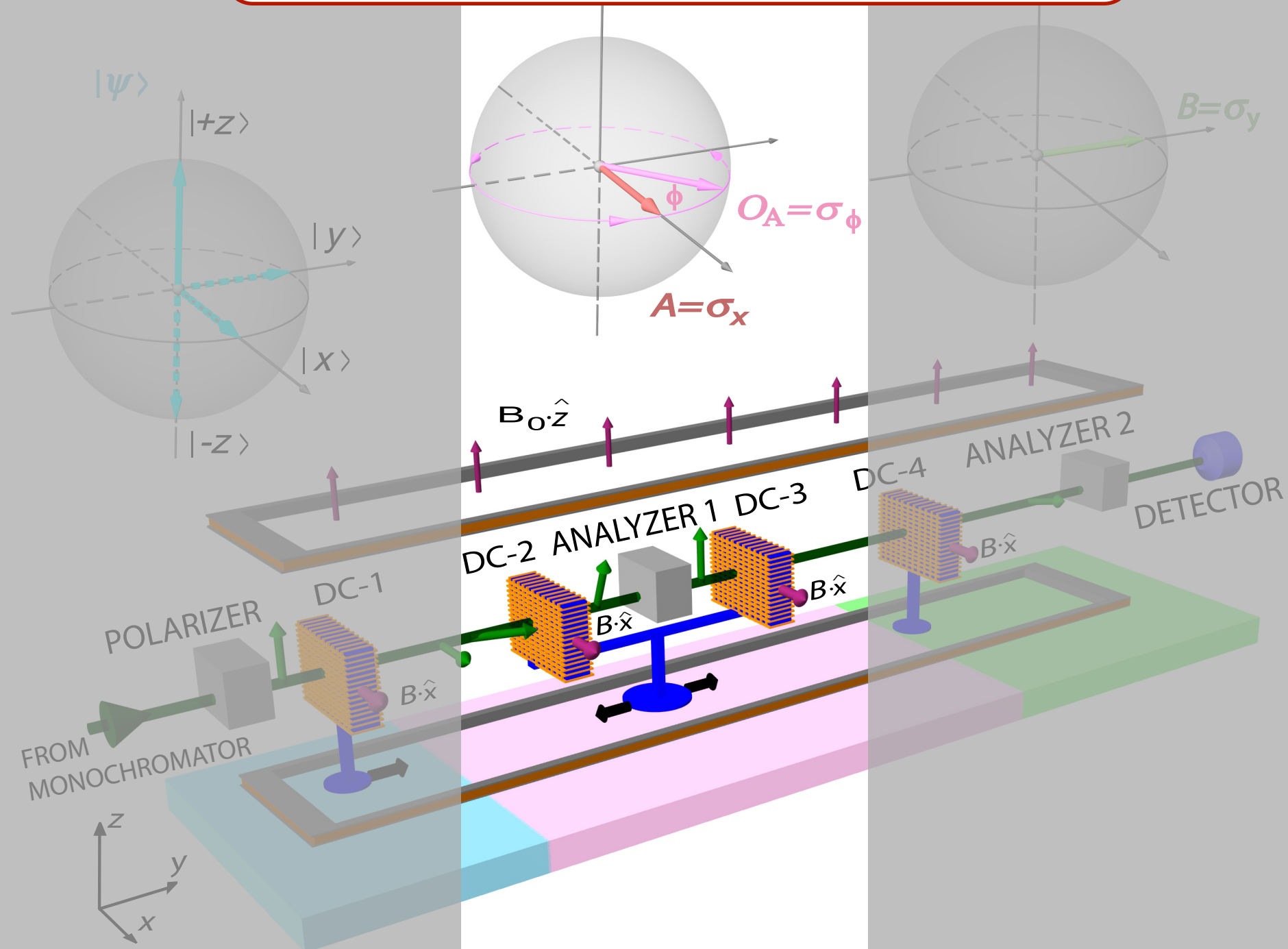
$$\vec{P}(t) = \langle \psi | \vec{\sigma} | \psi \rangle$$



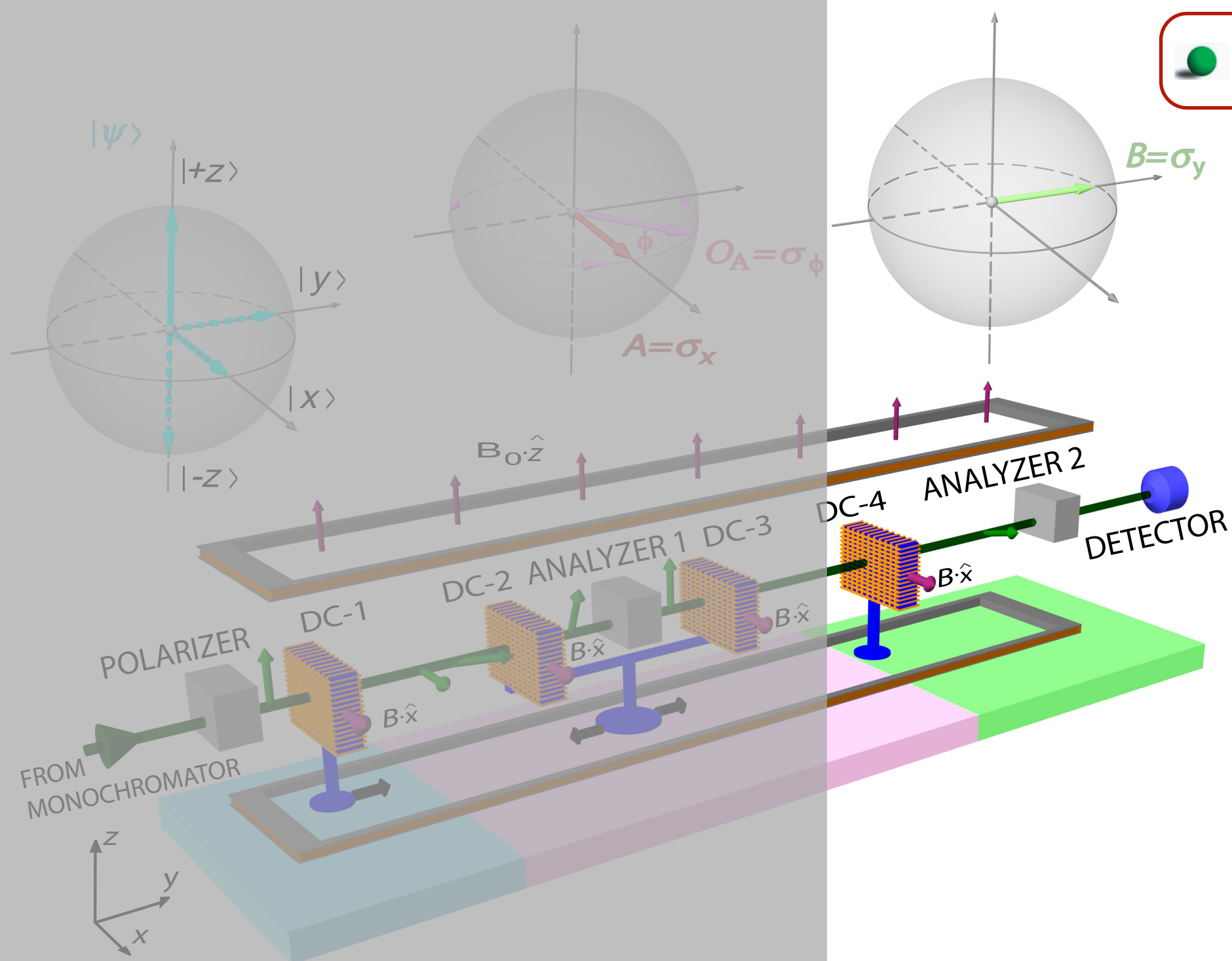
J. Erhart, S. Sponar, G. Sulyok, G. Badurek, M. Ozawa, Y. Hasegawa, *Nature Phys.* **8**, 185 (2012).

Experimental Setup: O_A - Measurement:

$$O_A = \sigma_\phi = \sigma_x \sin \phi + \sigma_y \cos \phi$$

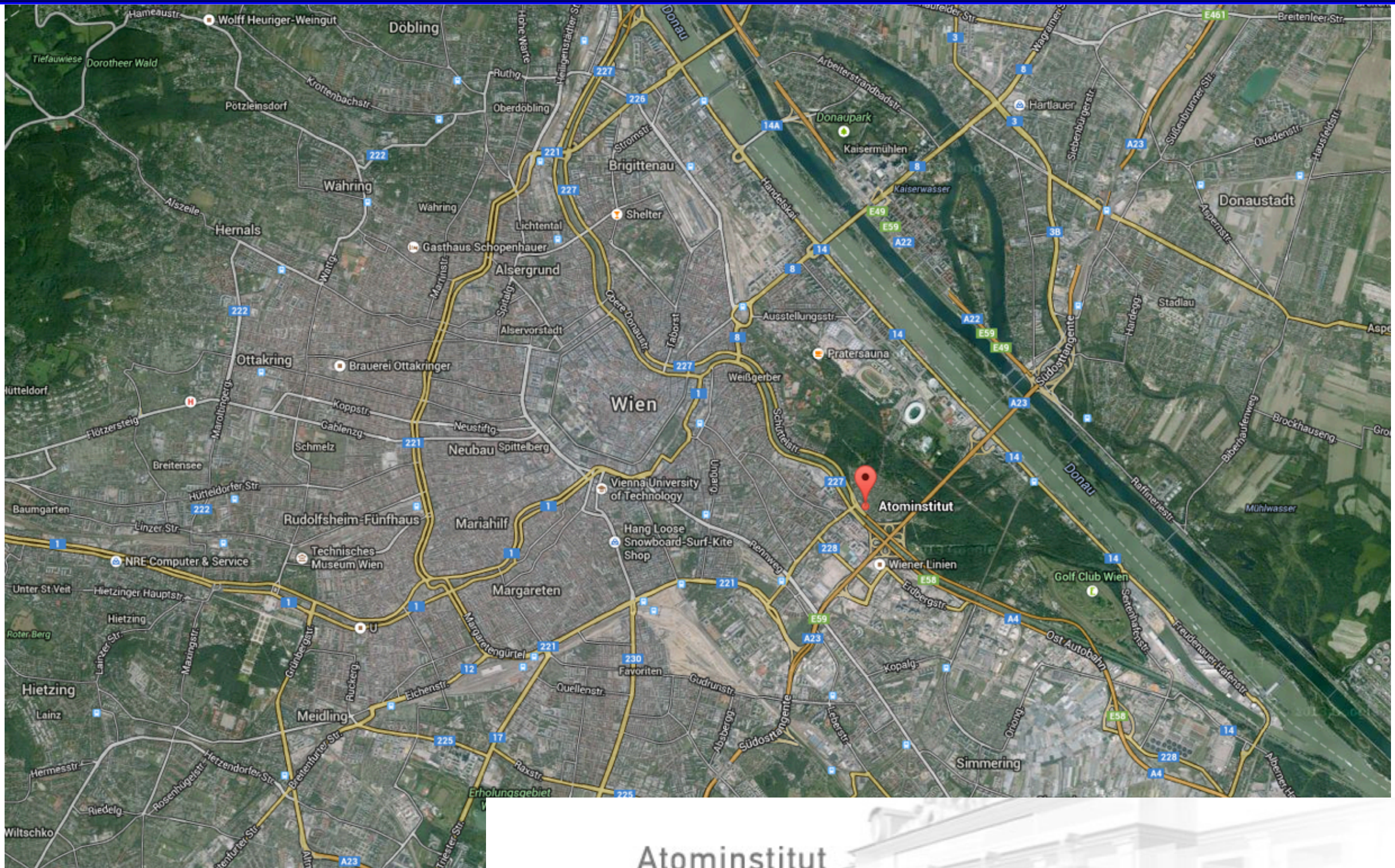


Experimental Setup: B - Measurement



$$B = \sigma_y$$

Experimental Setup @ Atominstitut - Vienna University of Technology



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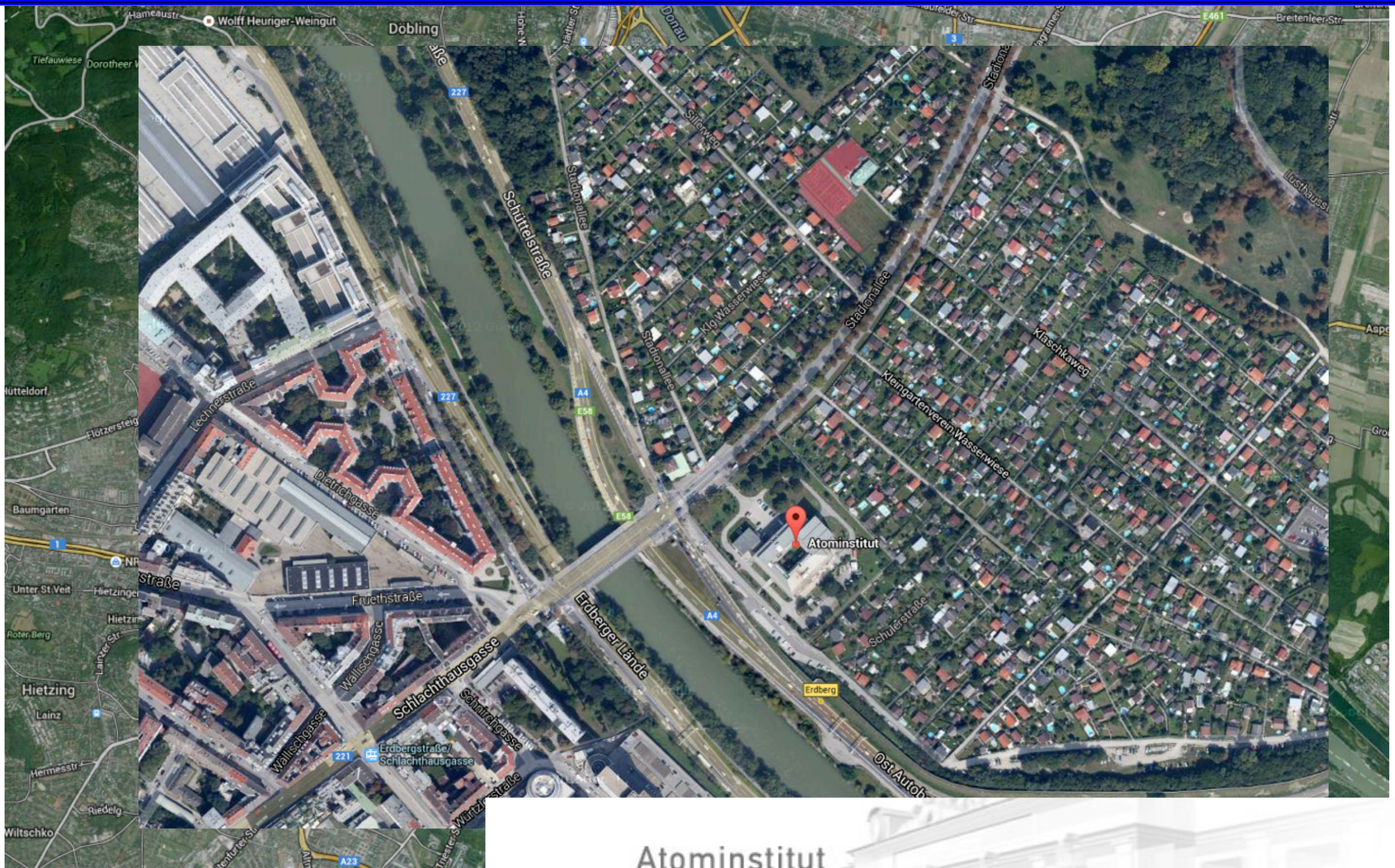
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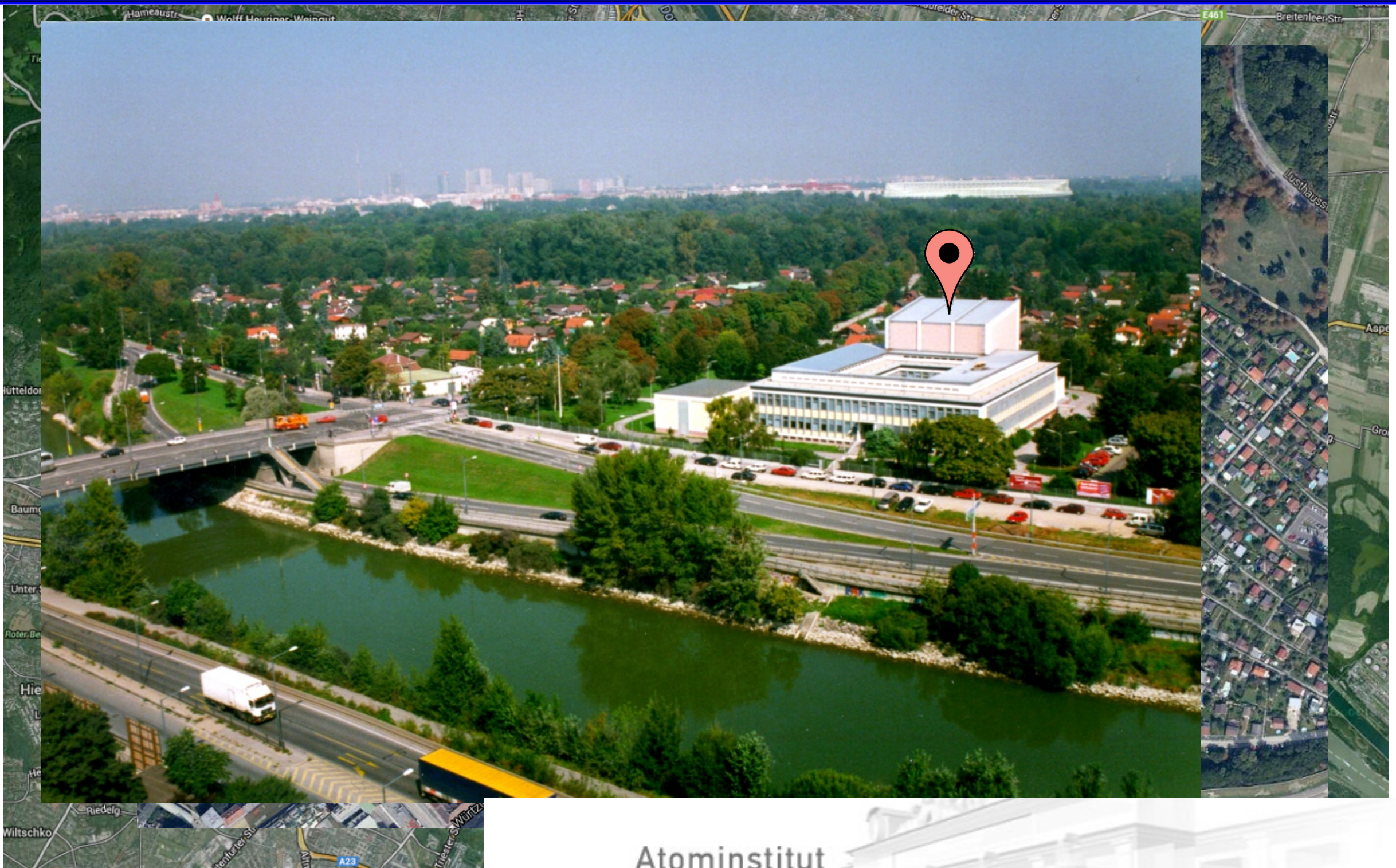
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Experimental Setup @ Atominstitut - Vienna University of Technology



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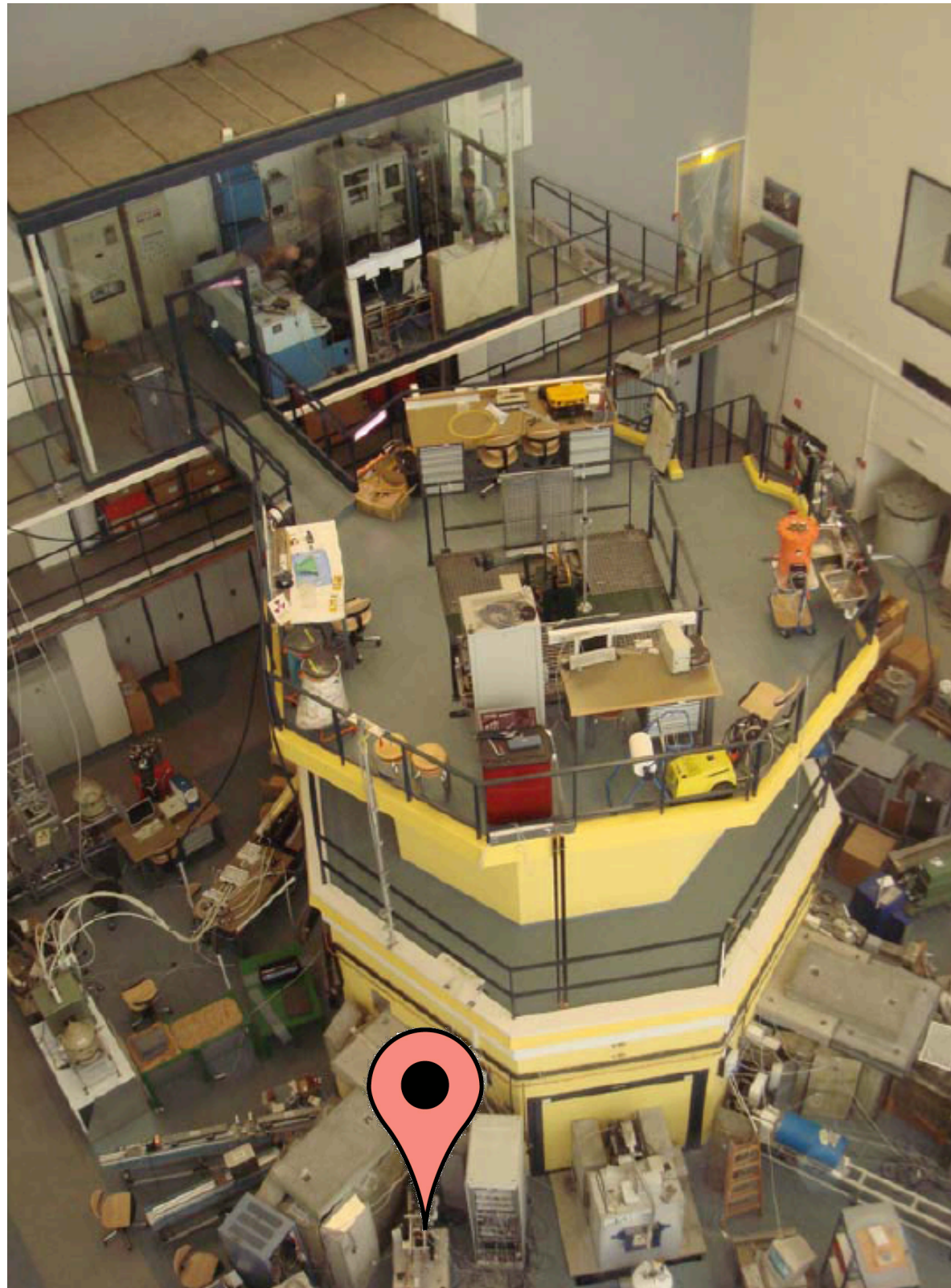
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p12/28

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Experimental Setup @ Atominstitut - Vienna University of Technology



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04.03.2014

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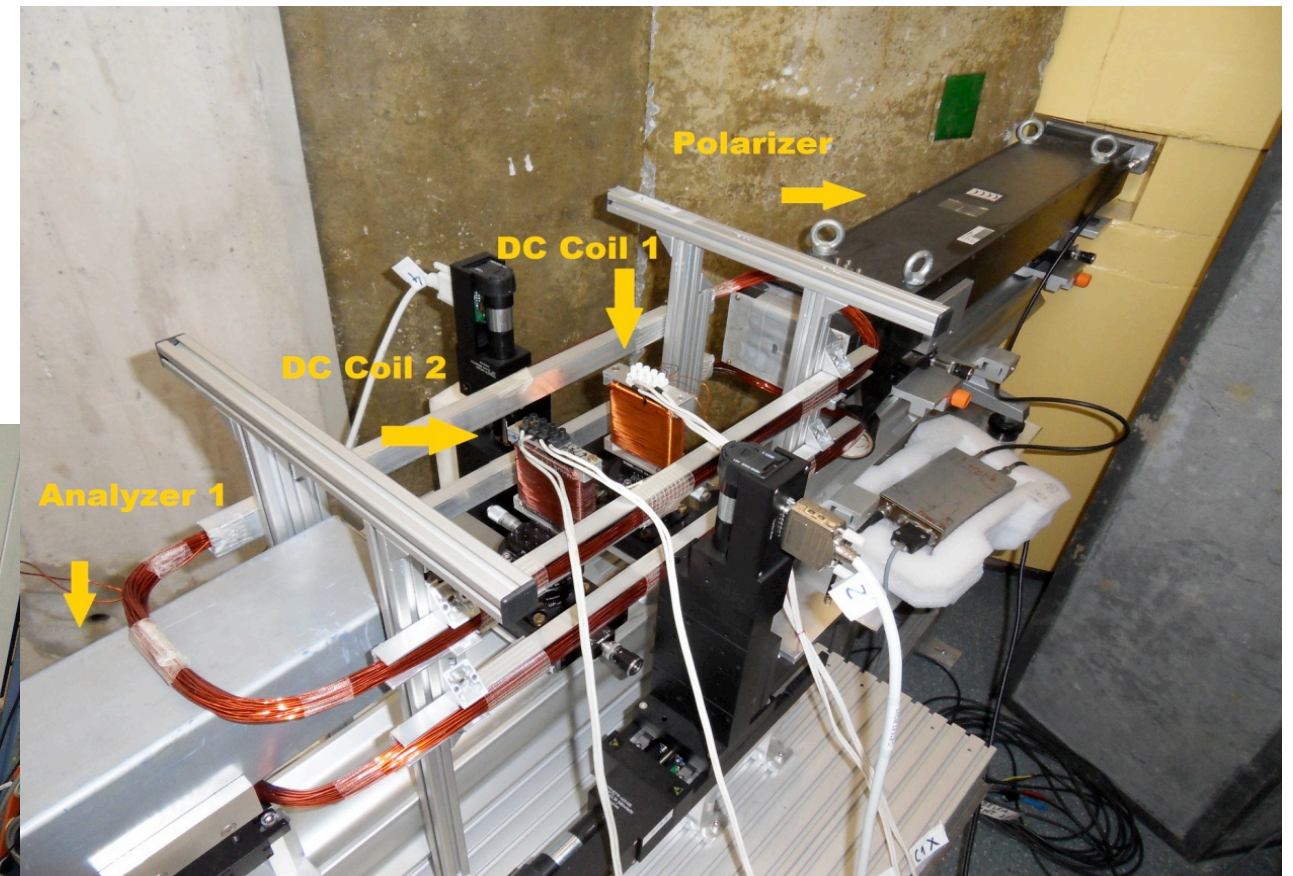
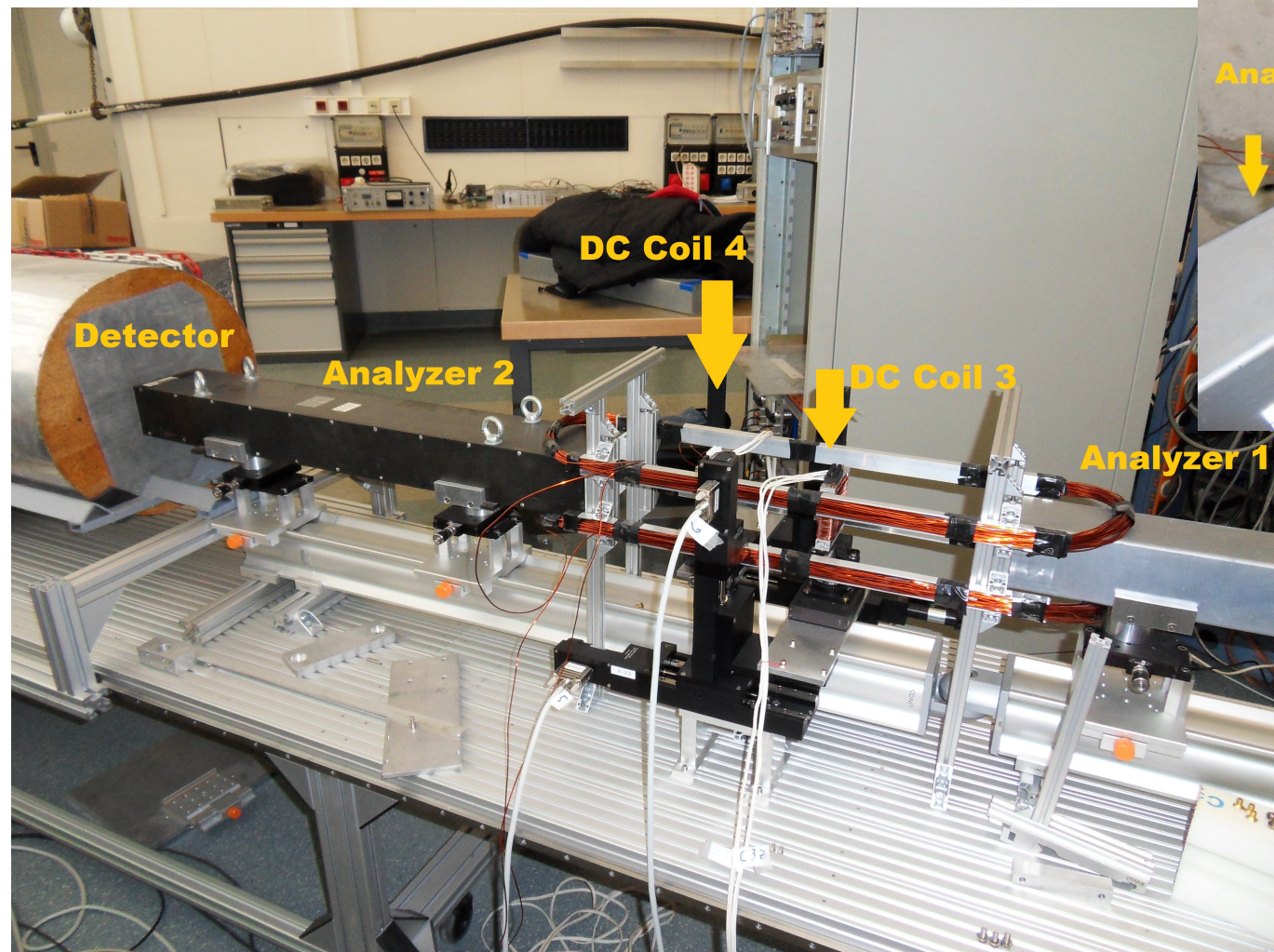
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Experimental Setup @ Atominstitut - Vienna University of Technology



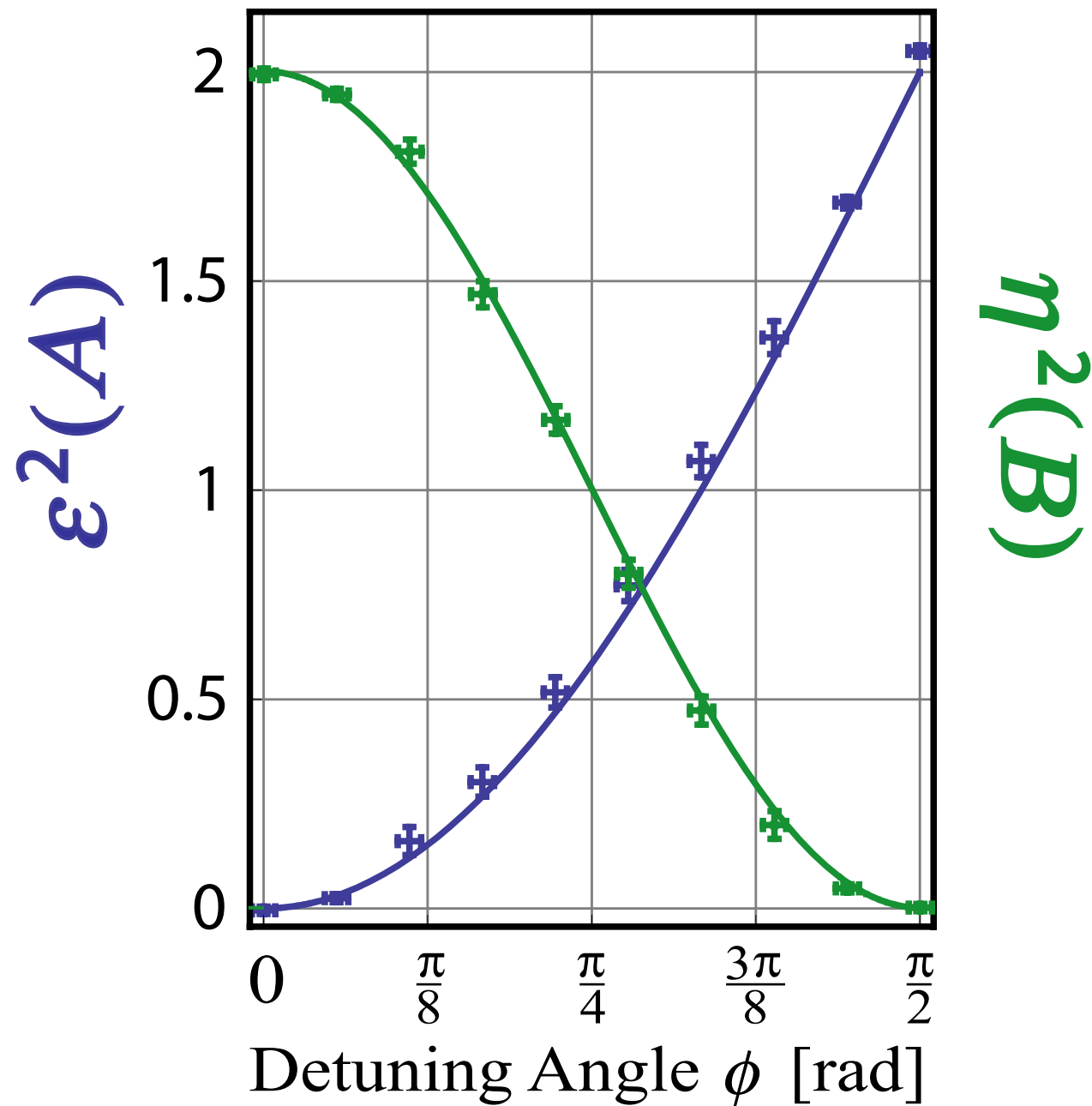
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WIEN
Vienna University of Technology



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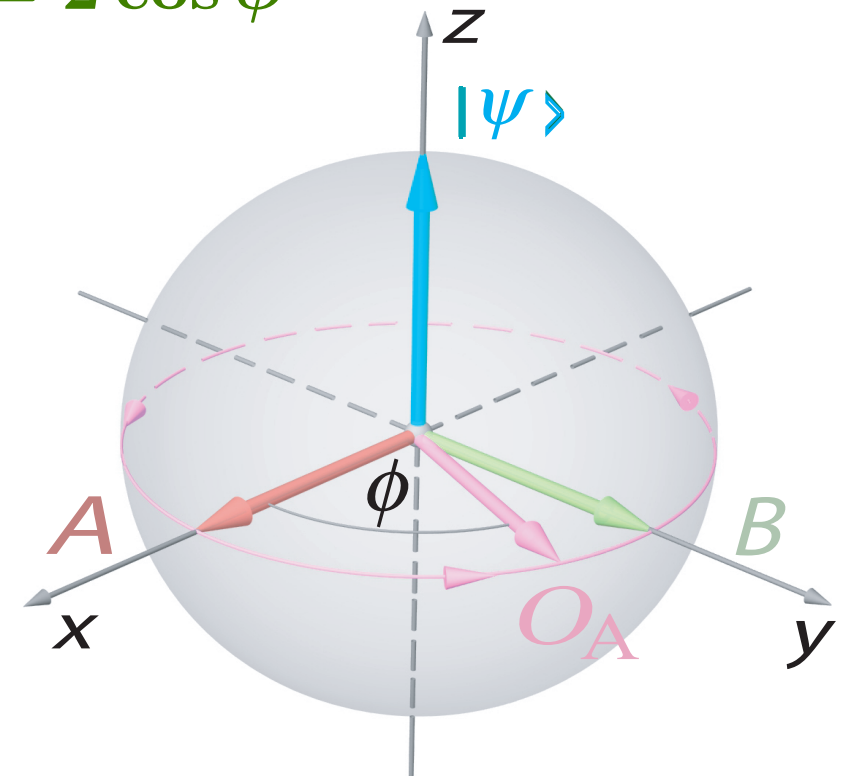


Experimental Results of Error and Disturbance



$$\epsilon(A)^2 = 4 \sin^2(\phi/2)$$

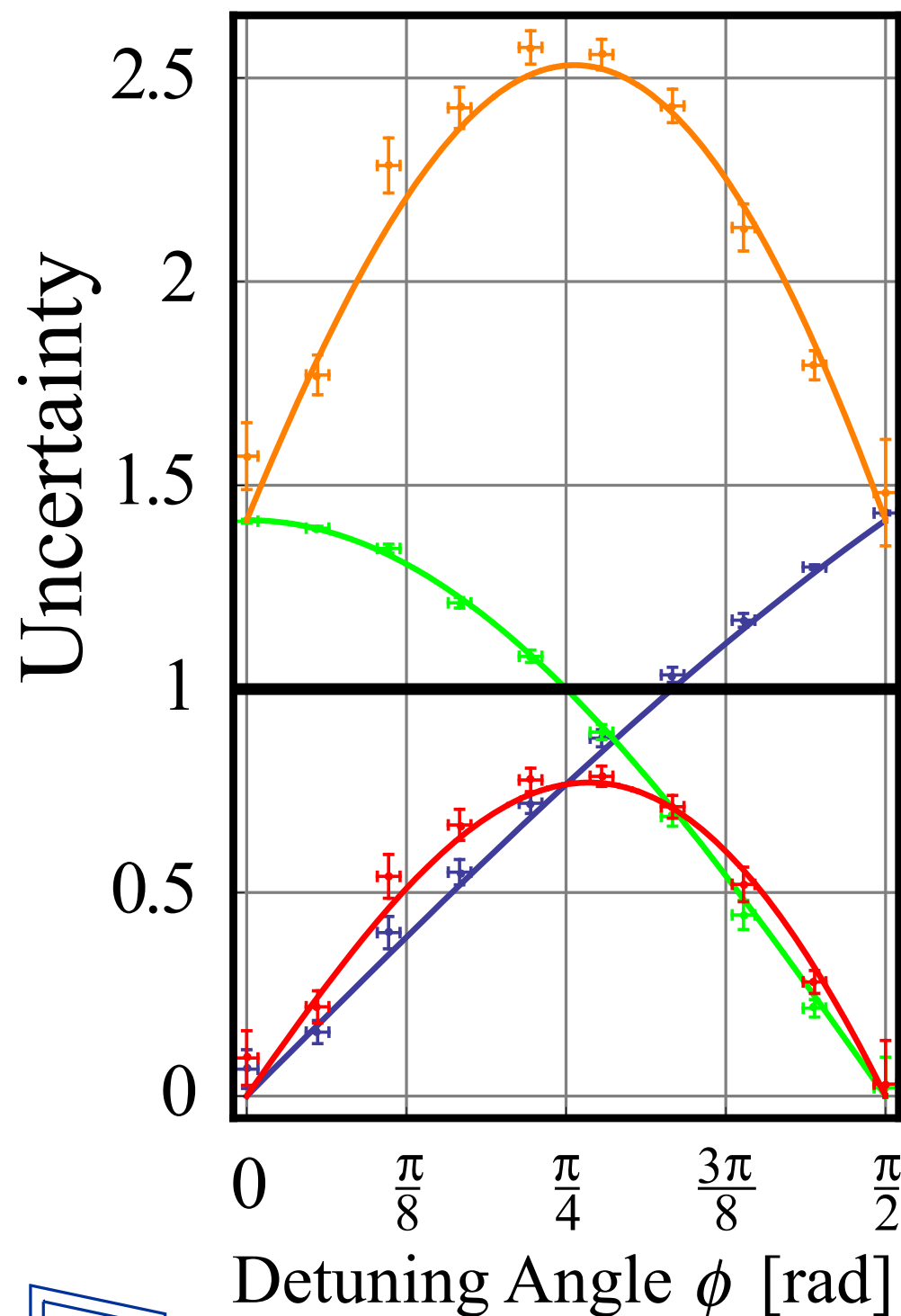
$$\eta(B)^2 = 2 \cos \phi$$



● error, disturbance: trade - off

J. Erhart, S. Sponar, G. Sulyok, G. Badurek, M. Ozawa, Y. Hasegawa, *Nature Phys.* **8**, 185 (2012).

Experimental Results $\phi \in [0, \pi/2]$



● error, disturbance & standard deviations

— $\varepsilon(A)\eta(B) + \varepsilon(A)\sigma(B) + \sigma(A)\eta(B)$

LHS of Ozawa's universally valid uncertainty relation (**UVUR**)

— $\varepsilon(A)\sigma(B)$

— $\sigma(A)\eta(B)$

standard deviations:

$\sigma(B) = 0.9999(1)$

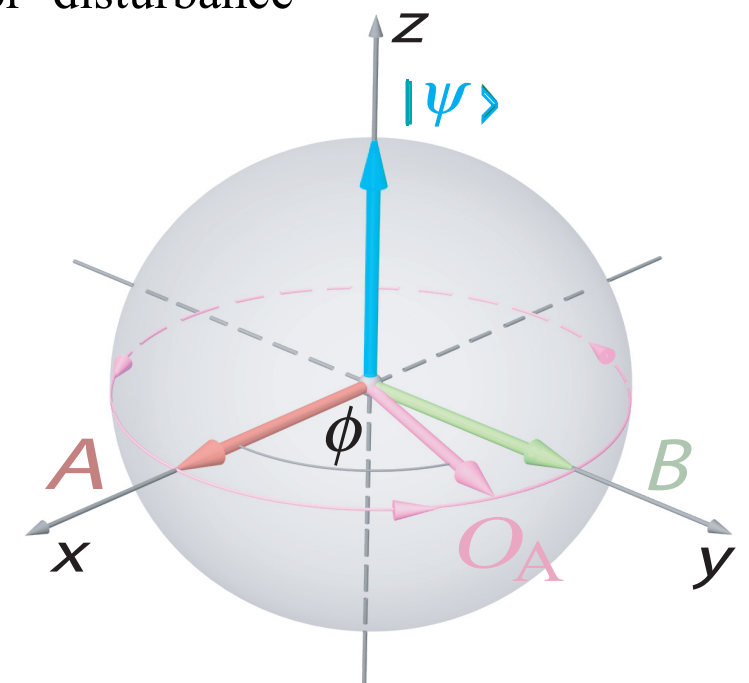
$\sigma(A) = 0.9994(3)$

— $\varepsilon(A)\eta(B)$

LHS of Heisenberg's original error- disturbance uncertainty relation (**EDUR**)

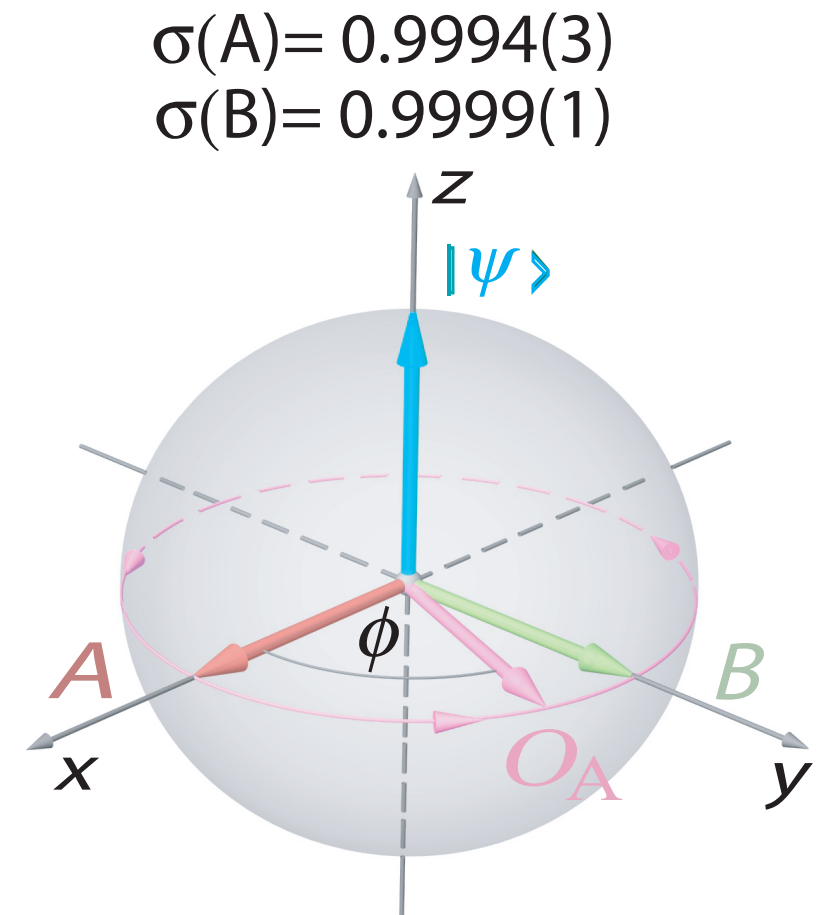
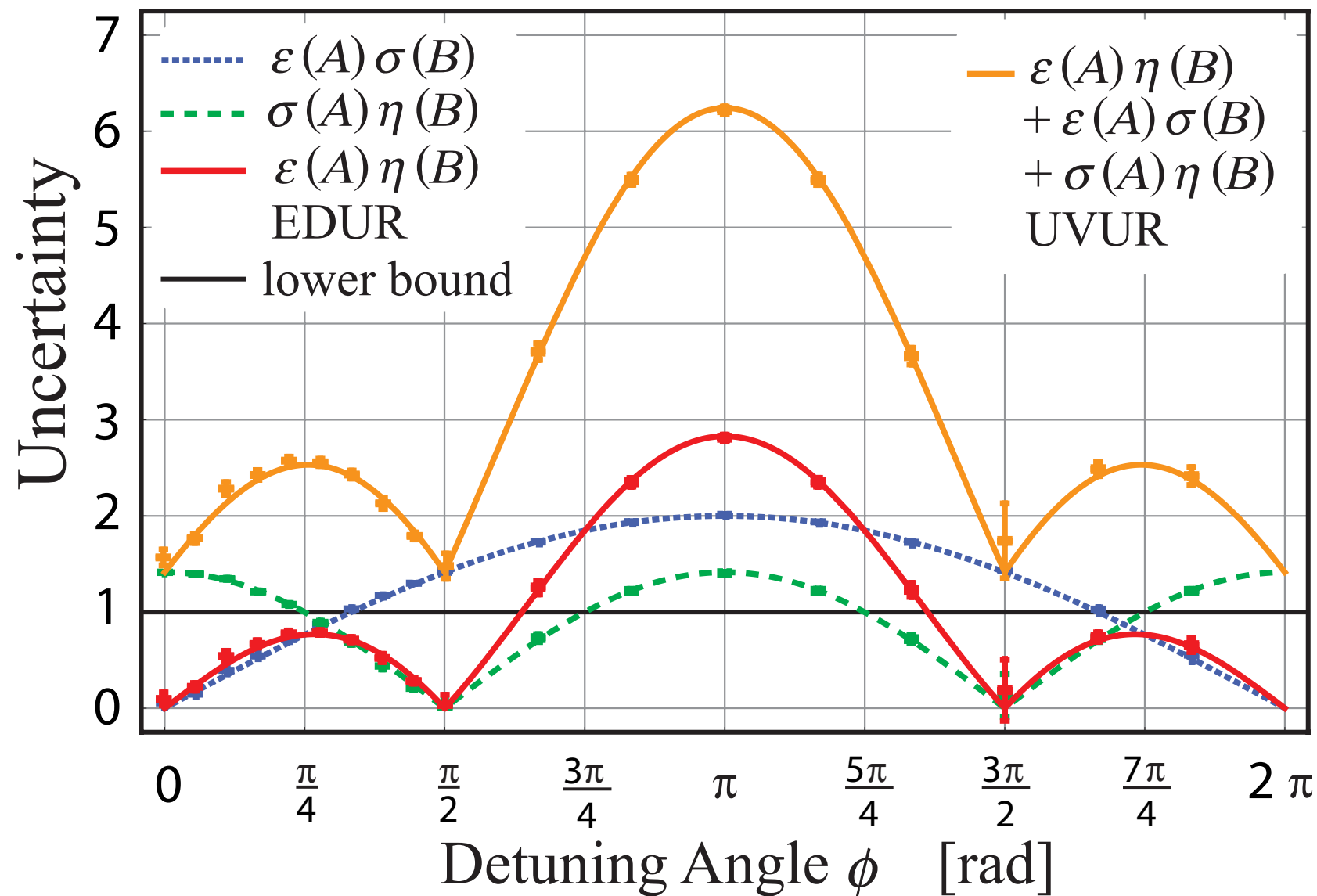
— lower bound

$\frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$



J. Erhart, S. Sponar, G. Sulyok, G. Badurek, M. Ozawa, Y. Hasegawa, *Nature Phys.* **8**, 185 (2012).

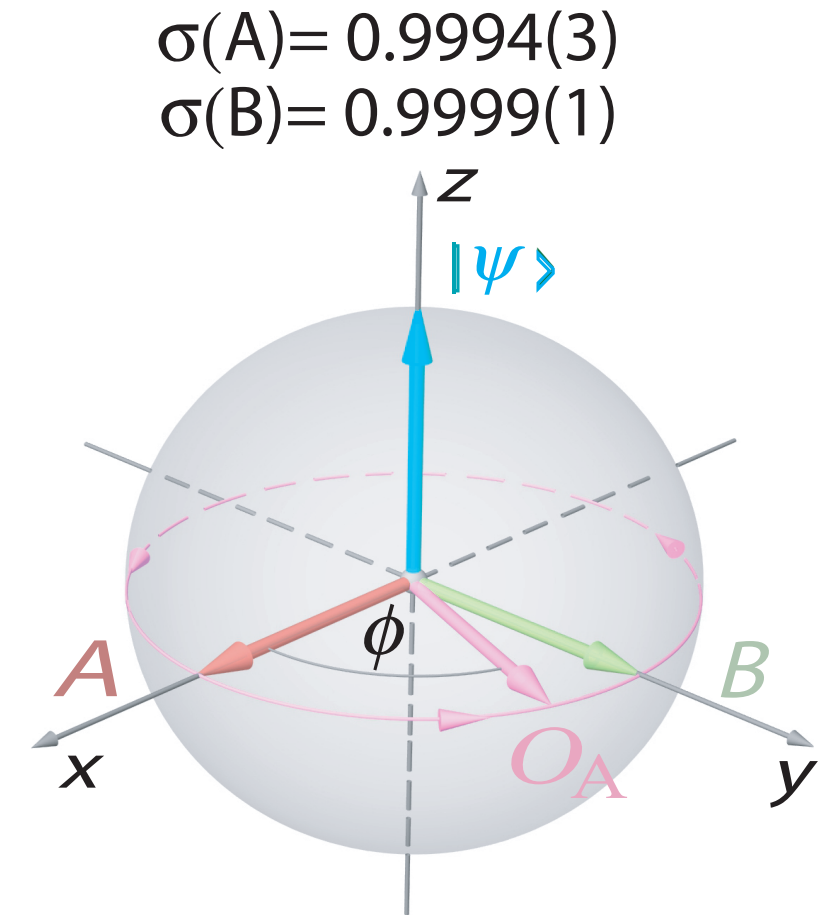
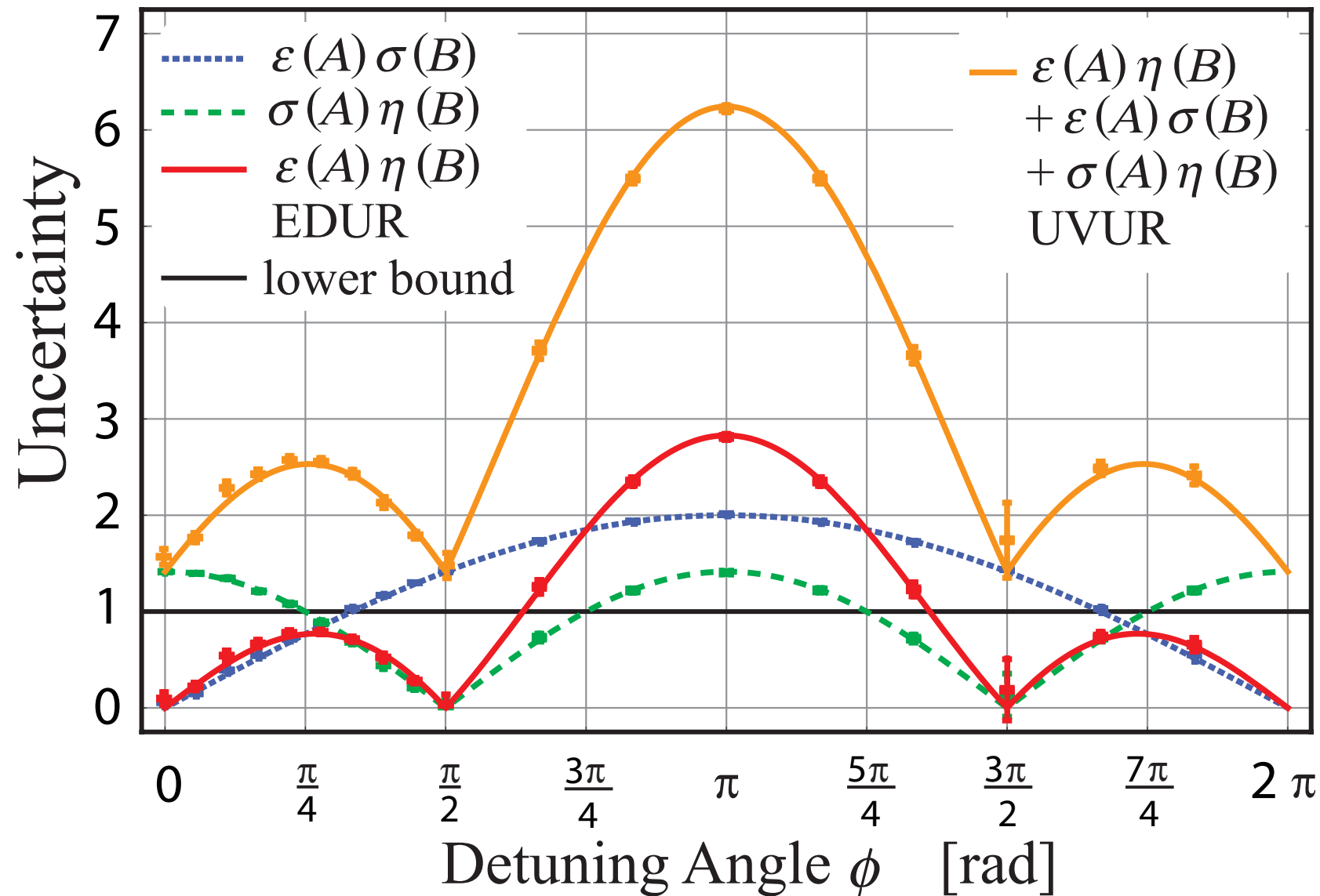
O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$



error, disturbance: trade - off not for all values of ϕ
Heisenberg's EDUR also above limit

Sulyok, Sponar, Erhart, Badurek, Ozawa, Hasegawa, *Phys. Rev. A* **88**, 022110 (2013).

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

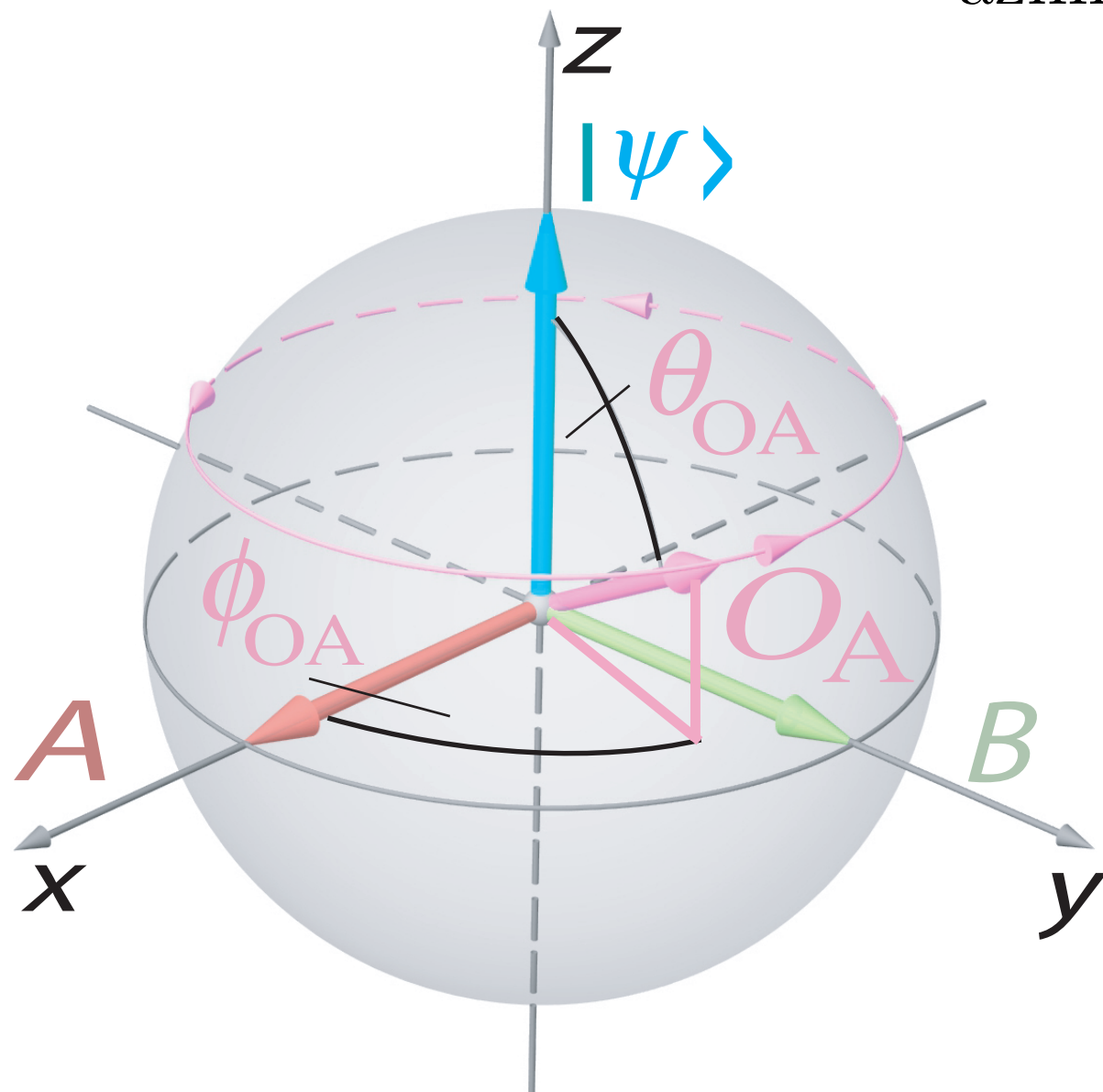


error, disturbance: trade - off not for all values of ϕ
 Heisenberg's EDUR also above limit

Sulyok, Sponar, Erhart, Badurek, Ozawa, Hasegawa, *Phys. Rev. A* **88**, 022110 (2013).

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

azimuthal angle OA : ϕ_{OA}
polar angle OA : θ_{OA}

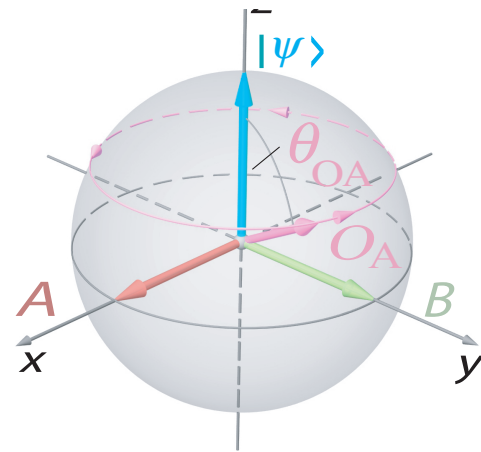


$$\epsilon(A) = \sqrt{2 - 2 \cos \phi_{OA} \sin \theta_{OA}}$$

$$\eta(B) = \sqrt{2 - 2 \sin^2 \phi_{OA} \sin^2 \theta_{OA}}$$

$$\sigma(A) = 1, \sigma(B) = 1$$

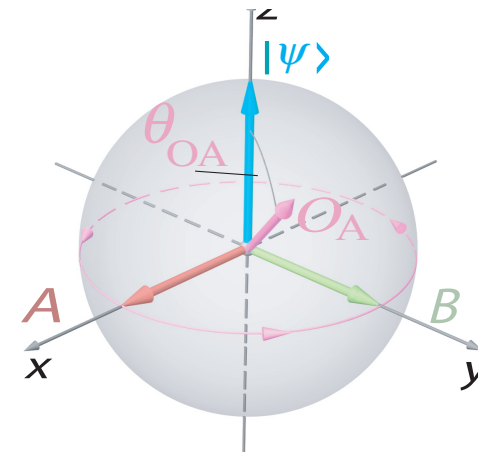
O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$



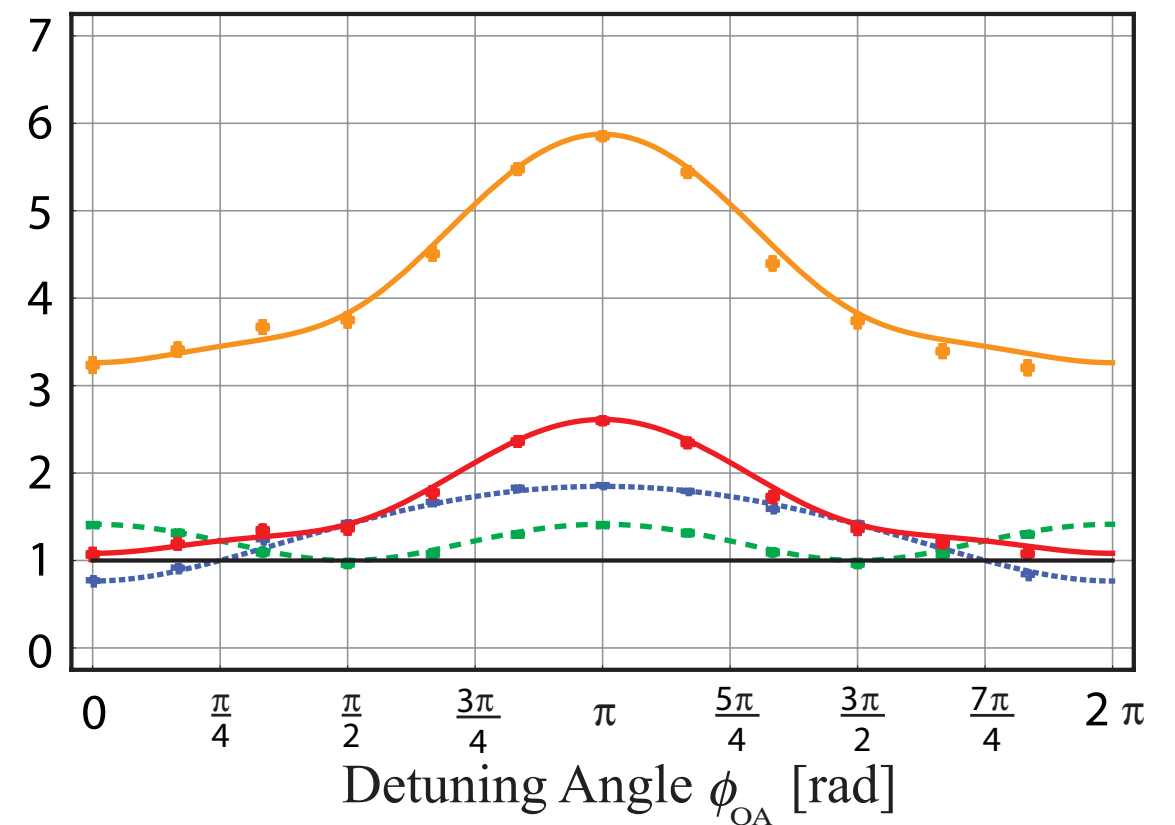
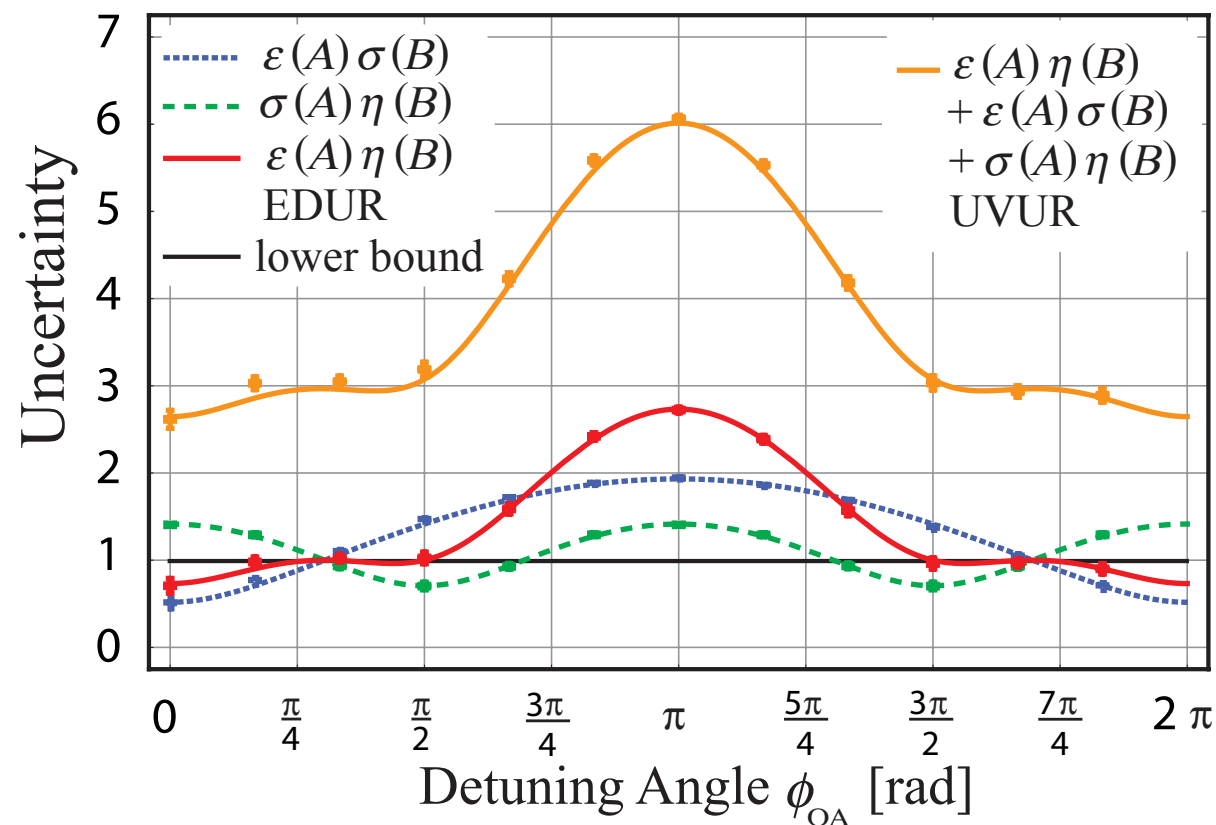
$$\theta_{OA} = \frac{\pi}{3}$$

$$\sigma(A) = 0.9994(3)$$

$$\sigma(B) = 0.9999(1)$$



$$\theta_{OA} = \frac{\pi}{4}$$



$\epsilon(A) \ \& \ \eta(B) \neq 0 \ \forall \ \phi_{OA}$ unless $\theta_{OA} = \pi/2$, limit remains unchanged

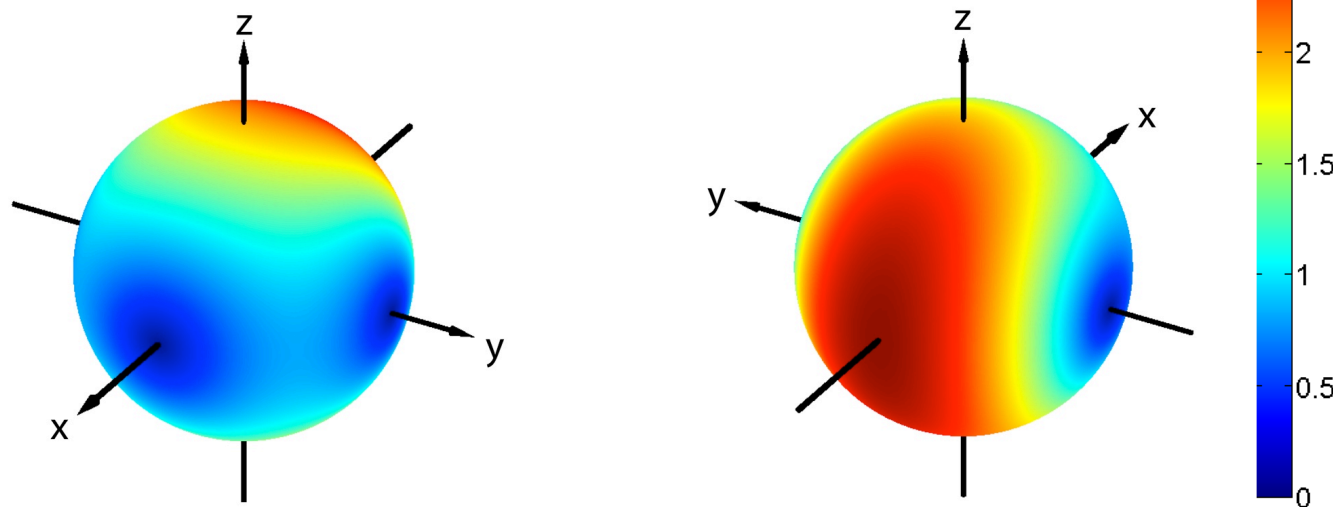
Sulyok, Sponar, Erhart, Badurek, Ozawa, Hasegawa, *Phys. Rev. A* **88**, 022110 (2013).

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

Heisenberg's original Error-Disturbance Uncertainty Relation :

$$\epsilon(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

● **Theory:**



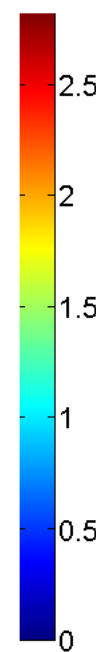
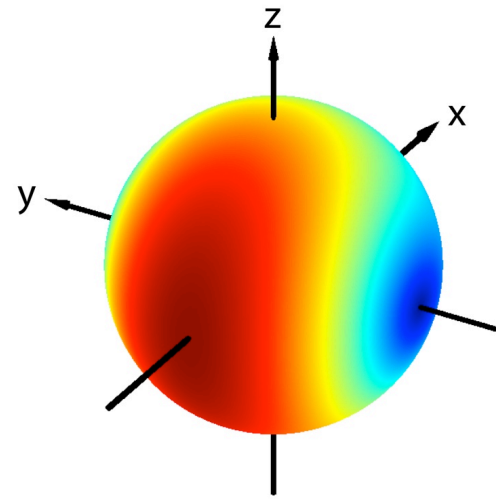
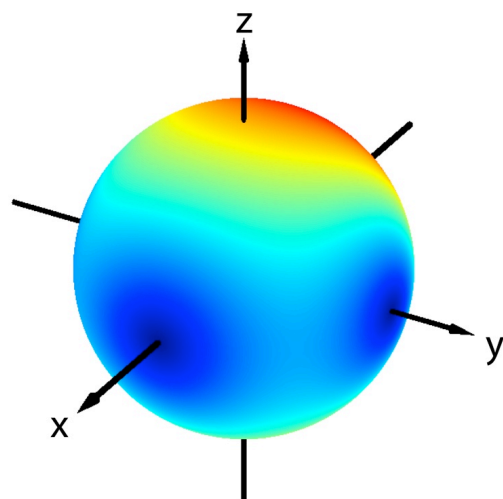
Sulyok, Sponar, Erhart, Badurek, Ozawa, Hasegawa, *Phys. Rev. A* **88**, 022110 (2013).

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

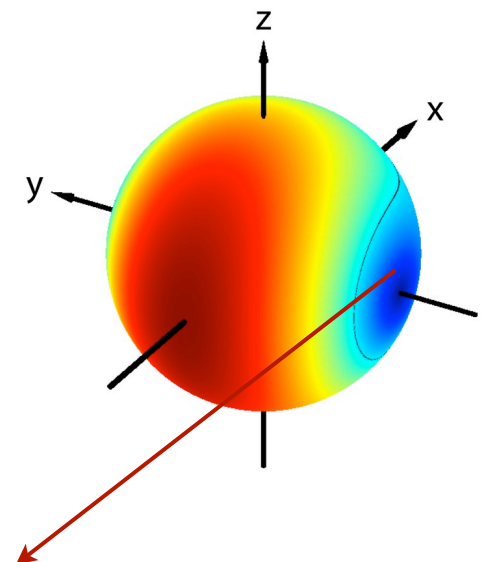
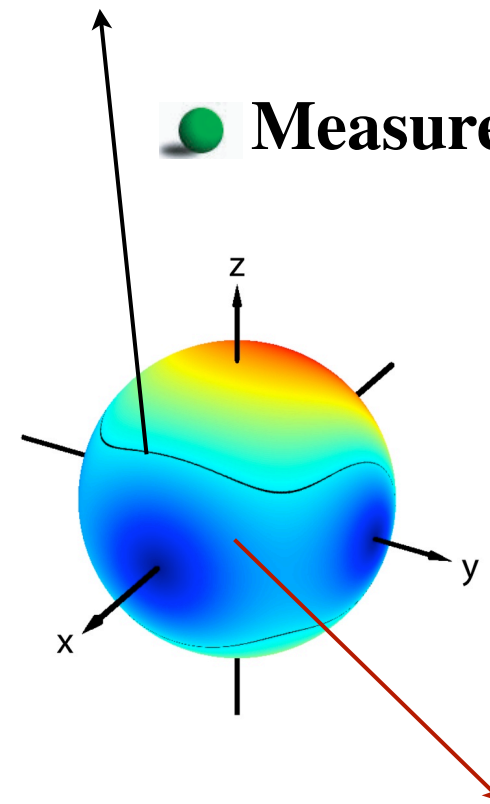
Heisenberg's original Error-Disturbance Uncertainty Relation :

$$\epsilon(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

● Theory:



● Measurement:



Violation

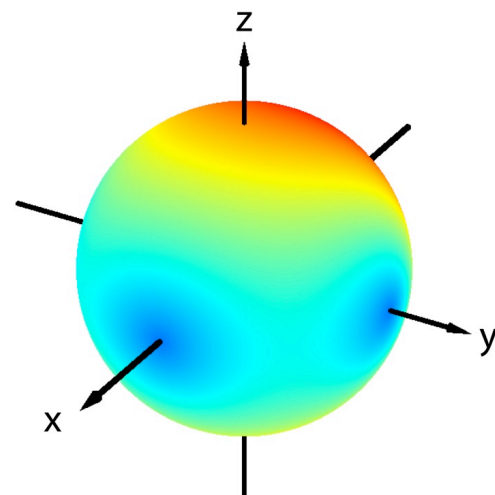
Sulyok, Sponar, Erhart, Badurek, Ozawa, Hasegawa, *Phys. Rev. A* **88**, 022110 (2013).

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

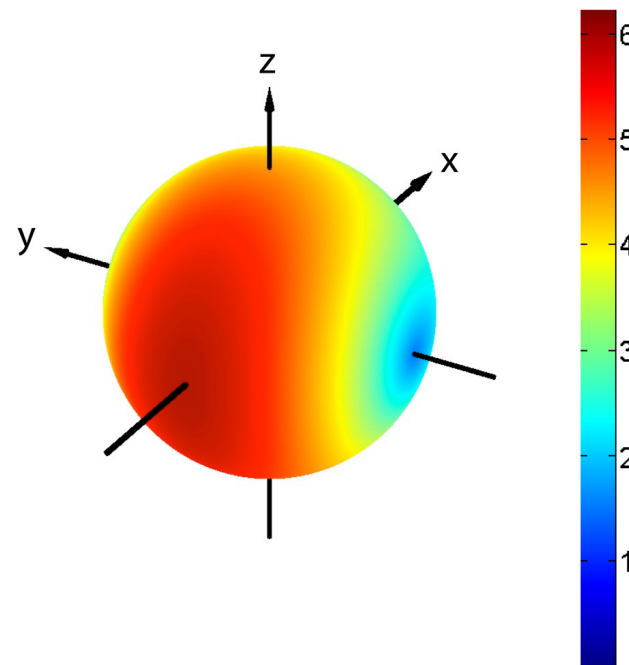
Ozawa's Universally Valid Uncertainty Relation :

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

● **Theory:**



● **Measurement:**

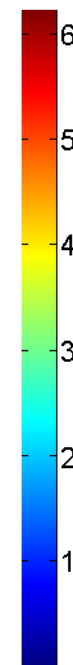
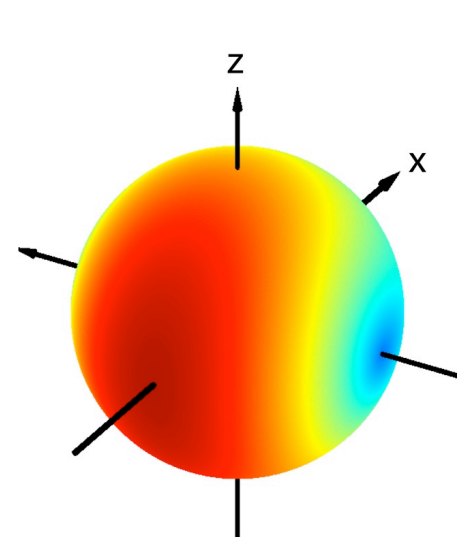
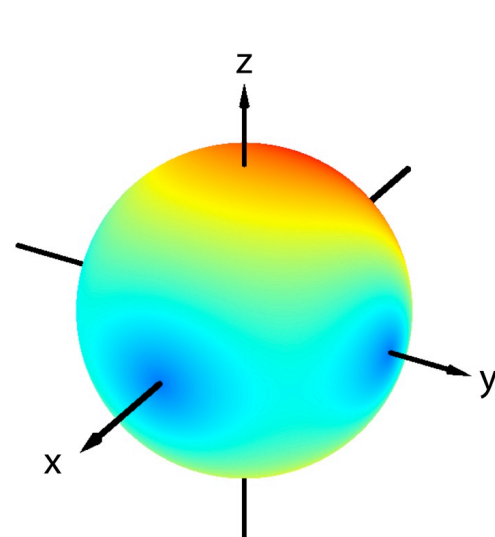


O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

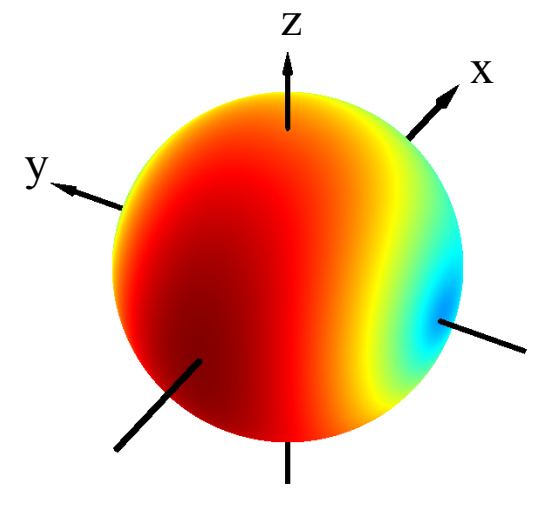
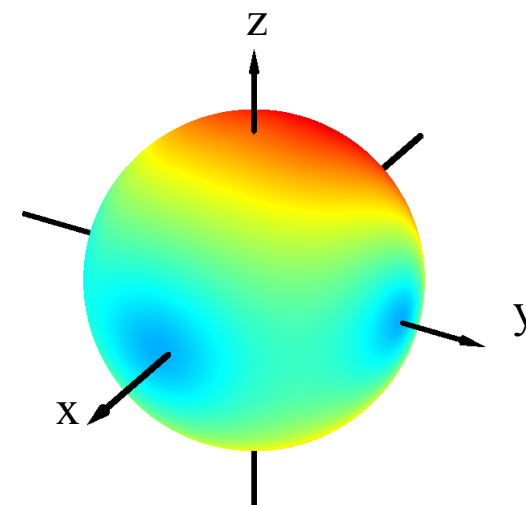
Ozawa's Universally Valid Uncertainty Relation :

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

● **Theory:**

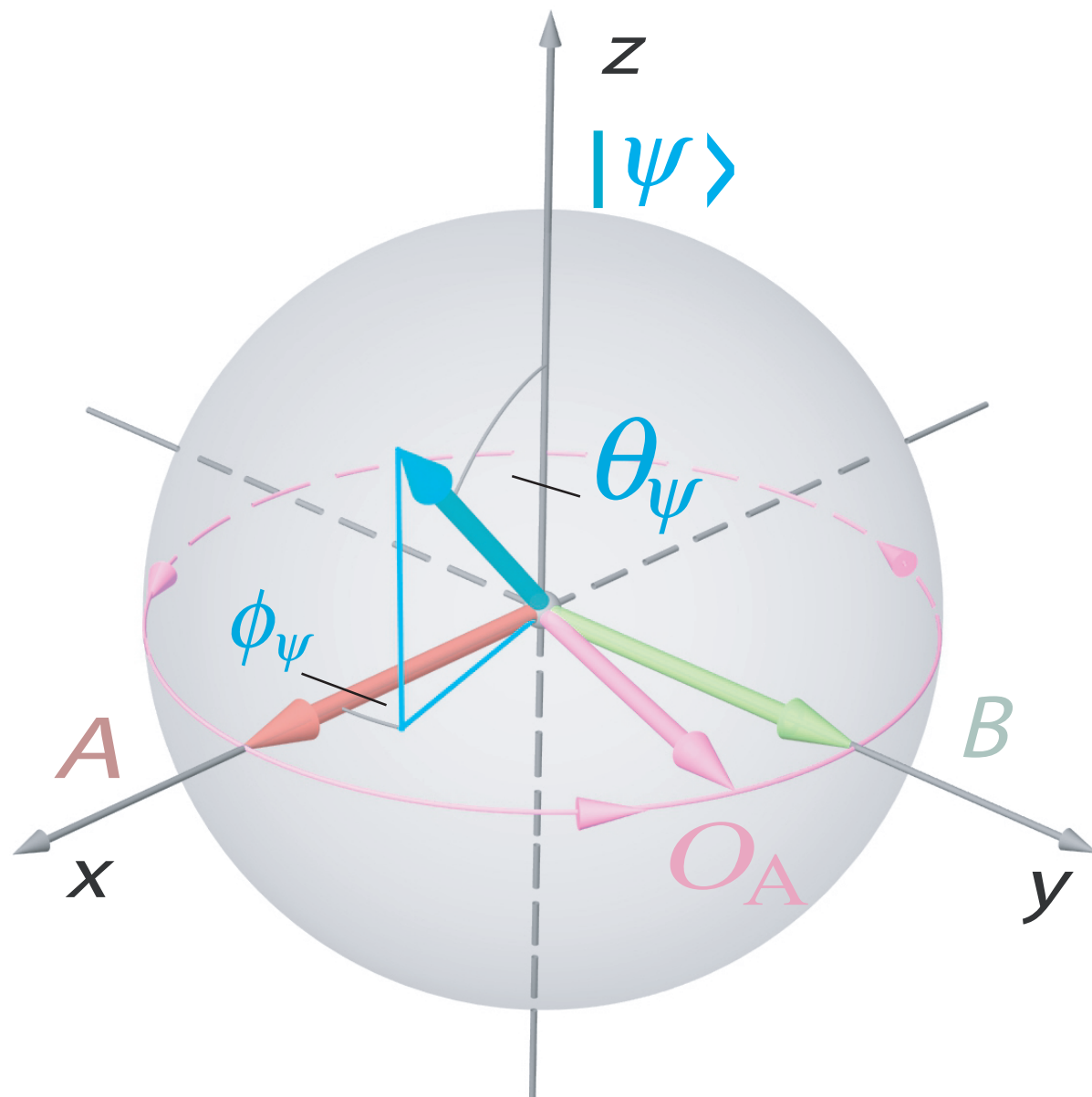


● **Measurement:**



No Violation

Variation of Initial State $|\psi\rangle$: Results $\phi_\psi \in [0, 2\pi]$, and $\theta_\psi \in [0, \pi]$



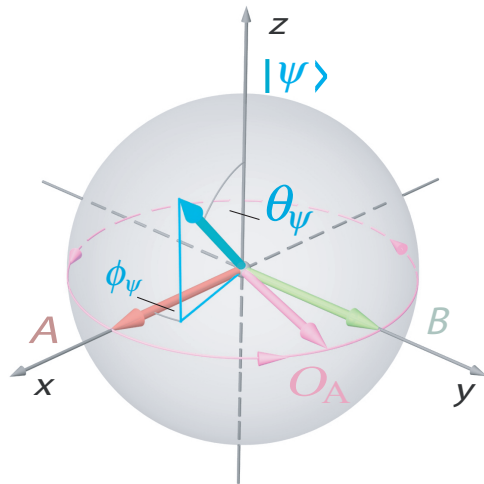
$$\left. \begin{aligned} \epsilon(A) &= 2 \sin \frac{\phi_{OA}}{2} \\ \eta(B) \sqrt{2} |\cos \phi_{OA}| \end{aligned} \right\} \text{state-independent}$$

$$\sigma(A) = \sqrt{1 - \cos^2 \phi_\psi \sin^2 \theta_\psi}$$

$$\sigma(B) = \sqrt{1 - \sin^2 \phi_\psi \sin^2 \theta_\psi}$$

$$\frac{1}{2} \langle \psi | [A, B] | \psi \rangle = |\cos \theta_\psi|$$

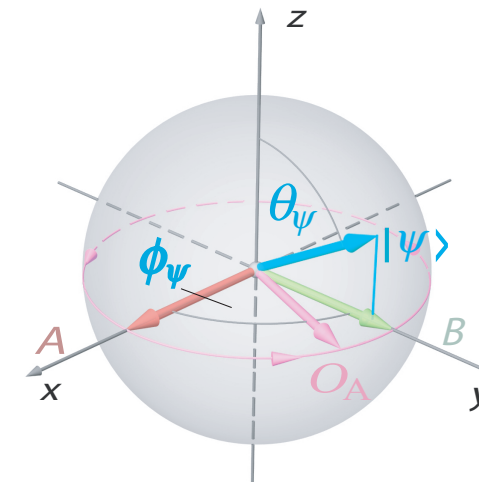
Variation of Initial State $|\psi\rangle$: Results $\phi_\psi \in [0, 2\pi]$, and $\theta_\psi \in [0, \pi]$



$$\theta_\psi = \frac{\pi}{4}, \phi_\psi = \frac{\pi}{12}$$

$$\sigma(A) = 0.714(9)$$

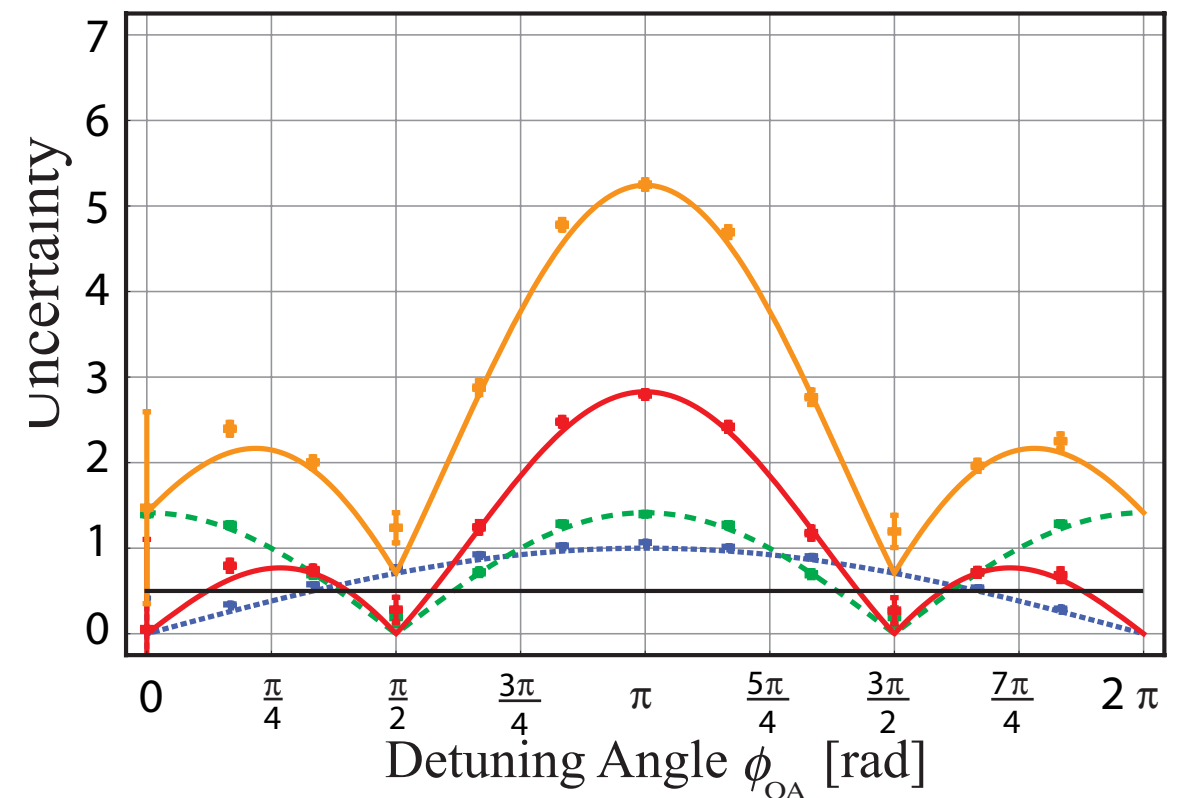
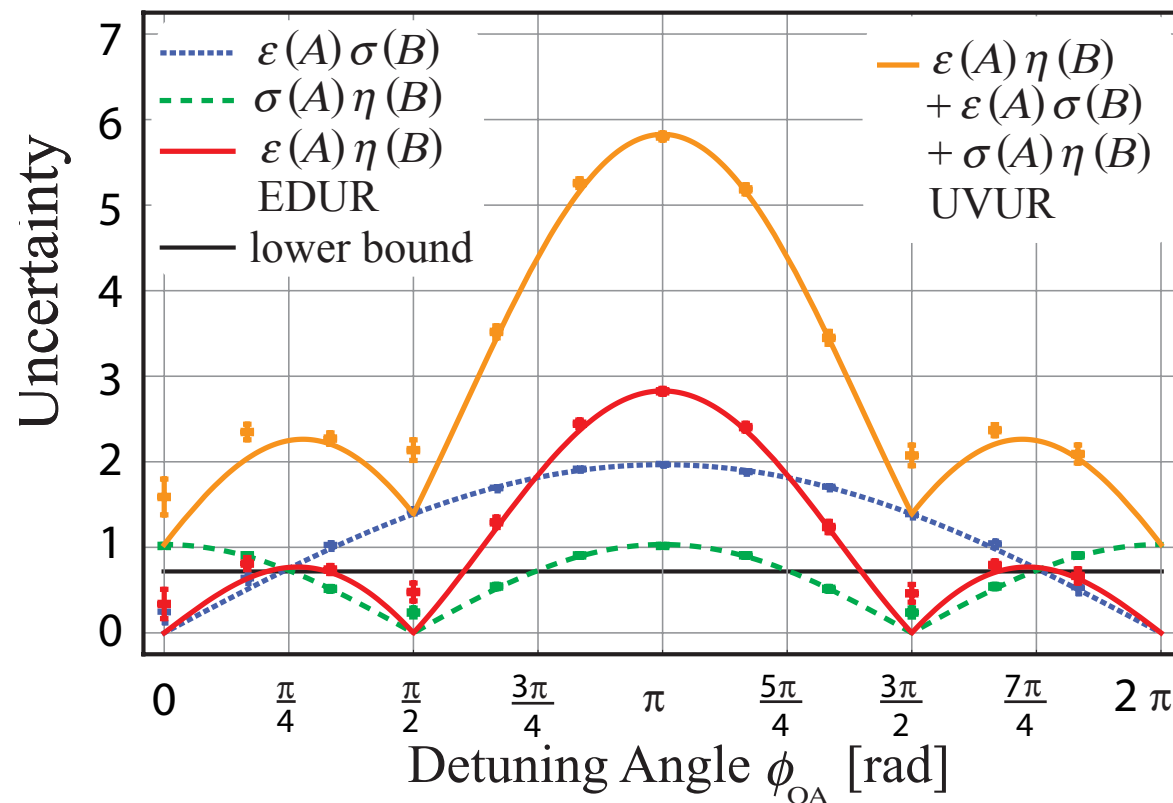
$$\sigma(B) = 0.988(2)$$



$$\theta_\psi = \frac{\pi}{3}, \phi_\psi = \frac{\pi}{2}$$

$$\sigma(A) = 1.000(1)$$

$$\sigma(B) = 0.528(11)$$



limit, standard deviations and UVUR change

error & disturbance state independent, EDUR unchanged

Sulyok, Sponar, Erhart, Badurek, Ozawa, Hasegawa, *Phys. Rev. A* **88**, 022110 (2013).

Steinberg Experiment - Weak Values of Single-Photon Polarization

PRL 109, 100404 (2012)

PHYSICAL REVIEW LETTERS

week ending
7 SEPTEMBER 2012



Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements

Lee A. Rozema, Ardavan Darabi, Dylan H. Mahler, Alex Hayat, Yasaman Soudagar, and Aeephraim M. Steinberg
Centre for Quantum Information & Quantum Control and Institute for Optical Sciences, Department of Physics, 60 St. George Street,
University of Toronto, Toronto, Ontario, Canada M5S 1A7
(Received 4 July 2012; published 6 September 2012)

While there is a rigorously proven relationship about uncertainties intrinsic to any quantum system, often referred to as “Heisenberg’s uncertainty principle,” Heisenberg originally formulated his ideas in terms of a relationship between the precision of measurement and the disturbance it must create. Although this latter relationship is not rigorously proven, it is commonly believed (and taught) as an aspect of the broader uncertainty principle. Here, we experimentally observe a violation of Heisenberg’s “measurement-disturbance relationship”, using weak measurements to characterize a quantum system before and after it interacts with a measurement apparatus. Our experiment implements a 2010 proposal of Lund and Wiseman to confirm a revised measurement-disturbance relationship derived by Ozawa in 2003. Its results have broad implications for the foundations of quantum mechanics and for practical issues in quantum measurement.

DOI: [10.1103/PhysRevLett.109.100404](https://doi.org/10.1103/PhysRevLett.109.100404)

PACS numbers: 03.65.Ta, 03.67.Ac, 42.50.Xa



A. M. Steinberg *et al.*, *Phys.Rev.Lett.* **109**, 100404 (2012).

04.03.2014

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Experimental Investigations (three state method & weak measurements)

PRL 109,

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SUBJECT AREAS:
QUANTUM MECHANICS
QUANTUM METROLOGY
QUANTUM INFORMATION
QUANTUM OPTICS

Received
7 August 2012

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2 July 2013

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17 July 2013

Experimental violation and reformulation of the Heisenberg's error-disturbance uncertainty relation

So-Young Baek^{1*}, Fumihiro Kaneda¹, Masanao Ozawa² & Keiichi Edamatsu¹

¹Research Institute of Electrical Communication, Tohoku University, Sendai 980-8577, Japan, ²Graduate School of Information Science, Nagoya University, Nagoya 464-8601, Japan.

The uncertainty principle formulated by Heisenberg in 1927 describes a trade-off between the error of a measurement of one observable and the disturbance caused on another complementary observable such that their product should be no less than the limit set by Planck's constant. However, Ozawa in 1988 showed a model of position measurement that breaks Heisenberg's relation and in 2003 revealed an alternative relation for error and disturbance to be proven universally valid. Here, we report an experimental test of Ozawa's relation for a single-photon polarization qubit, exploiting a more general class of quantum measurements than the class of projective measurements. The test is carried out by linear optical devices and realizes an indirect measurement model that breaks Heisenberg's relation throughout the range of our experimental parameter and yet validates Ozawa's relation.

K Edamatsu *et al.*, *Scientific Reports* **3**, 2221 (2013).

04.03.2014

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Experimental Investigations (three state method & weak measurements)

PRL 109,

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PRL 112, 020402 (2014)

PHYSICAL REVIEW LETTERS

week ending
17 JANUARY 2014



Experimental Test of Error-Disturbance Uncertainty Relations by Weak Measurement

Fumihiro Kaneda,^{1,*} So-Young Baek,^{1,†} Masanao Ozawa,² and Keiichi Edamatsu¹

¹Research Institute of Electrical Communication, Tohoku University, Sendai 980-8577, Japan

²Graduate School of Information Science, Nagoya University, Nagoya 464-8601, Japan

(Received 27 August 2013; published 15 January 2014)

We experimentally test the error-disturbance uncertainty relation (EDR) in generalized, strength-variable measurement of a single photon polarization qubit, making use of weak measurement that keeps the initial signal state practically unchanged. We demonstrate that the Heisenberg EDR is violated, yet the Ozawa and Branciard EDRs are valid throughout the range of our measurement strength.

DOI: 10.1103/PhysRevLett.112.020402

PACS numbers: 03.65.Ta, 03.67.-a, 42.50.Xa

The error-disturbance uncertainty relation (EDR) is one of the most fundamental issues in quantum mechanics since the EDR describes a peculiar limitation on measurements of quantum mechanical observables. In 1927, Heisenberg [1] argued that any measurement of the position Q of a particle with the error $\epsilon(Q)$ causes the disturbance $\eta(P)$ on its momentum P so that the product $\epsilon(Q)\eta(P)$ has a lower bound set by the Planck constant. The generalized form of the Heisenberg EDR for an arbitrary pair of observ-

where the average is taken in the state $|\psi\rangle_s \otimes |\xi\rangle_p$ of the signal-probe composite system, U is a unitary operator that provides interaction between the signal and probe systems, and M is the meter observable in the probe to be directly observed. The definition of $\epsilon(A)$ is uniquely derived from the classical notion of root-mean-square error if $U^\dagger(I \otimes M)U$ and $A \otimes I$ commute [11], and otherwise, it is considered as a natural quantization of the notion of classical root-mean-square error (see Supplemental

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Experimental Investigations (three state method & weak measurements)

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PRL 112, 020402 (2014)

PHYSICAL REVIEW LETTERS

week ending
17 JANUARY 2014



Experimental Test of Error-Disturbance Uncertainty Relations by Weak Measurement

Fumihiro Kaneda,^{1,*} So-Young Baek,^{1,†} Masanao Ozawa,² and Keiichi Edamatsu¹

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PRL 112, 020401 (2014)

PHYSICAL REVIEW LETTERS

week ending
17 JANUARY 2014



Experimental Joint Quantum Measurements with Minimum Uncertainty

Martin Ringbauer,^{1,2,*} Devon N. Biggerstaff,^{1,2} Matthew A. Broome,^{1,2} Alessandro Fedrizzi,^{1,2}
Cyril Branciard,¹ and Andrew G. White^{1,2}

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University of Queensland, Brisbane QLD 4072, Australia

(Received 27 August 2013; published 15 January 2014)

Quantum physics constrains the accuracy of joint measurements of incompatible observables. Here we test tight measurement-uncertainty relations using single photons. We implement two independent, idealized uncertainty-estimation methods, the three-state method and the weak-measurement method, and adapt them to realistic experimental conditions. Exceptional quantum state fidelities of up to 0.999 98(6) allow us to verge upon the fundamental limits of measurement uncertainty.

DOI: 10.1103/PhysRevLett.112.020401

PACS numbers: 03.65.Ta, 03.67.-a, 42.50.Ex, 42.50.Xa

The error-disturbance of the most fundamental the EDR describes a p of quantum mechanics [1] argued that any m particle with the error on its momentum P s lower bound set by th form of the Heisenberg

Measurement—assigning a number to a property of a physical system—is the keystone of the natural sciences. Our belief in perfect measurement precision was shattered by the paradigm shift heralded by quantum physics almost

given measurement apparatus; hence, their inaccuracies and disturbances are in general defined for different states. When both quantities are defined on the same quantum state, the relation $\epsilon_n > \hbar/2$ does not necessarily hold [5, 6]

04.03.2014

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Recent Development: alternative Definitions for Error & Disturbance



PHYSICAL REVIEW LETTERS

week ending
18 OCTOBER 2013



Proof of Heisenberg's Error-Disturbance Relation

Paul Busch,^{1,*} Pekka Lahti,^{2,†} and Reinhard F. Werner^{3,‡}

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²Turku Centre for Quantum Physics, Department of Physics and Astronomy, University of Turku, FI-20014 Turku, Finland

³Institut für Theoretische Physik, Leibniz Universität, D-30167 Hannover, Germany

(Received 7 June 2013; published 17 October 2013)

While the slogan “no measurement without disturbance” has established itself under the name of the Heisenberg effect in the consciousness of the scientifically interested public, a precise statement of this fundamental feature of the quantum world has remained elusive, and serious attempts at rigorous formulations of it as a consequence of quantum theory have led to seemingly conflicting preliminary results. Here we show that despite recent claims to the contrary [L. Rozemaal, Phys. Rev. Lett. 109, 100404 (2012)], Heisenberg-type inequalities can be proven that describe a tradeoff between the precision of a position measurement and the necessary resulting disturbance of momentum (and vice versa). More generally, these inequalities are instances of an uncertainty relation for the imprecisions of joint measurement of position and momentum. Measures of error and disturbance are here defined as figures of merit characteristic of measuring devices. As such they are **state independent** each giving worst-case estimates across all states, in contrast to previous work that is concerned with the relationship between error and disturbance in an individual state.

DOI: [10.1103/PhysRevLett.111.160405](https://doi.org/10.1103/PhysRevLett.111.160405)

PACS numbers: 03.65.Ta, 03.65.Db, 03.67. -a

P. Busch *et al.*, *Phys. Rev. Lett.* **111**, 160405 (2013).

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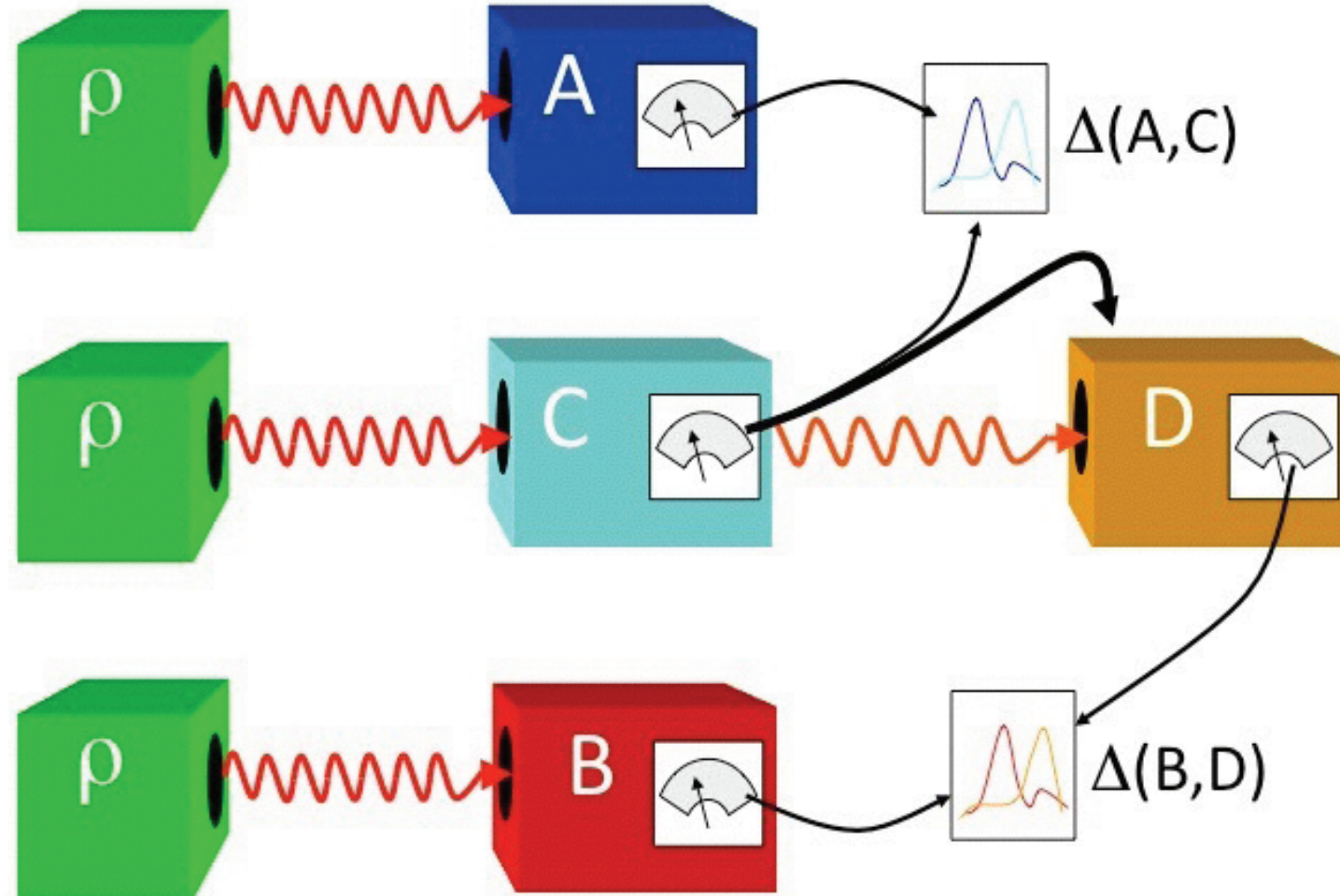
PACS numbers: 03.65.Ta, 03.65.Db, 03.67. -a

P. Busch *et al.*, *Phys. Rev. Lett.* **111**, 160405 (2013).

Theory-Paper: alternative Definitions for Error & Disturbance

state-independent model:

- *individual measurements (ideal) compared with approximate measurements;*
- a definition for the *distance of two general probability distributions* is required



P. Busch *et al.*, *Phys. Rev. Lett.* **111**, 160405 (2013).

Outline

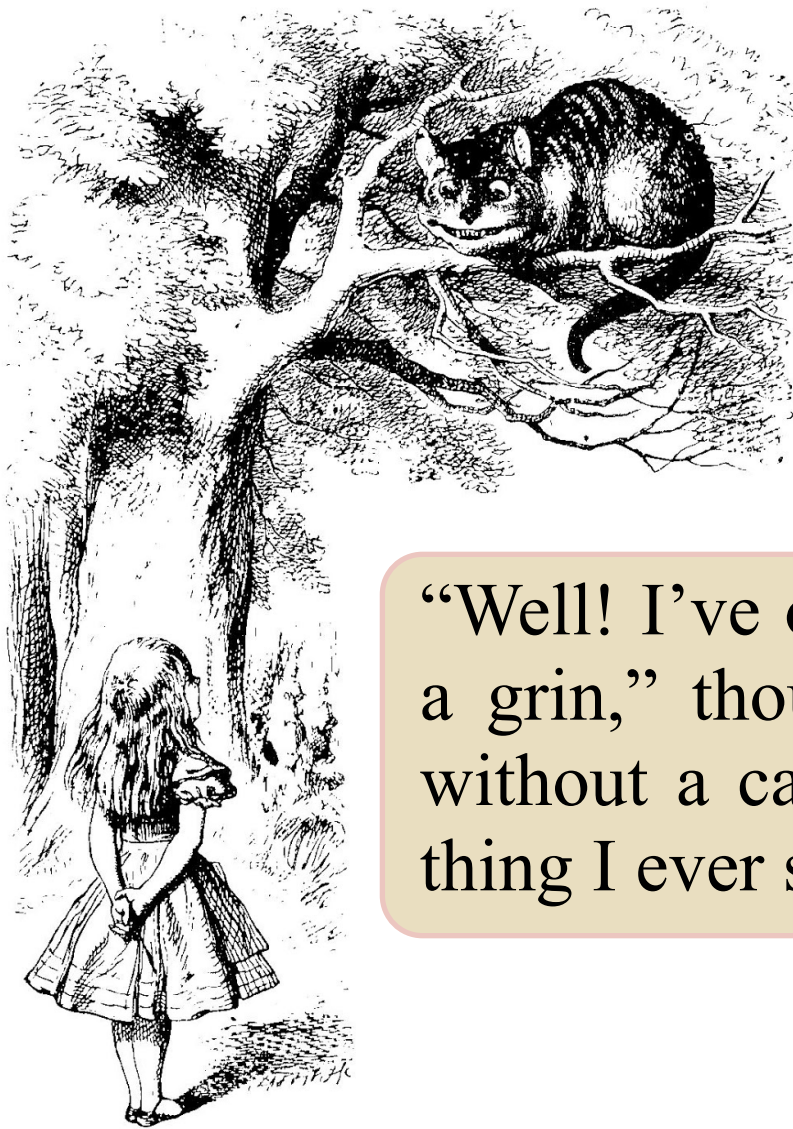
- **Part I: Violation Error-Disturbance Uncertainty Relation** ✓
 - Theory: Ozawa's universally valid Error-Disturbance Uncertainty Relation (UVUR) ✓
 - Violation of Heisenberg's Error-Disturbance Uncertainty Relation & Validity of Ozawa's UVUR in Neutron Spin Measurements ✓
- **Part II: Hunting for the *Quantum Cheshire Cat***
 - New Quantum Paradox & Neutron Interferometric Observation
- **Summary**

Cheshire Cat & Alice in Wonderland



L. Carroll [C. L. Dodgson], *Alice's Adventures In Wonderland* (1965) pp. 90-91

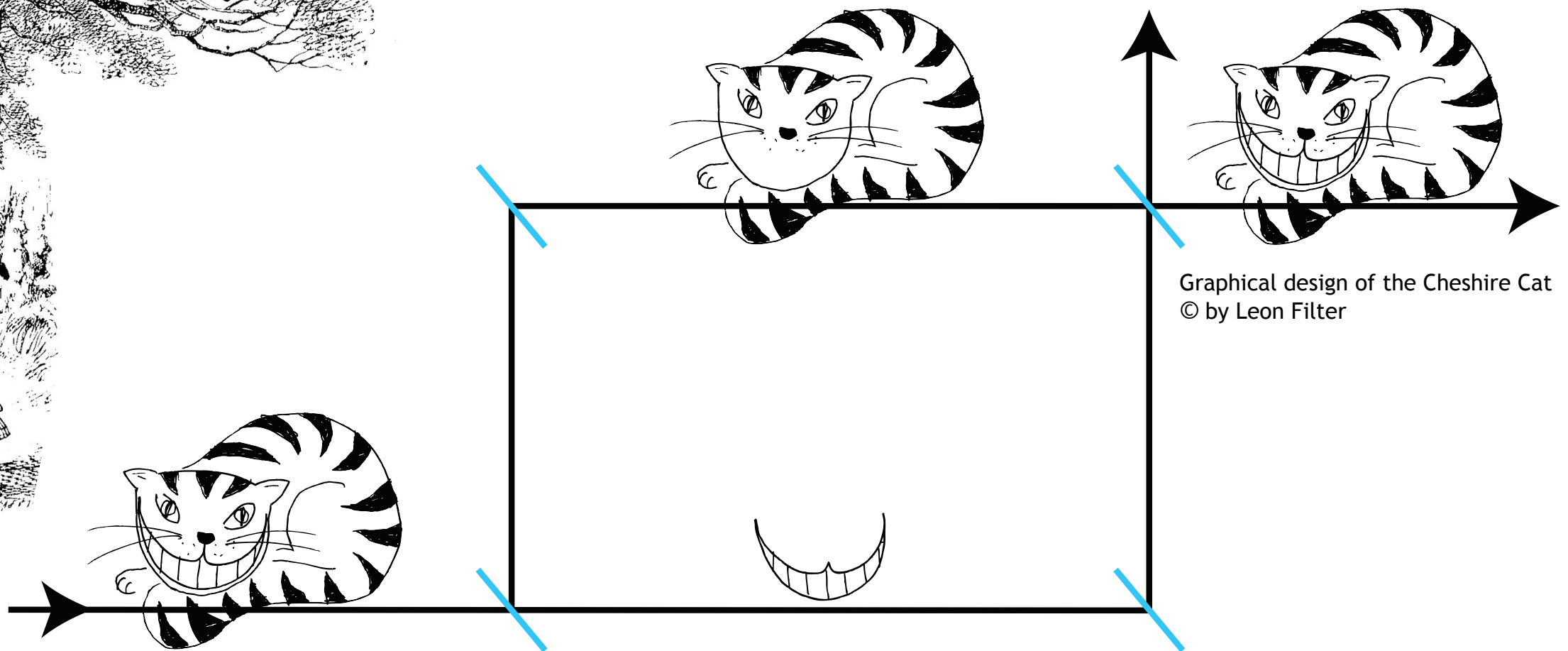
Cheshire Cat & Alice in Wonderland



“Well! I’ve often seen a cat without a grin,” thought Alice; “but a grin without a cat! It’s the most curious thing I ever saw in all my life!”

Cheshire Cat & Alice in Wonderland

Quantum Cheshire Cat



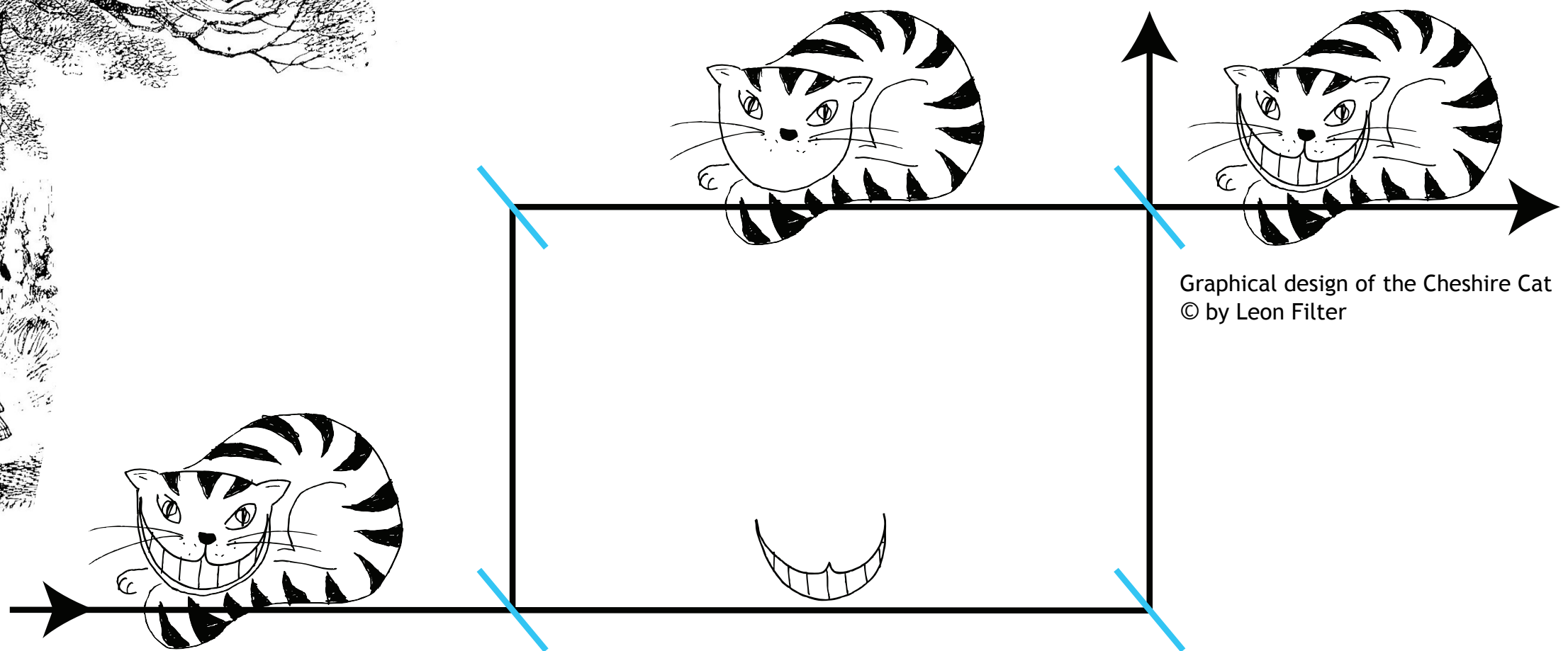
L. Carroll [C. L. Dodgson], *Alice's Adventures In Wonderland* (1965) pp. 90-91; Y. Aharonov, et al., *New J. Phys.***15** (2013) 113018

Cheshire Cat & Alice in Wonderland



Quantum Cheshire Cat

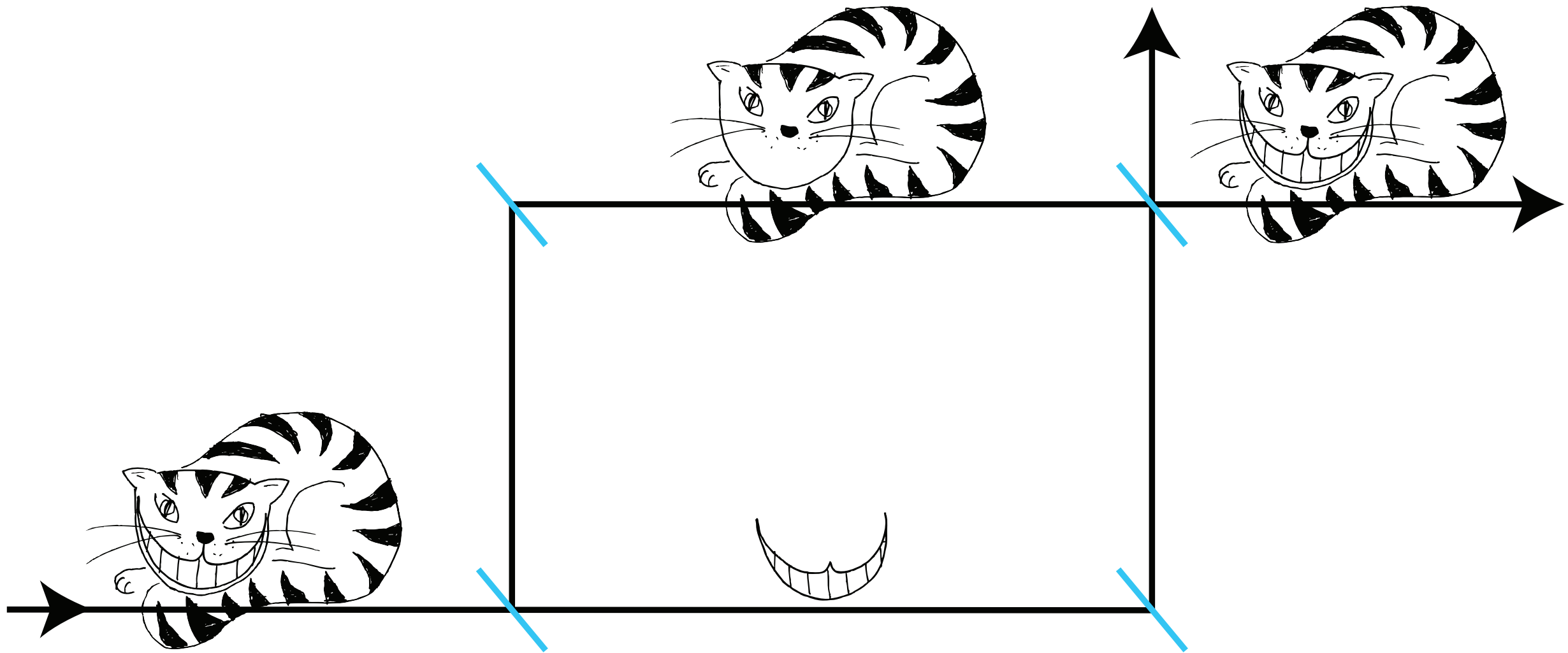
Detanglement of particle & property!



Graphical design of the Cheshire Cat
© by Leon Filter

L. Carroll [C. L. Dodgson], *Alice's Adventures In Wonderland* (1965) pp. 90-91; Y. Aharonov, et al., *New J. Phys.* **15** (2013) 113018

Quantum Cheshire Cat



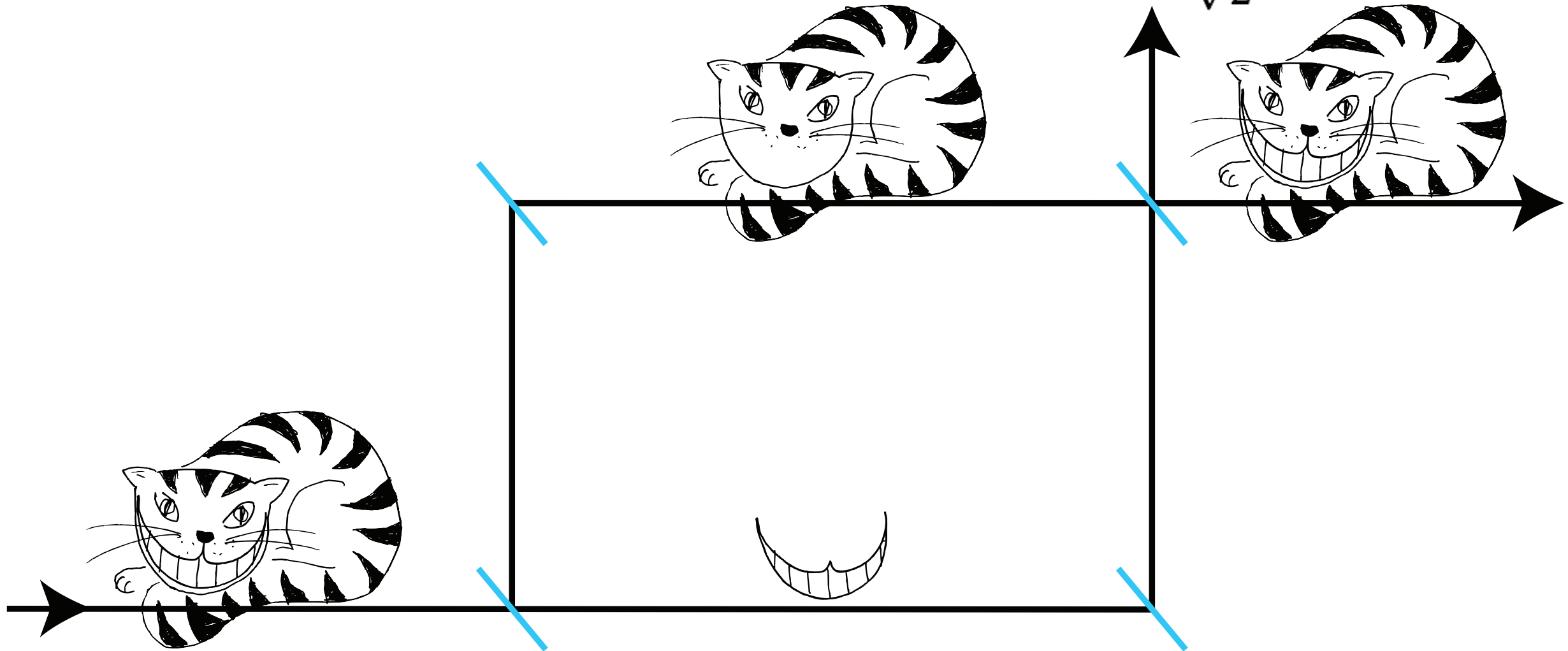
● **Pre-Selection:** $|\psi_i\rangle = \frac{1}{\sqrt{2}} (|S_x; +\rangle |I\rangle + |S_x; -\rangle |II\rangle)$

Y. Aharonov, et al., *New J. Phys.* **15** (2013) 113018

Quantum Cheshire Cat

● **Post-Selection:**

$$\langle \psi_f | = \frac{1}{\sqrt{2}} \langle S_x; - | (\langle I | + \langle II |)$$



● **Pre-Selection:** $|\psi_i\rangle = \frac{1}{\sqrt{2}} (|S_x; +\rangle |I\rangle + |S_x; -\rangle |II\rangle)$

Y. Aharonov, et al., *New J. Phys.* **15** (2013) 113018

Quantum Cheshire Cat

$$\langle \hat{\Pi}_{II} \rangle_{\text{w}} = 1$$

$$\langle \hat{\sigma}_z^S \hat{\Pi}_{II} \rangle_{\text{w}} = 0$$

w denotes

Weak Values

● **Post-Selection:**

$$\langle \psi_f | = \frac{1}{\sqrt{2}} \langle S_x; - | (\langle I | + \langle II |)$$

$$\langle \hat{A} \rangle_{\text{w}} := \frac{\langle \psi_f | \hat{A} | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle}$$

$$\langle \hat{\Pi}_I \rangle_{\text{w}} = 0$$

$$\langle \hat{\sigma}_z^S \hat{\Pi}_I \rangle_{\text{w}} = 1$$

● **Pre-Selection:** $|\psi_i\rangle = \frac{1}{\sqrt{2}} (|S_x; +\rangle |I\rangle + |S_x; -\rangle |II\rangle)$

How the Result of a Measurement of a Component of the Spin of a Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

*Physics Department, University of South Carolina, Columbia, South Carolina 29208, and
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel*

(Received 30 June 1987)



New Journal of Physics

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Quantum Cheshire Cats

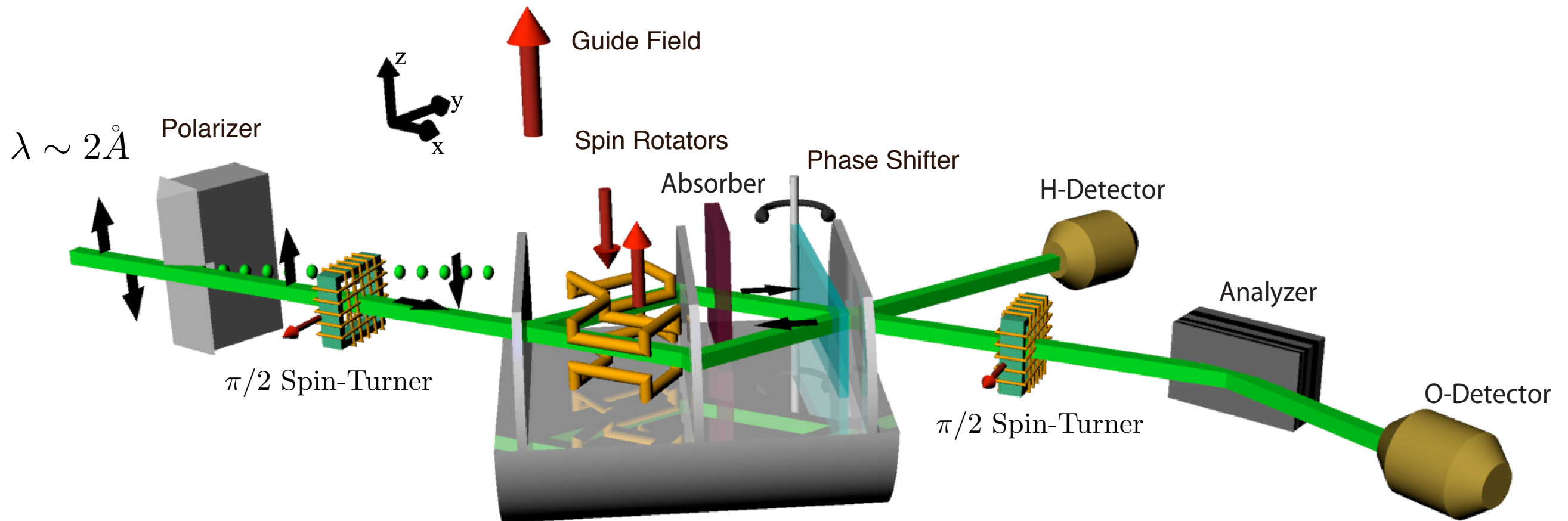
Yakir Aharonov^{1,2}, Sandu Popescu^{3,6}, Daniel Rohrlich^{4,6}
and Paul Skrzypczyk⁵

Abstract. In this paper we present a quantum Cheshire Cat. In a pre- and post-selected experiment we find the Cat in one place, and its grin in another. The Cat is a photon, while the grin is its circular polarization.

New Journal of Physics **15** (2013) 113015 (8pp)

National Medal of Science, awarded and presented by President Barak Obama @ the White House (1.Feb. 2013).

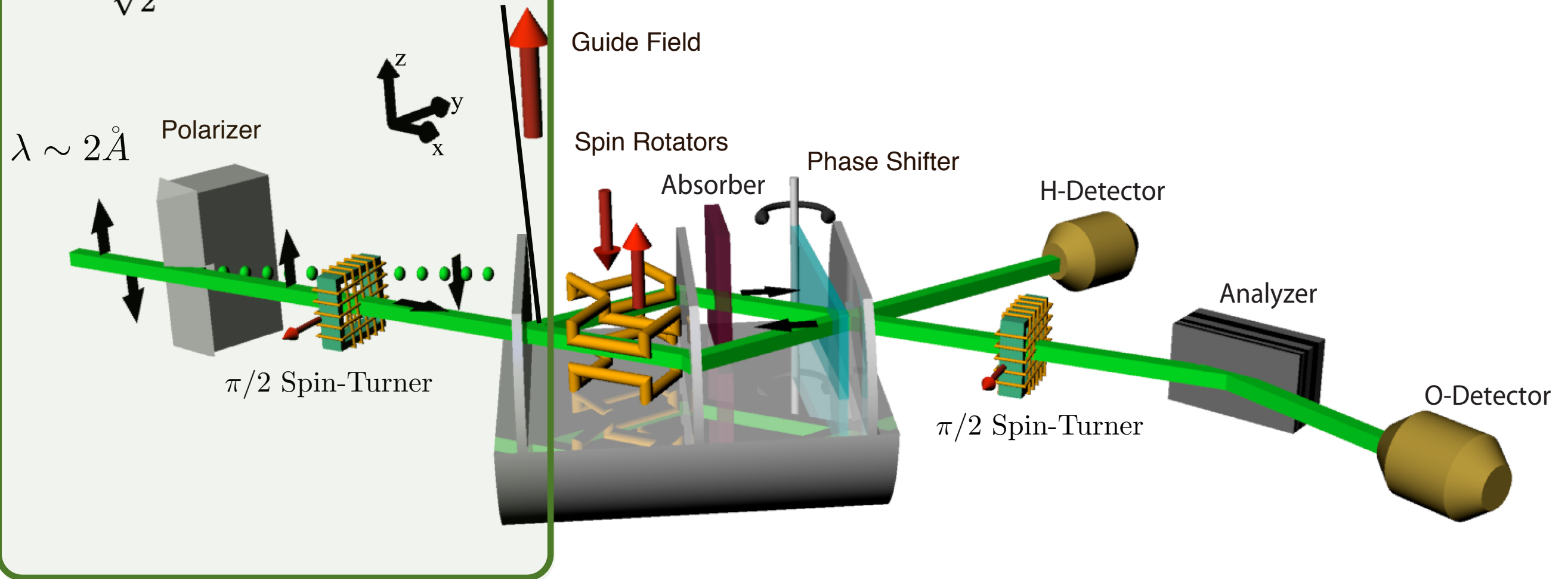
Quantum Cheshire Cat & Weak Measurements



Quantum Cheshire Cat & Weak Measurements

● Pre-Selection:

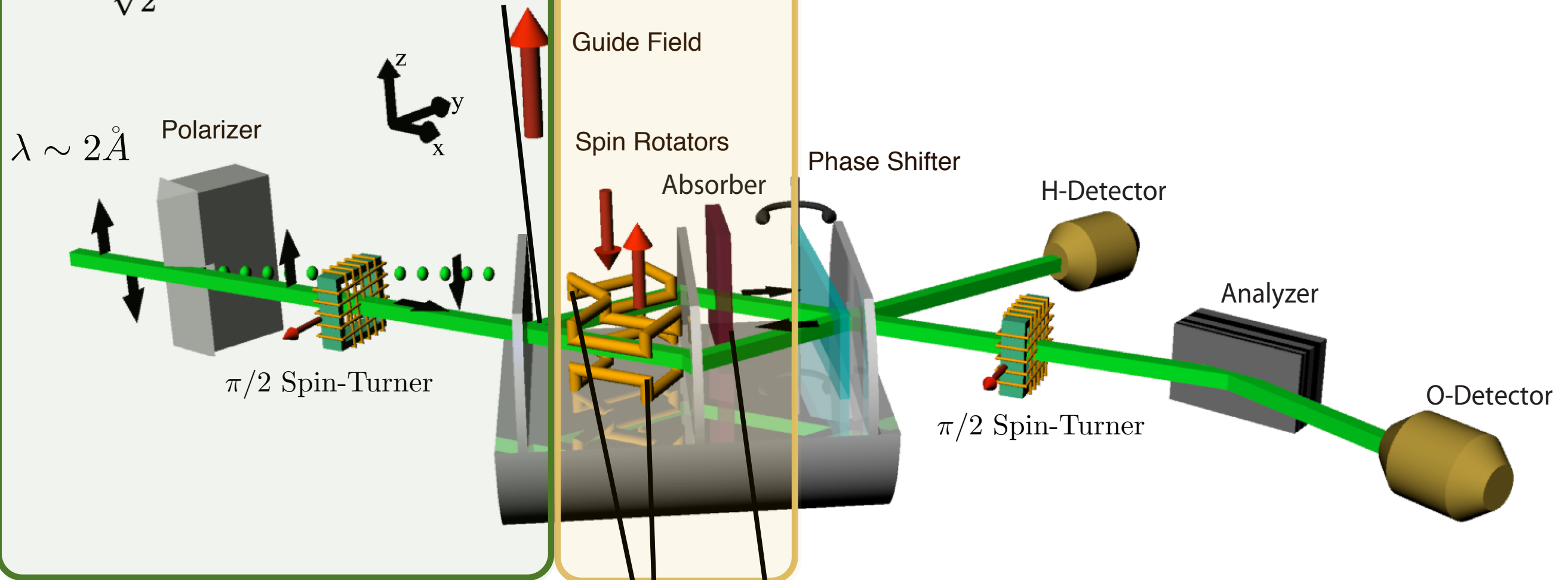
$$|\psi_i\rangle = \frac{1}{\sqrt{2}} (|S_x; +\rangle |I\rangle + |S_x; -\rangle |II\rangle)$$



Quantum Cheshire Cat & Weak Measurements

Pre-Selection:

$$|\psi_i\rangle = \frac{1}{\sqrt{2}} (|S_x; +\rangle |I\rangle + |S_x; -\rangle |II\rangle)$$



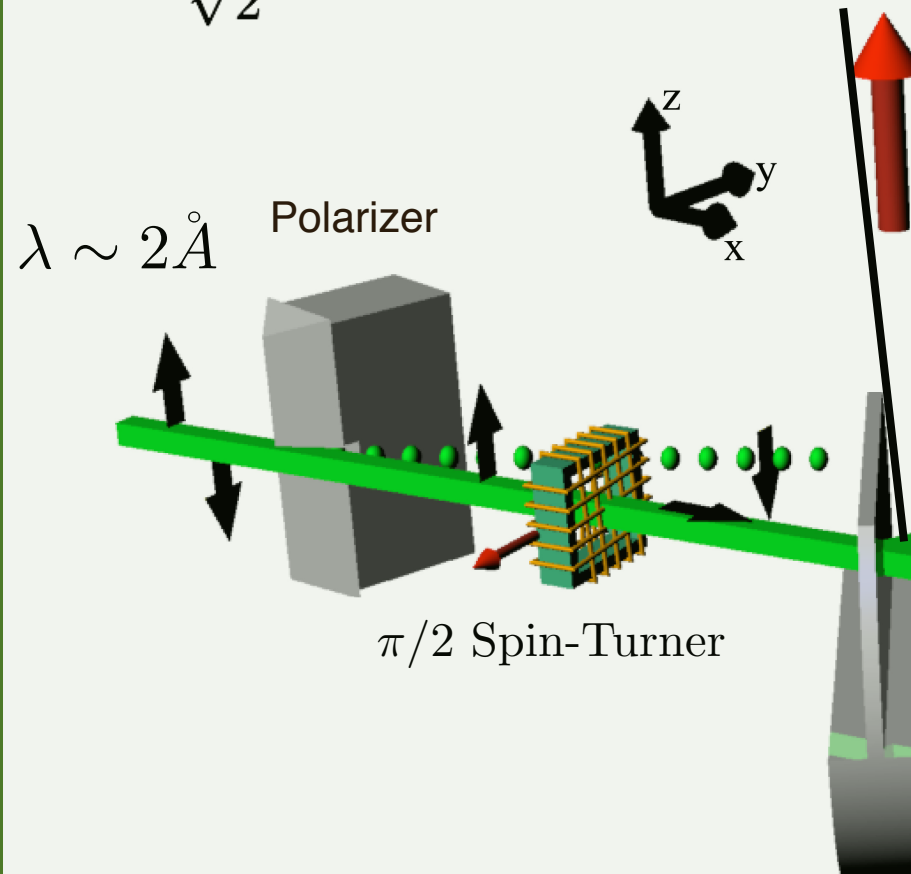
imaginary optical potential $\hat{V}_j = -i\mu_j(r)\hat{\Pi}_j$ (path j)

path conditioned interaction Hamiltonian $\hat{H}_j = -\gamma \frac{\hat{\sigma}_z}{2} B_z \hat{\Pi}_j$

Quantum Cheshire Cat & Weak Measurements

Pre-Selection:

$$|\psi_i\rangle = \frac{1}{\sqrt{2}} (|S_x; +\rangle |I\rangle + |S_x; -\rangle |II\rangle)$$

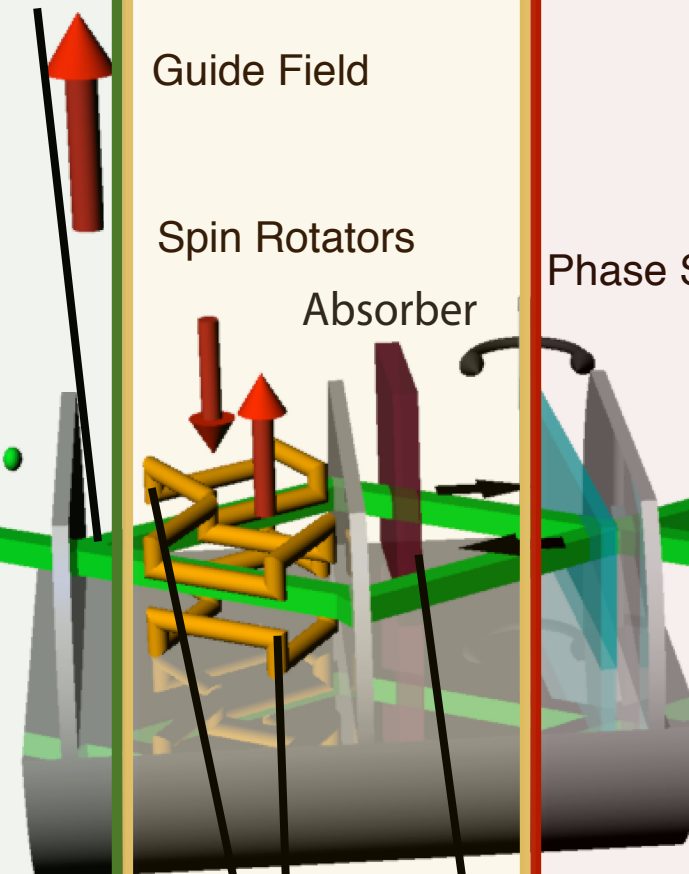


Weak Interaction

Guide Field

Spin Rotators

Absorber



Phase Shifter

Post-Selection:

$$\langle\psi_f| = \frac{1}{\sqrt{2}} \langle S_x; - | (\langle I | + \langle II |)$$

$$\langle\hat{A}\rangle_w := \frac{\langle\psi_f|\hat{A}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle}$$

H-Detector

Analyzer

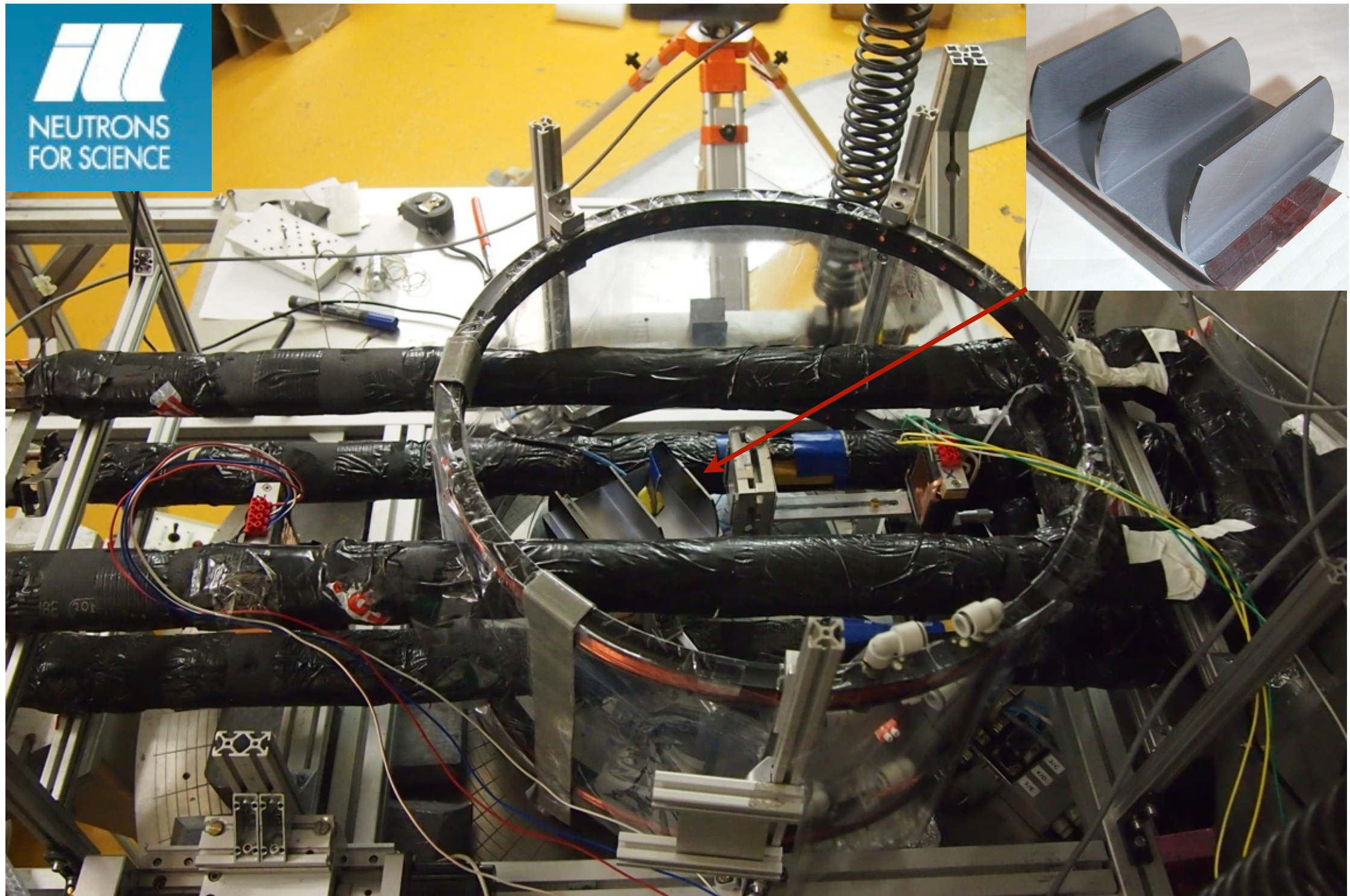
π/2 Spin-Turner

O-Detector

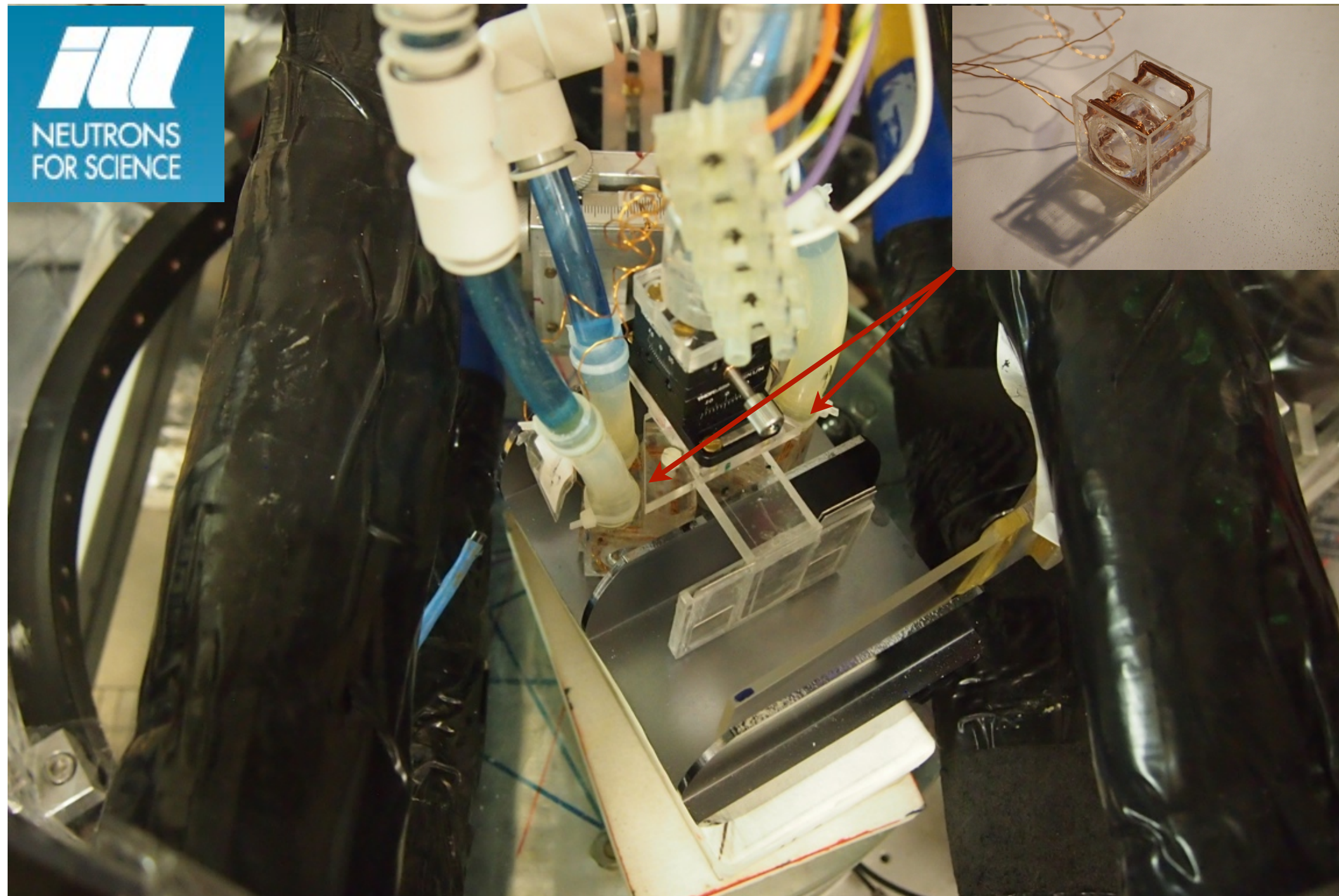
imaginary optical potential $\hat{V}_j = -i\mu_j(r)\hat{\Pi}_j$ (path j)

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Quantum Cheshire Cat: Experimental Setup @ SI 8 - ILL

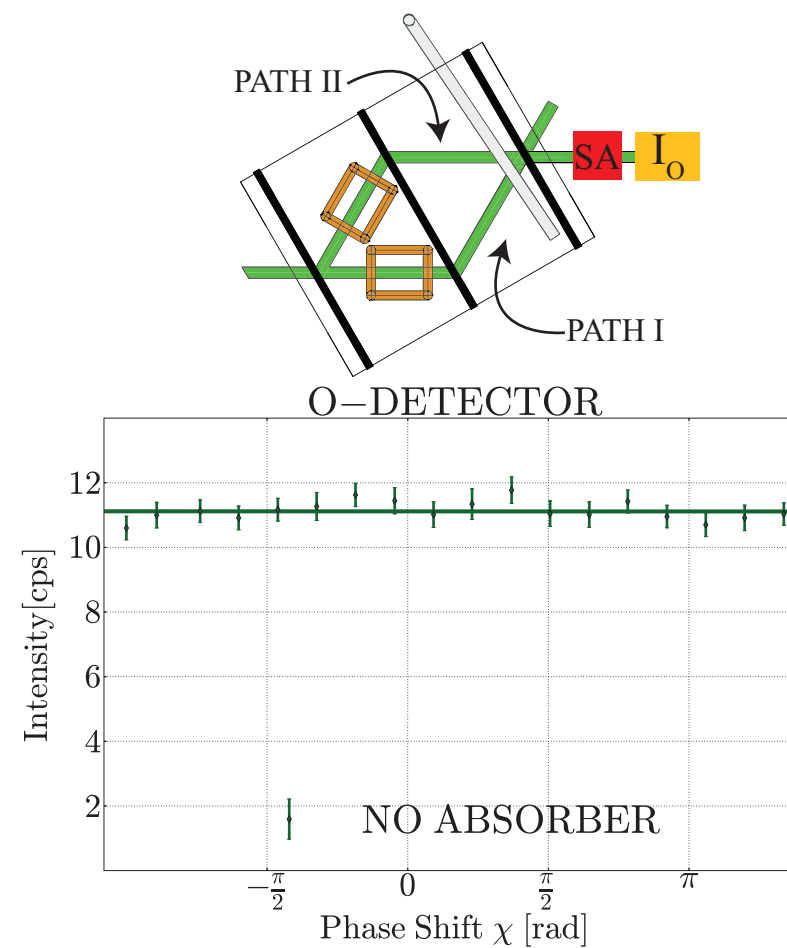


Quantum Cheshire Cat: Experimental Setup

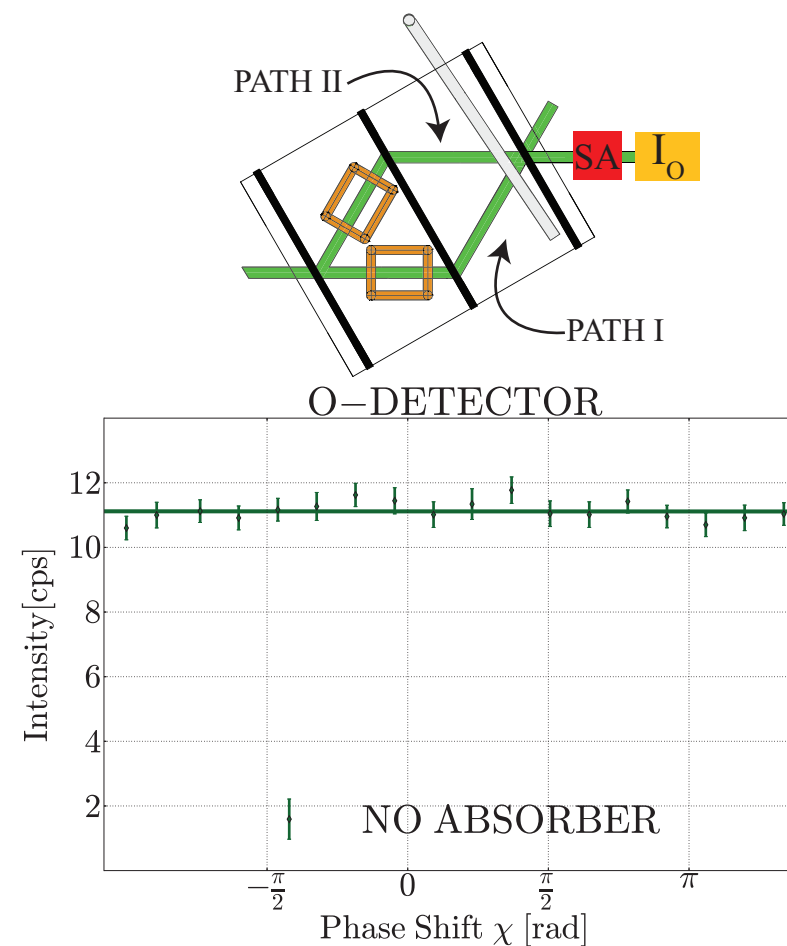
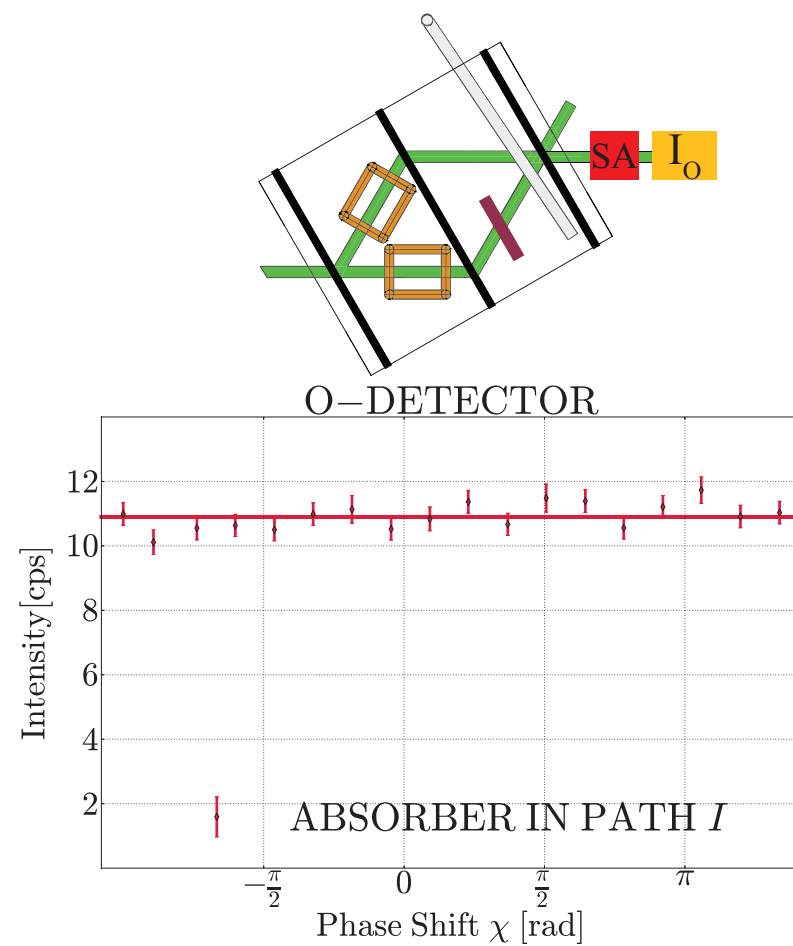


Experimental Results: Weak Path Measurement using Absorber

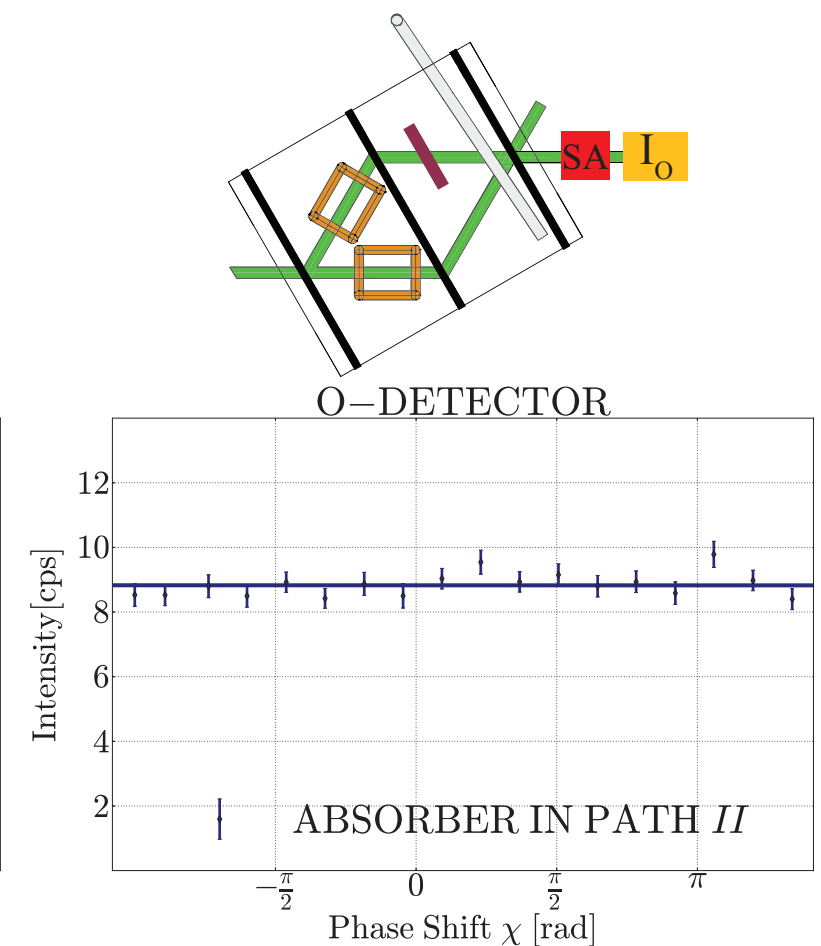
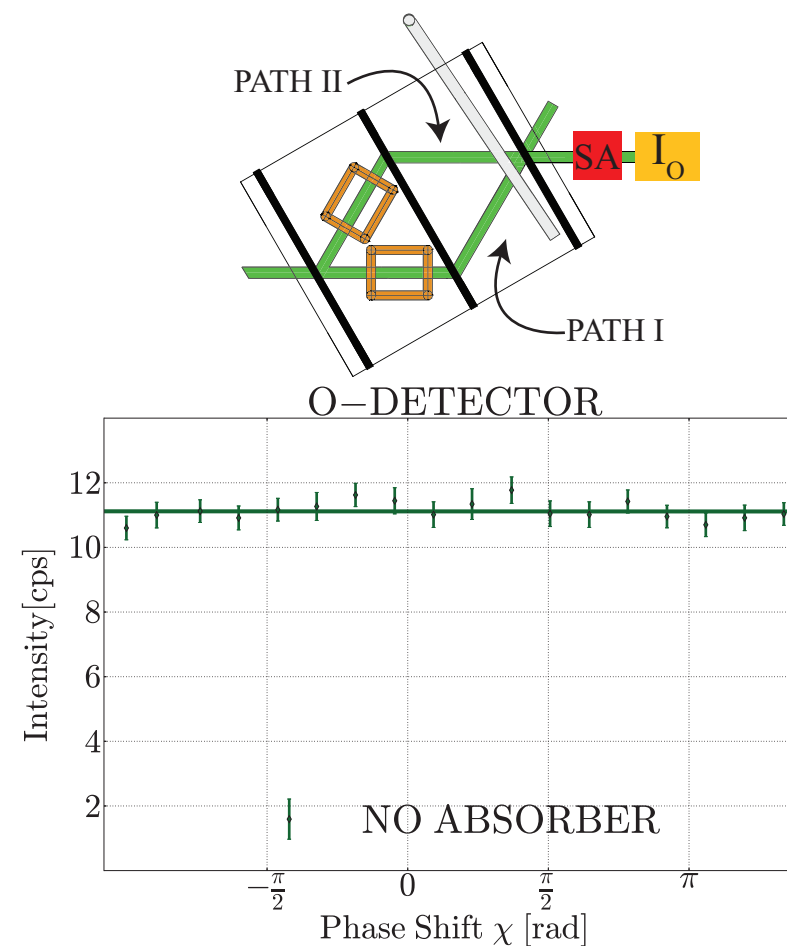
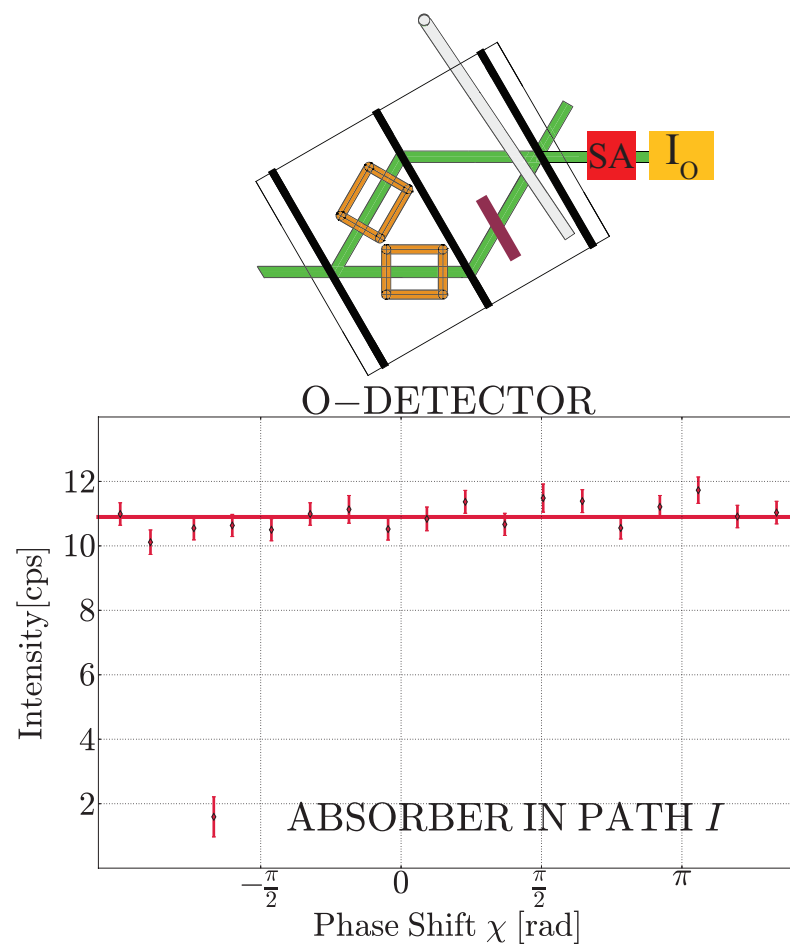
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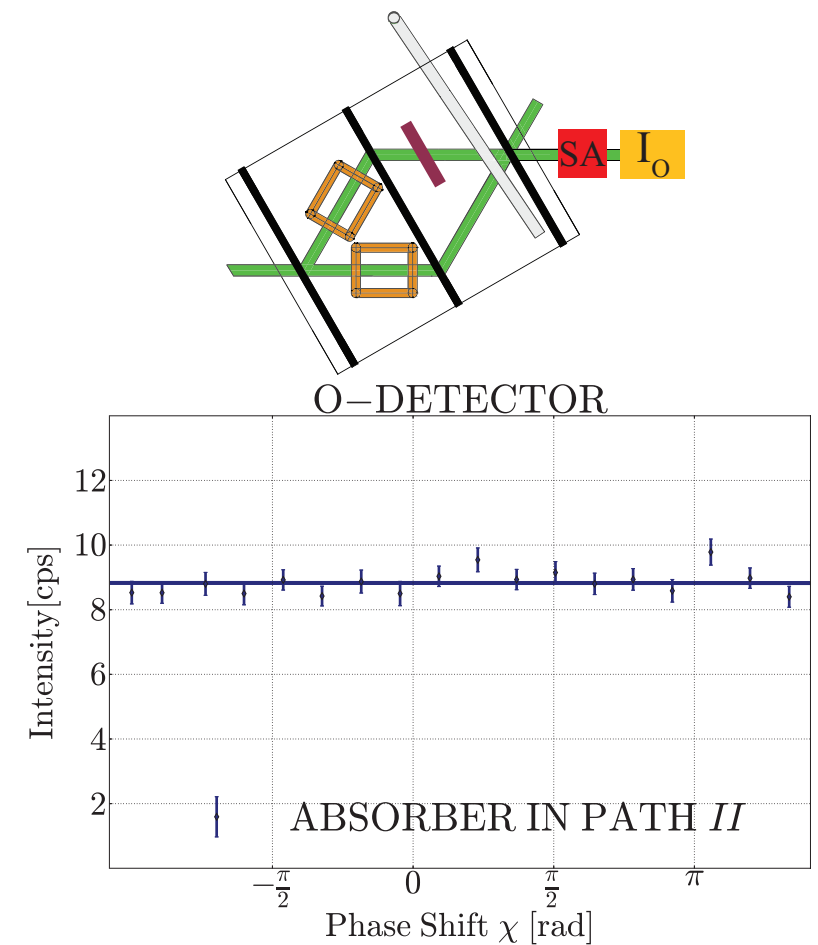
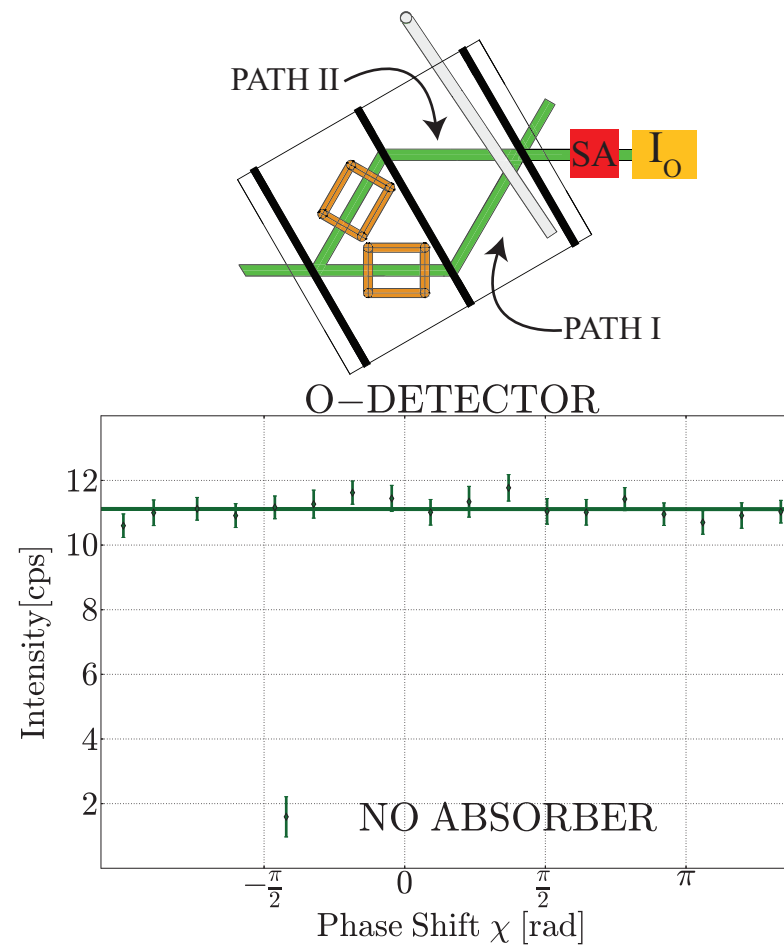
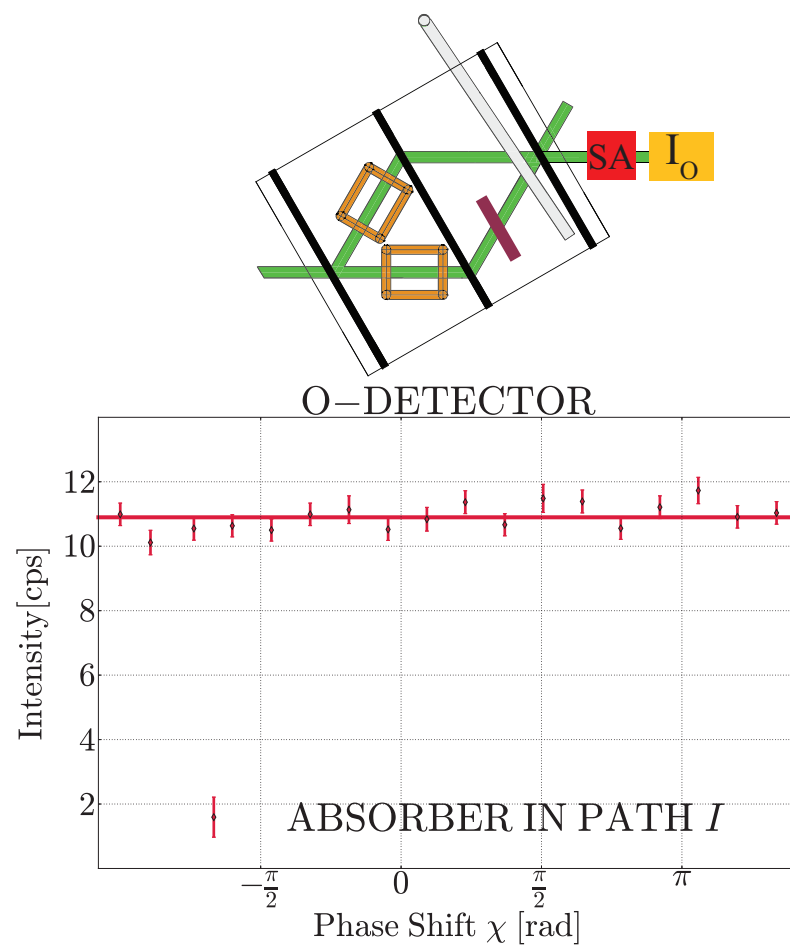
Experimental Results: Weak Path Measurement using Absorber



Experimental Results: Weak Path Measurement using Absorber



Experimental Results: Weak Path Measurement using Absorber



$$\begin{aligned} \langle \hat{\Pi}_I \rangle_w &= 0.14 \pm 0.04 \\ \langle \hat{\Pi}_{II} \rangle_w &= 0.96 \pm 0.06 \end{aligned}$$



arXiv:1312.3775 [quant-ph]

04.03.2014

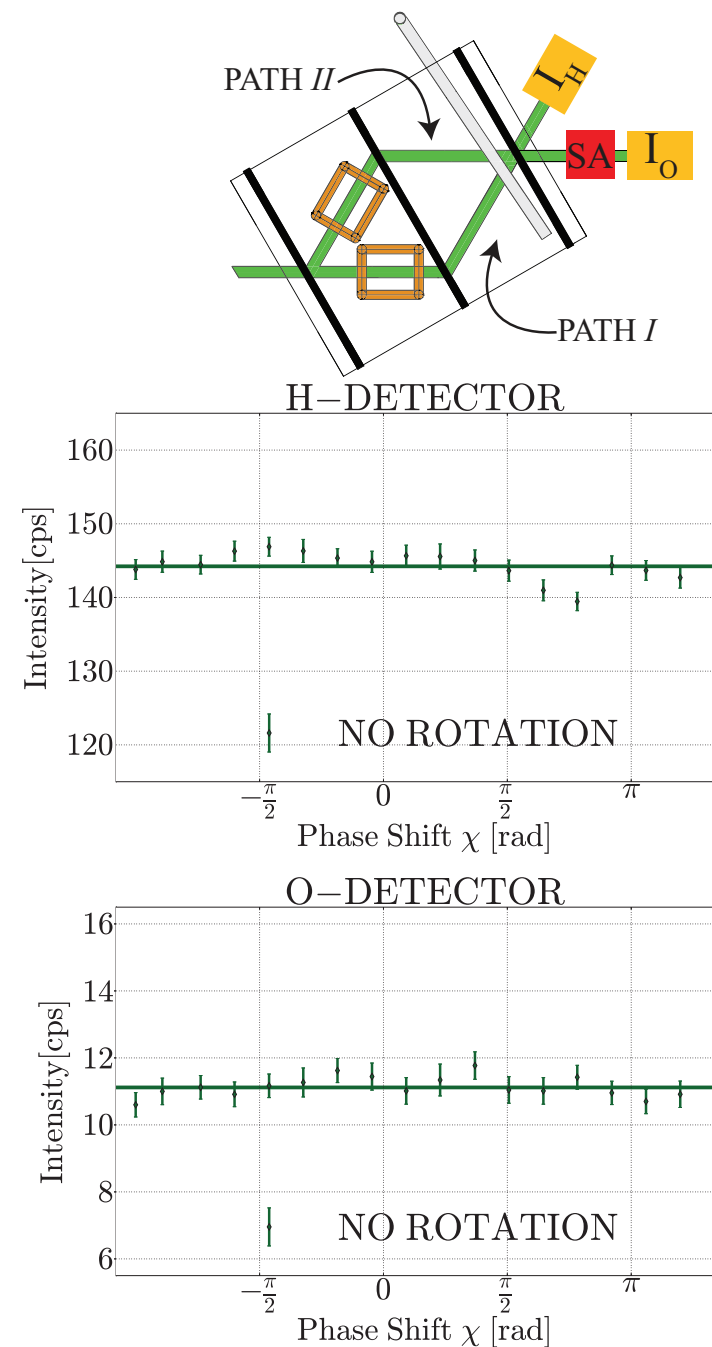
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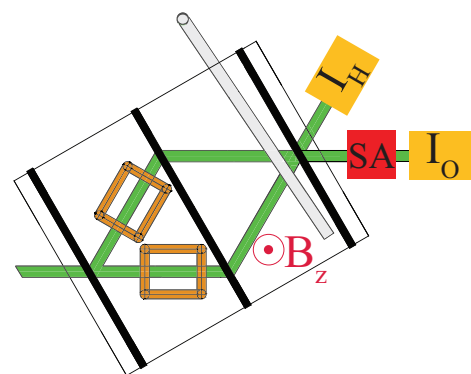
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Experimental Results: Weak Spin Measurement along Path I,II

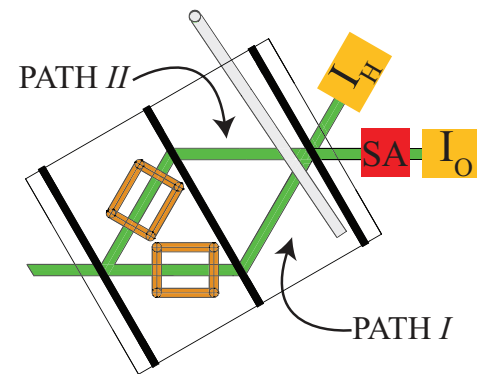
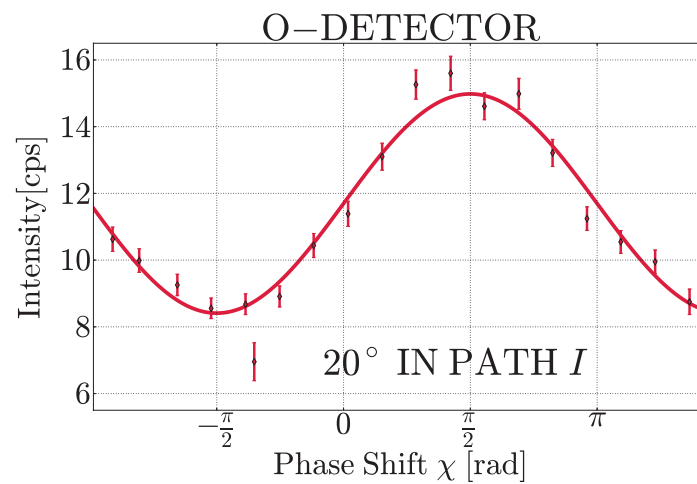
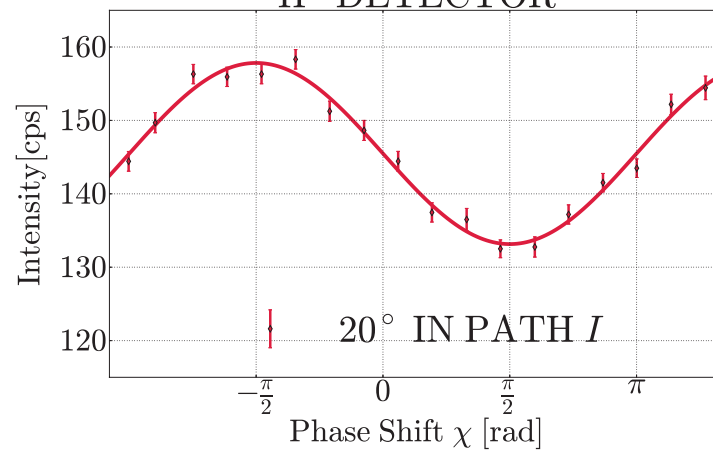
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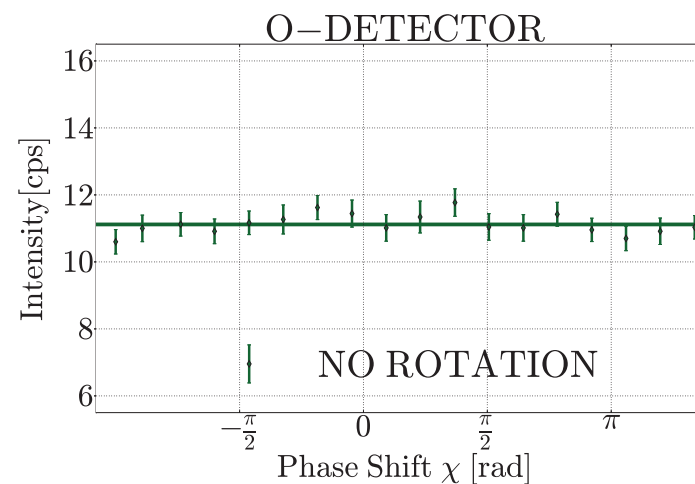
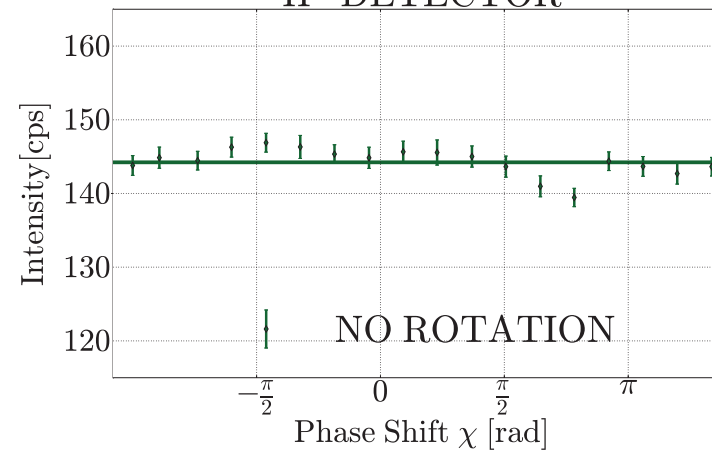
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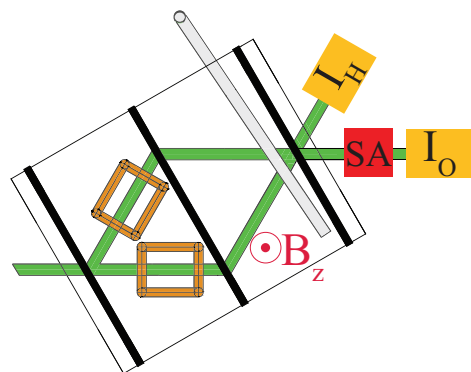
H-DETECTOR



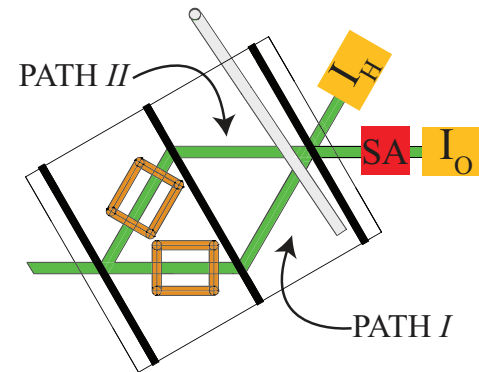
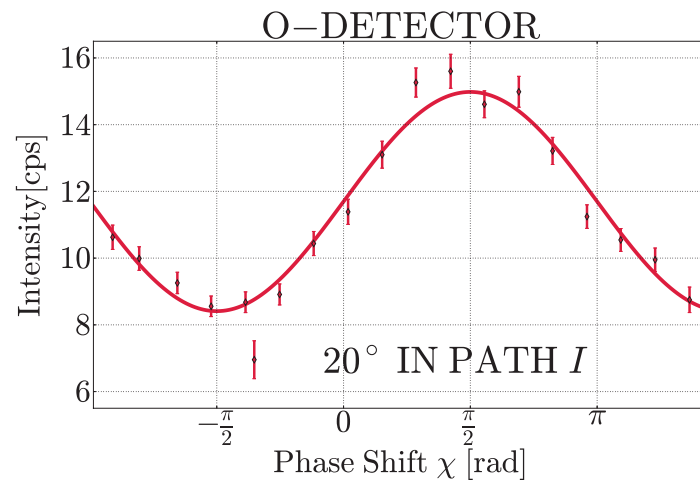
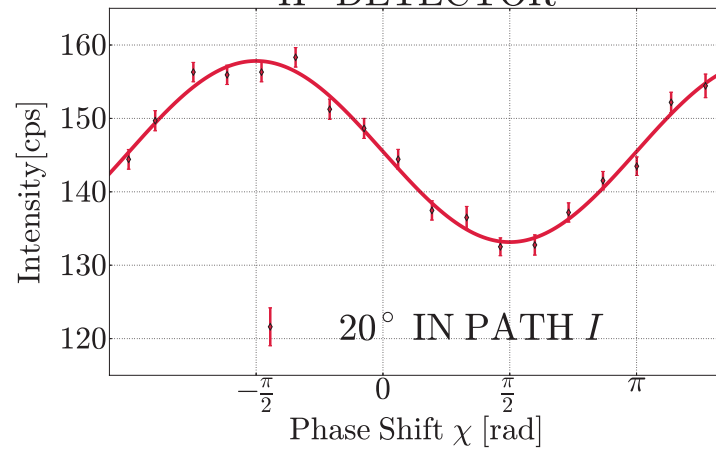
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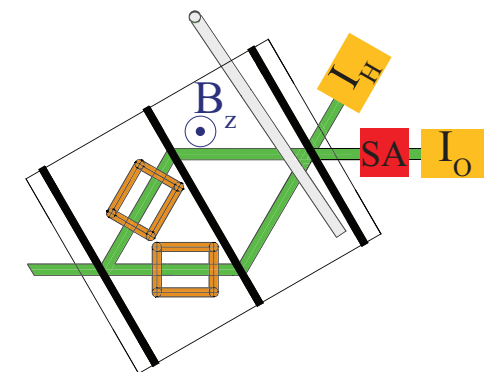
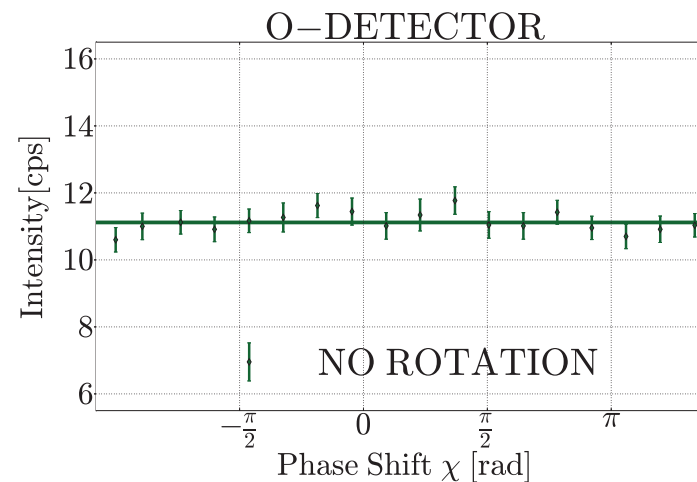
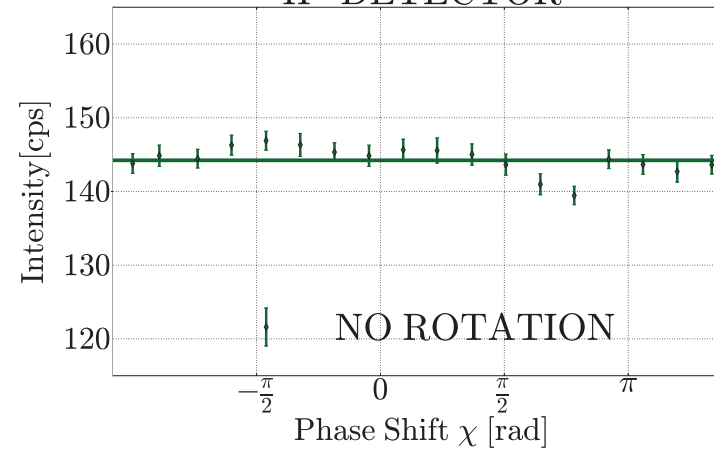
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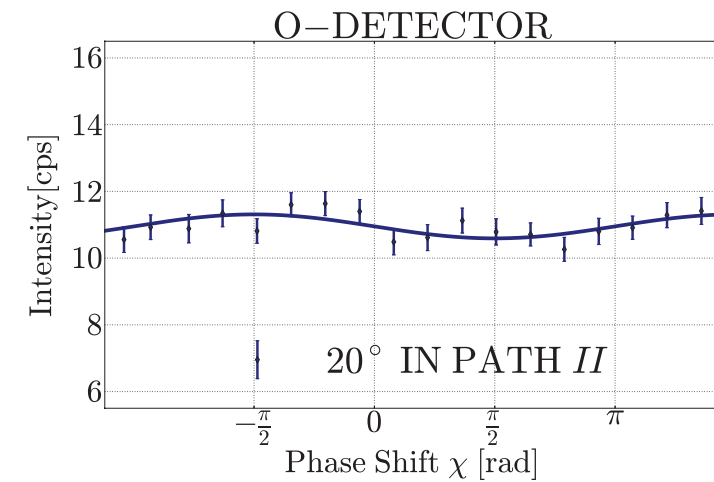
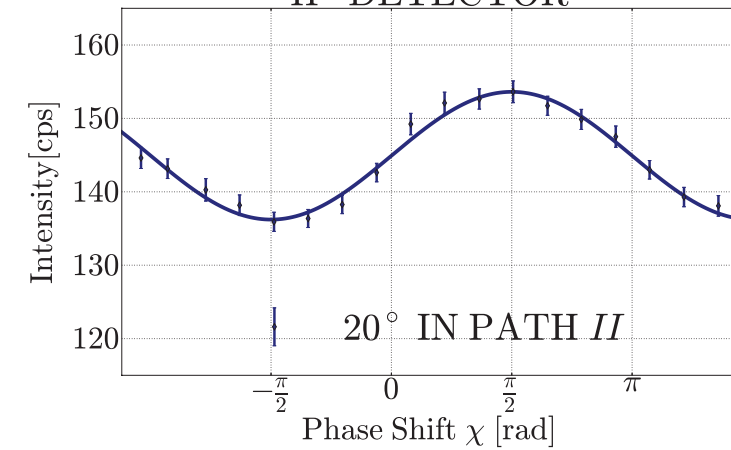
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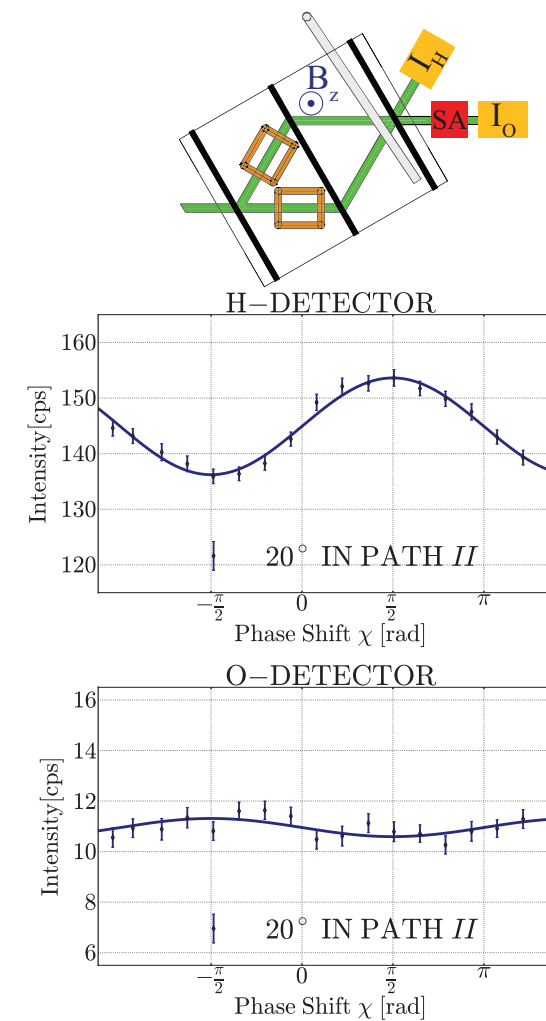
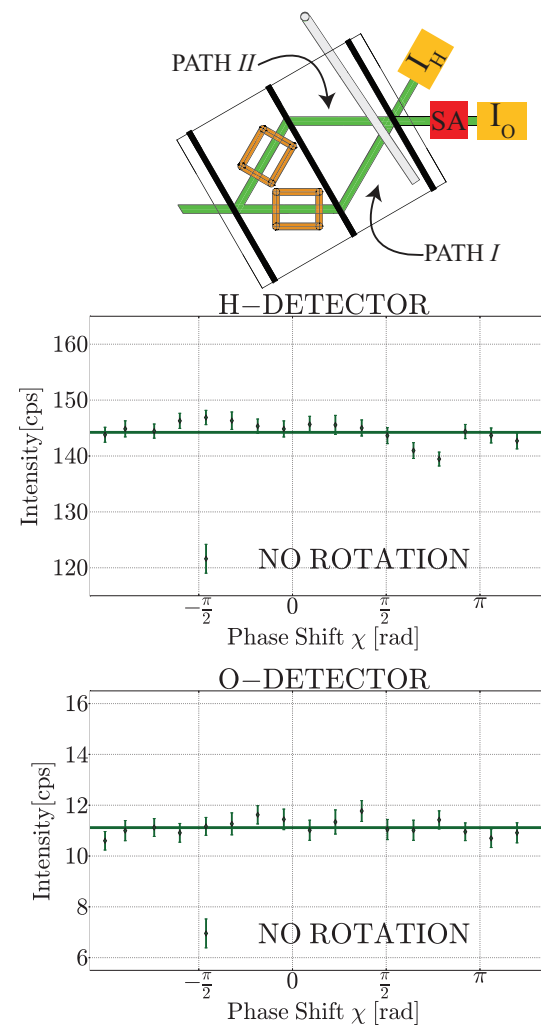
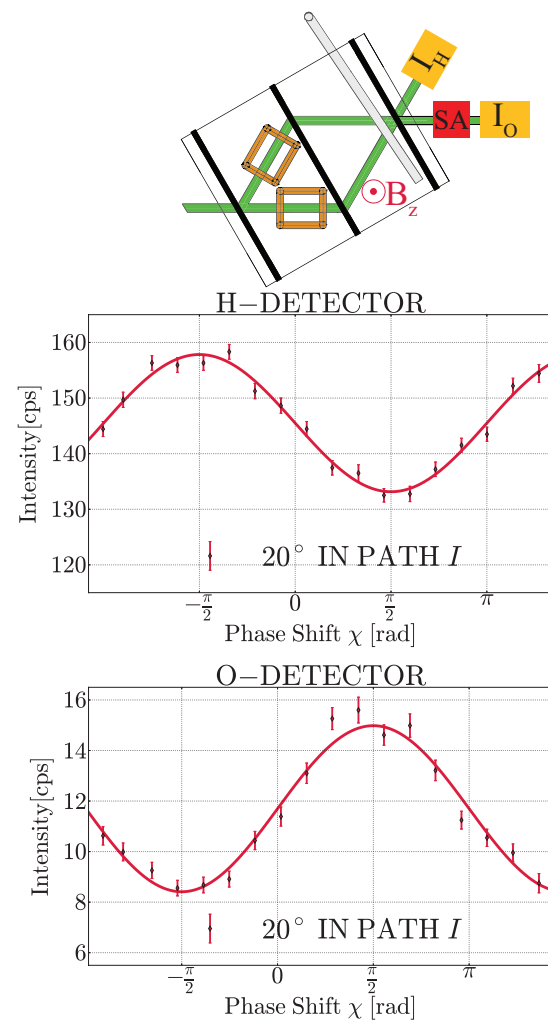
H-DETECTOR



H-DETECTOR



Experimental Results: Weak Spin Measurement along Path I,II



$$\left| \left\langle \hat{\sigma}_z \hat{\Pi}_I \right\rangle_w \right|^2 = 0.93 \pm 0.24$$

$$\left| \left\langle \hat{\sigma}_z \hat{\Pi}_{II} \right\rangle_w \right|^2 = 0.06 \pm 0.23$$

arXiv:1312.3775 [quant-ph]

04.03.2014

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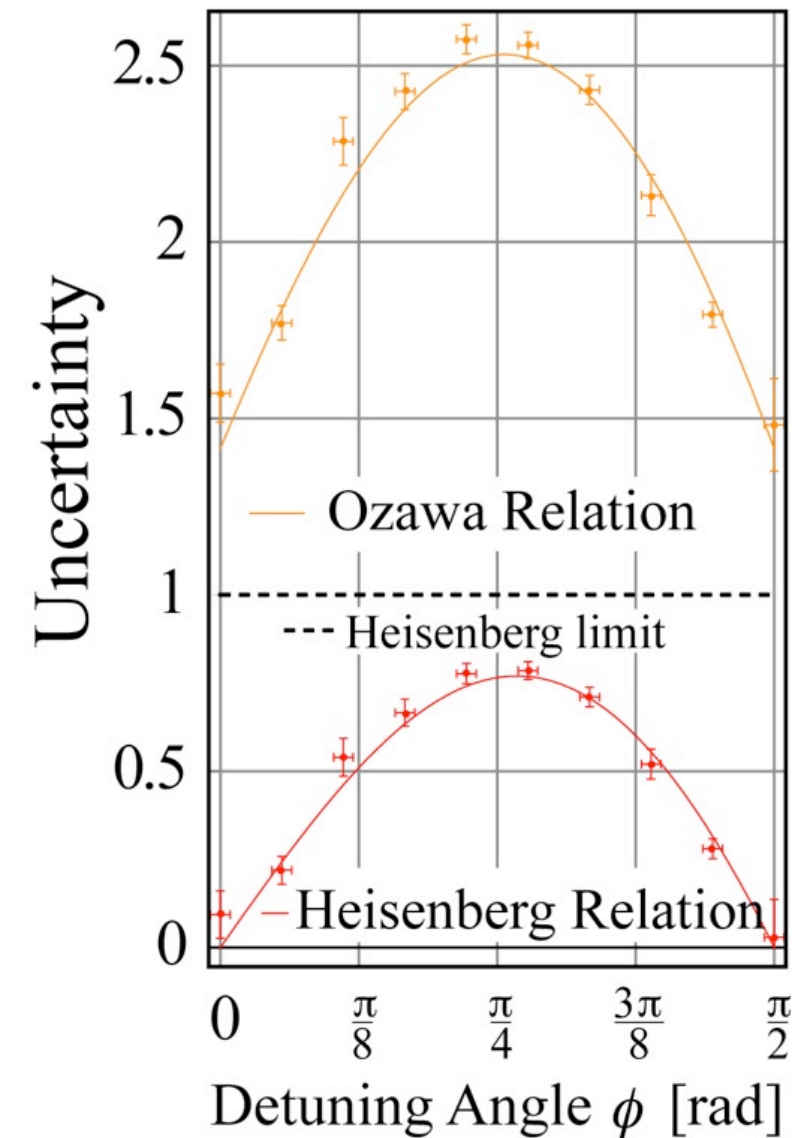
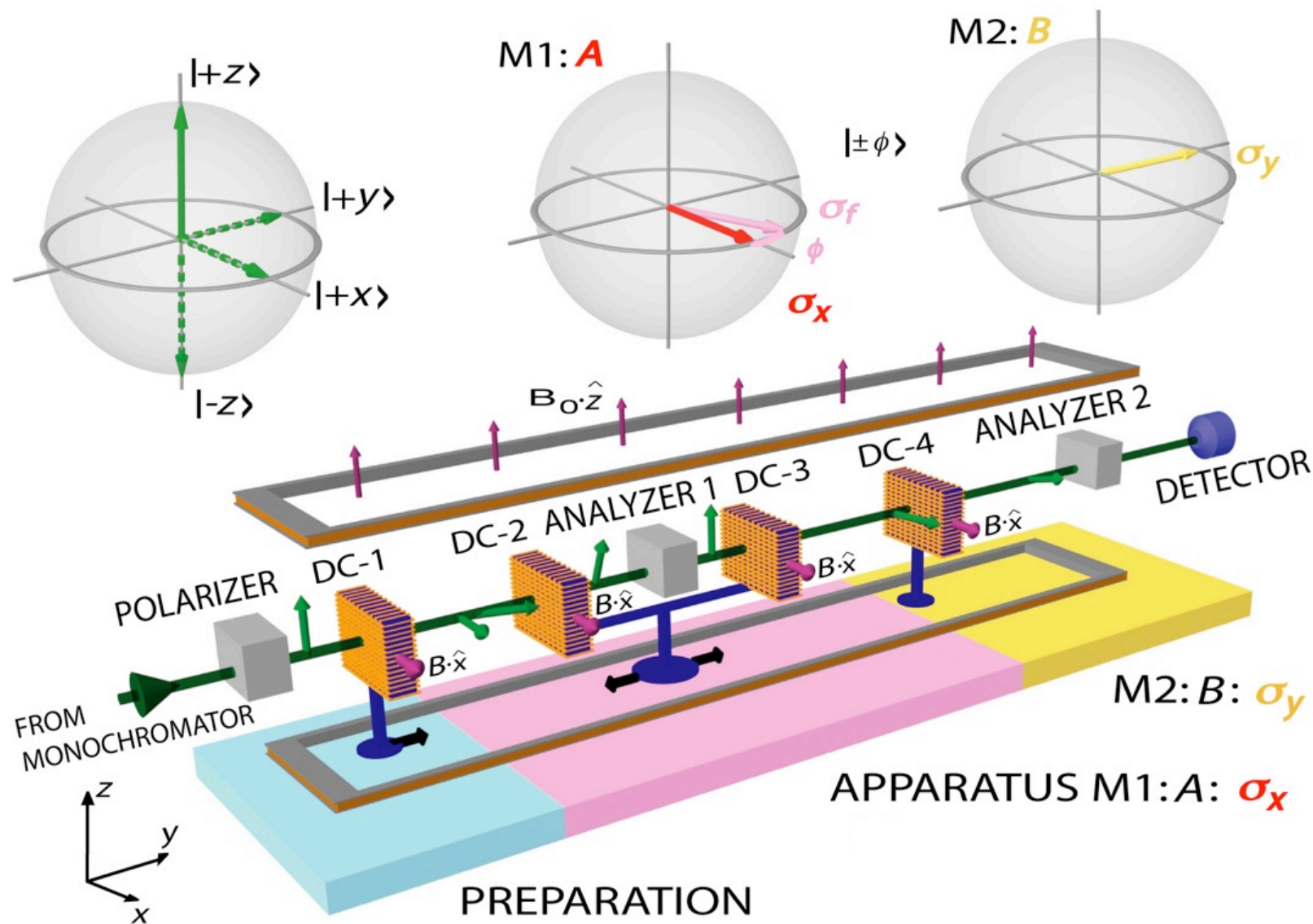
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Summary

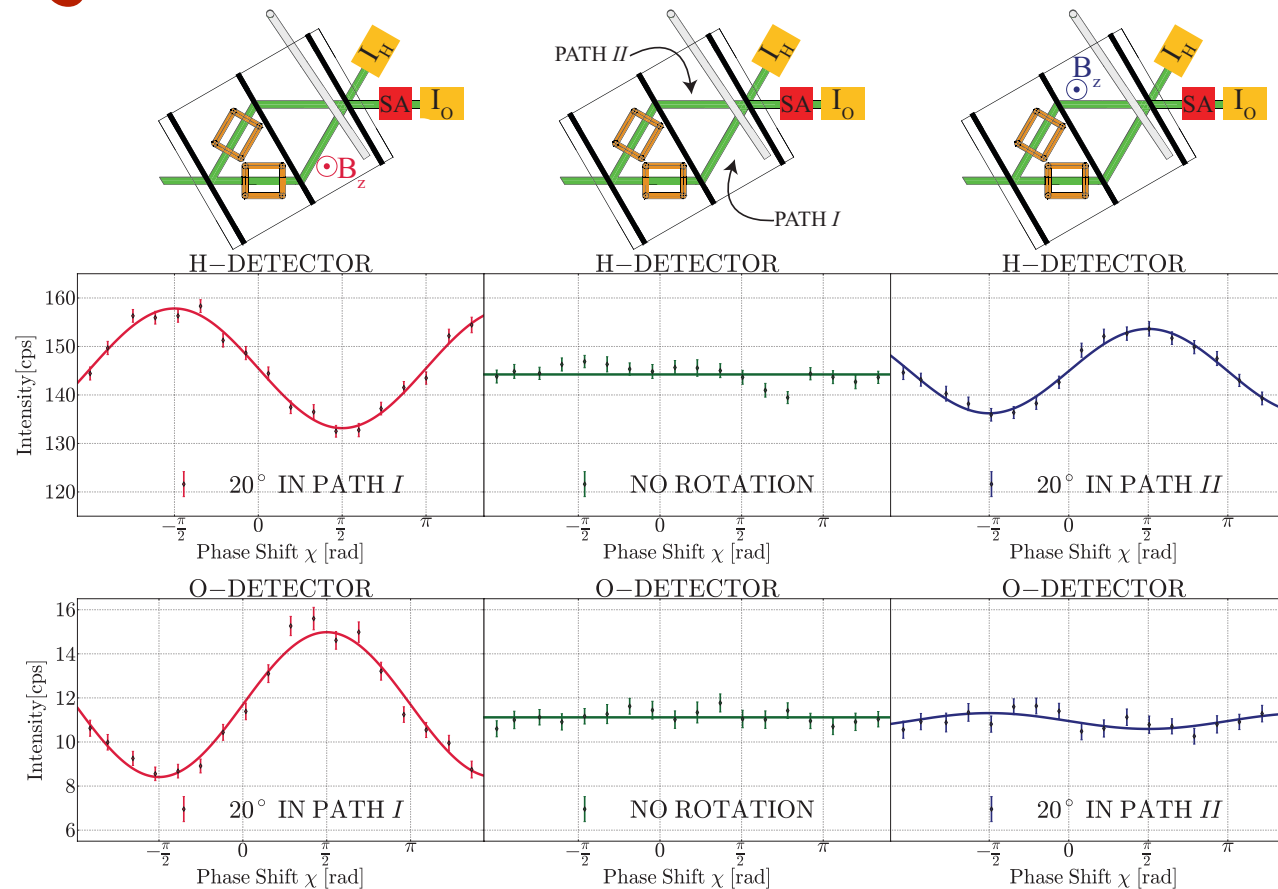
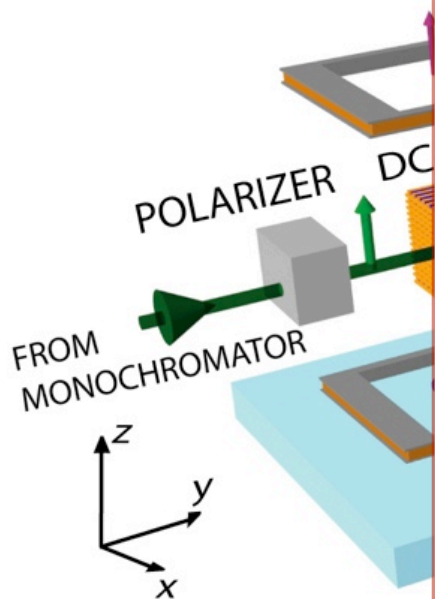
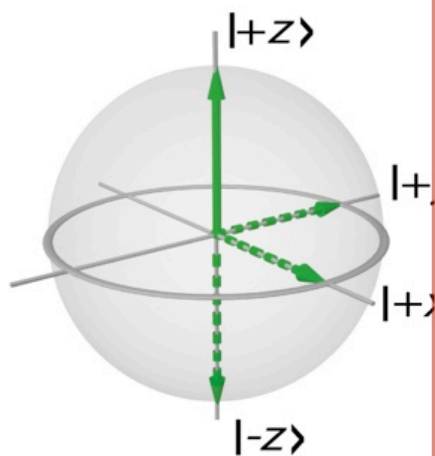
● Ozawa's Universally Valid Uncertainty Relation (UVUR)



Summary

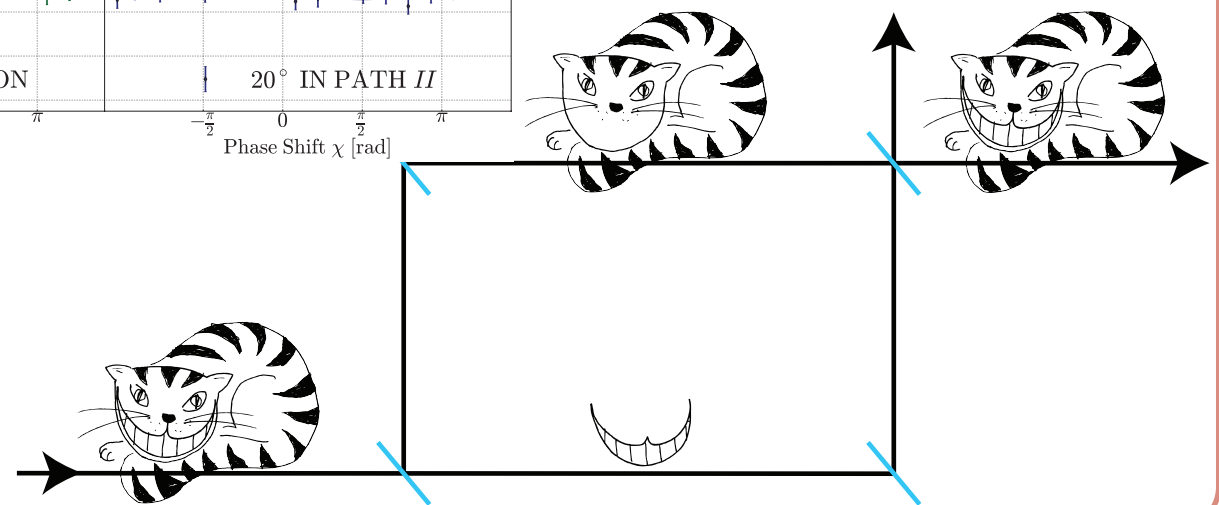
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● Quantum Cheshire Cat

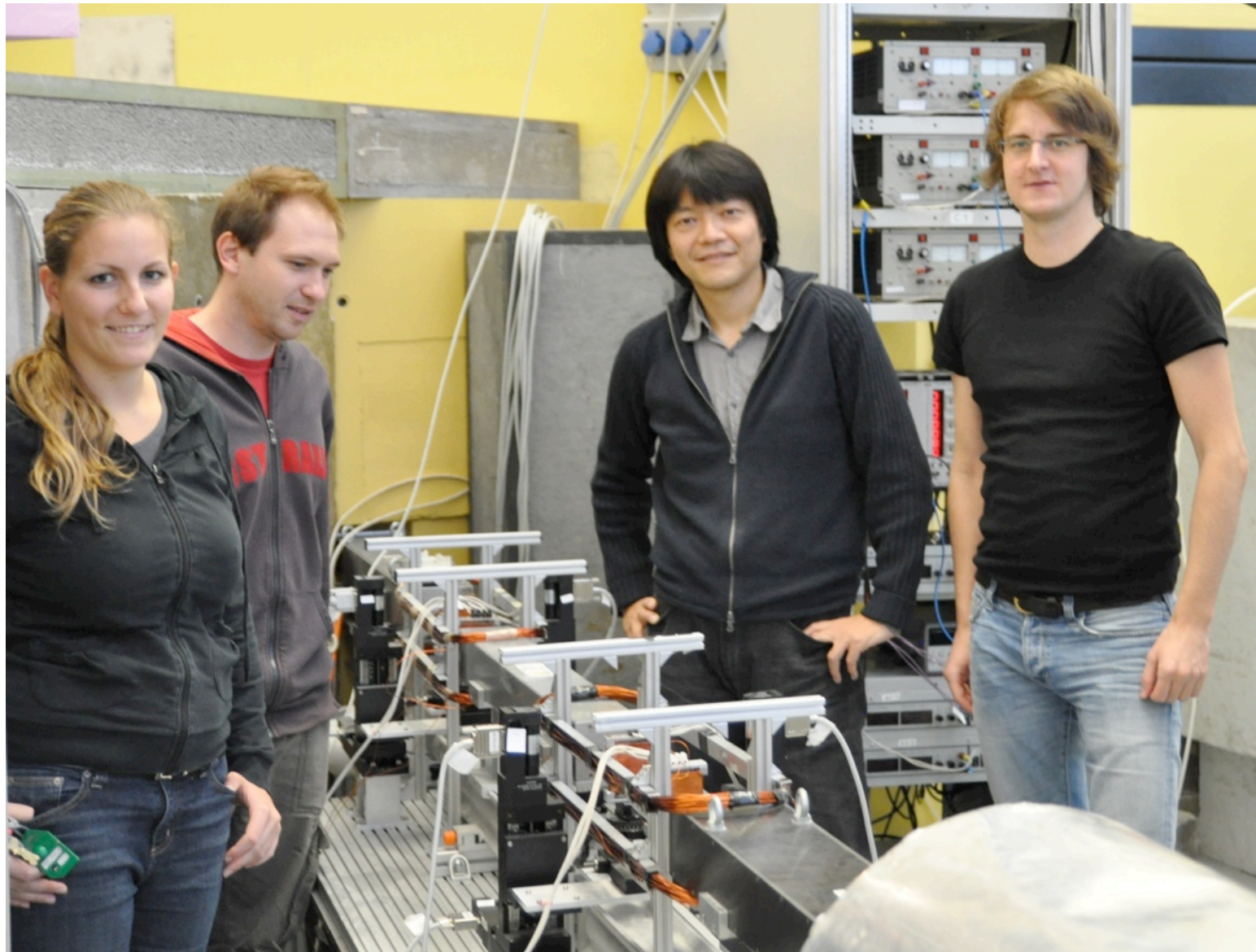


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$$\begin{aligned}\left| \langle \hat{\sigma}_z \hat{\Pi}_I \rangle_w \right|^2 &= 0.93 \pm 0.24 \\ \left| \langle \hat{\sigma}_z \hat{\Pi}_{II} \rangle_w \right|^2 &= 0.06 \pm 0.23\end{aligned}$$



Ozawa's Universally Valid Uncertainty Relation (UVUR)



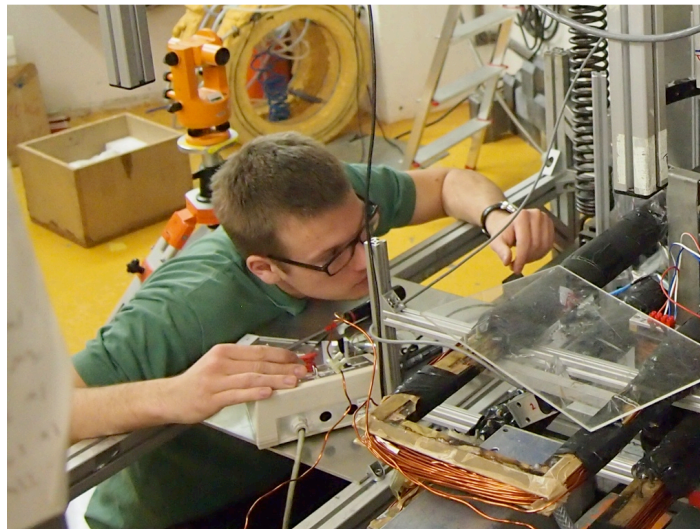
Experimental Team:

J. Erhart, S. Sponar, Y. Hasegawa, G. Sulyok



Y. Hasegawa, & M. Ozawa

Quantum Cheshire Cat



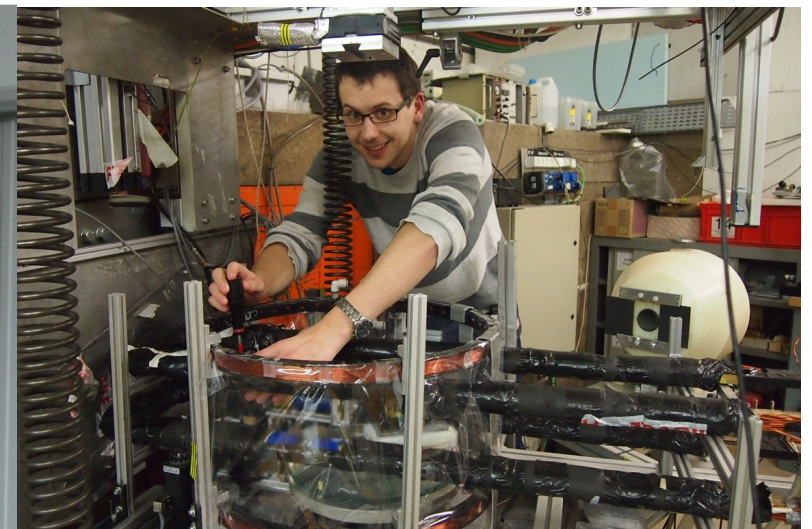
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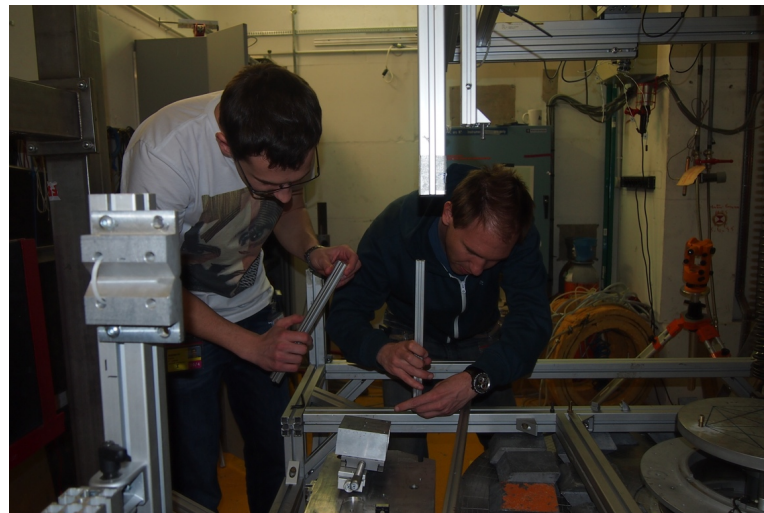
A. Matzkin



Y. Hasegawa



T. Denkmayr



T. Denkmayr, S. Sponar



H. Lemmel

Thank you for your attention



How the Result of a Measurement of a Component of the Spin of a Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

*Physics Department, University of South Carolina, Columbia, South Carolina 29208, and
School of Physics and Astronomy, Tel-Aviv University, Ramat Aviv 69978, Israel*

(Received 30 June 1987)

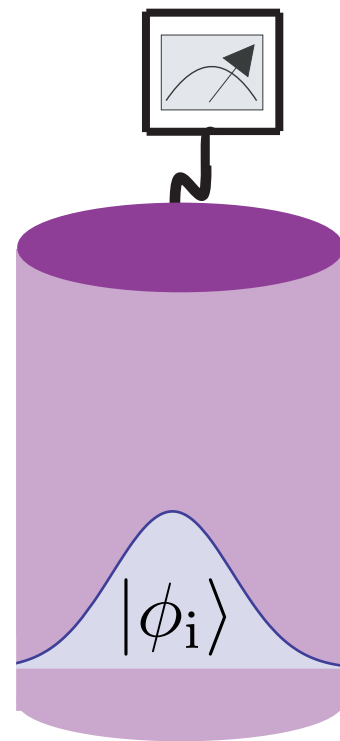
We have found that the usual measuring procedure for preselected and postselected ensembles of quantum systems gives unusual results. Under some natural conditions of weakness of the measurement, its result consistently defines a new kind of value for a quantum variable, which we call the weak value. A description of the measurement of the weak value of a component of a spin for an ensemble of preselected and postselected spin- $\frac{1}{2}$ particles is presented.

PACS numbers: 03.65.Bz

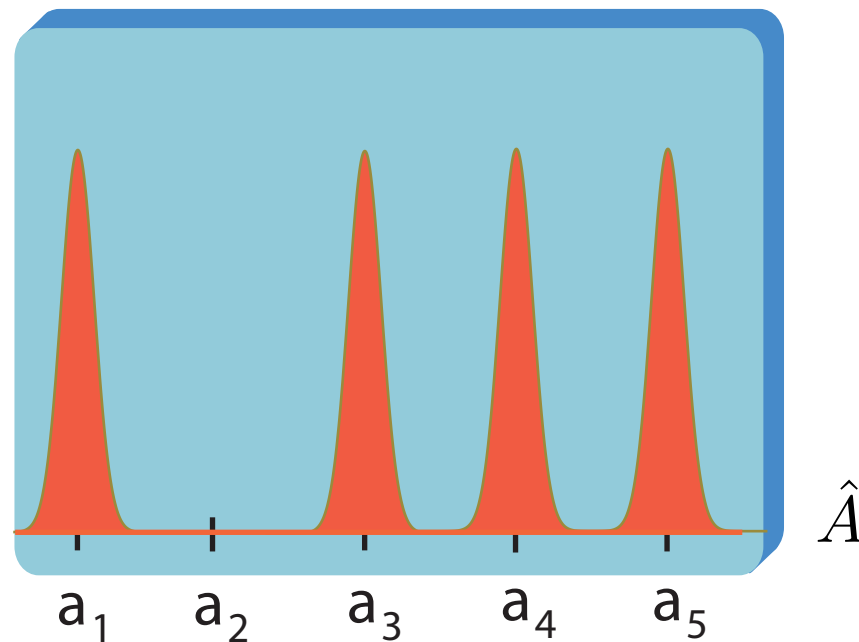
National Medal of Science, awarded and presented by President Barak Obama (2010).

Weak Measurements

measurement device
(probe system)



initial state of **probe**: $|\phi_i\rangle$
(broad Gaussian)

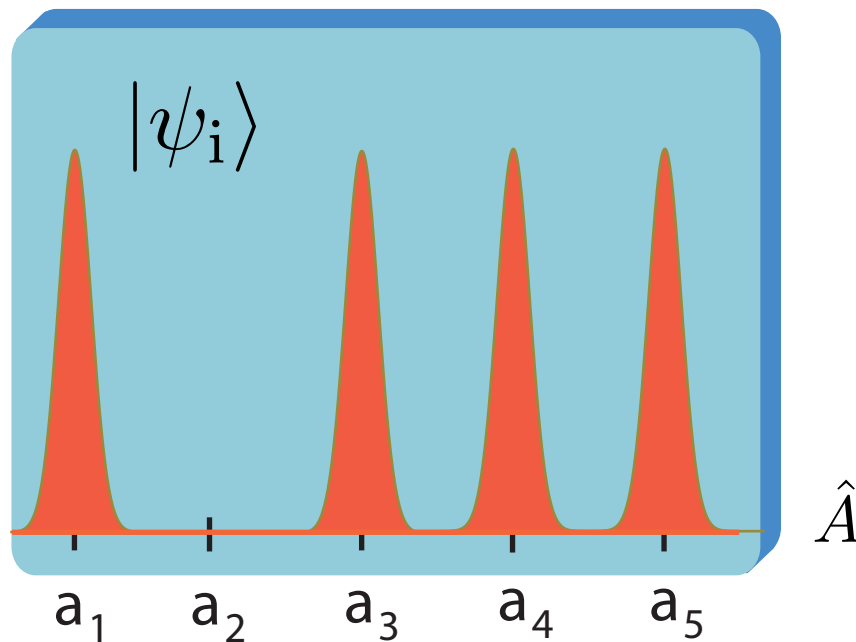
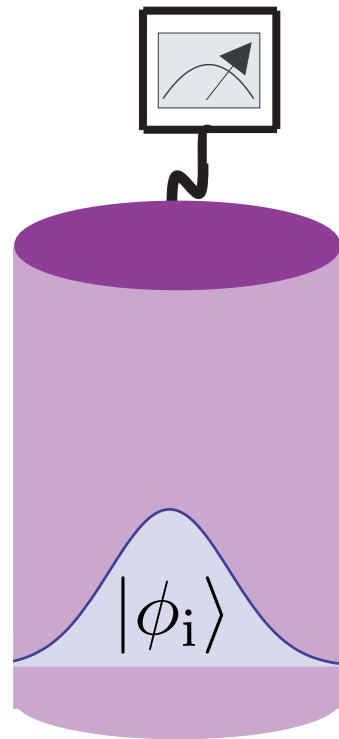


system whose observable $\hat{A}$ is to be measured

Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Weak Measurements: i) Pre-Selection

measurement device
(probe system)



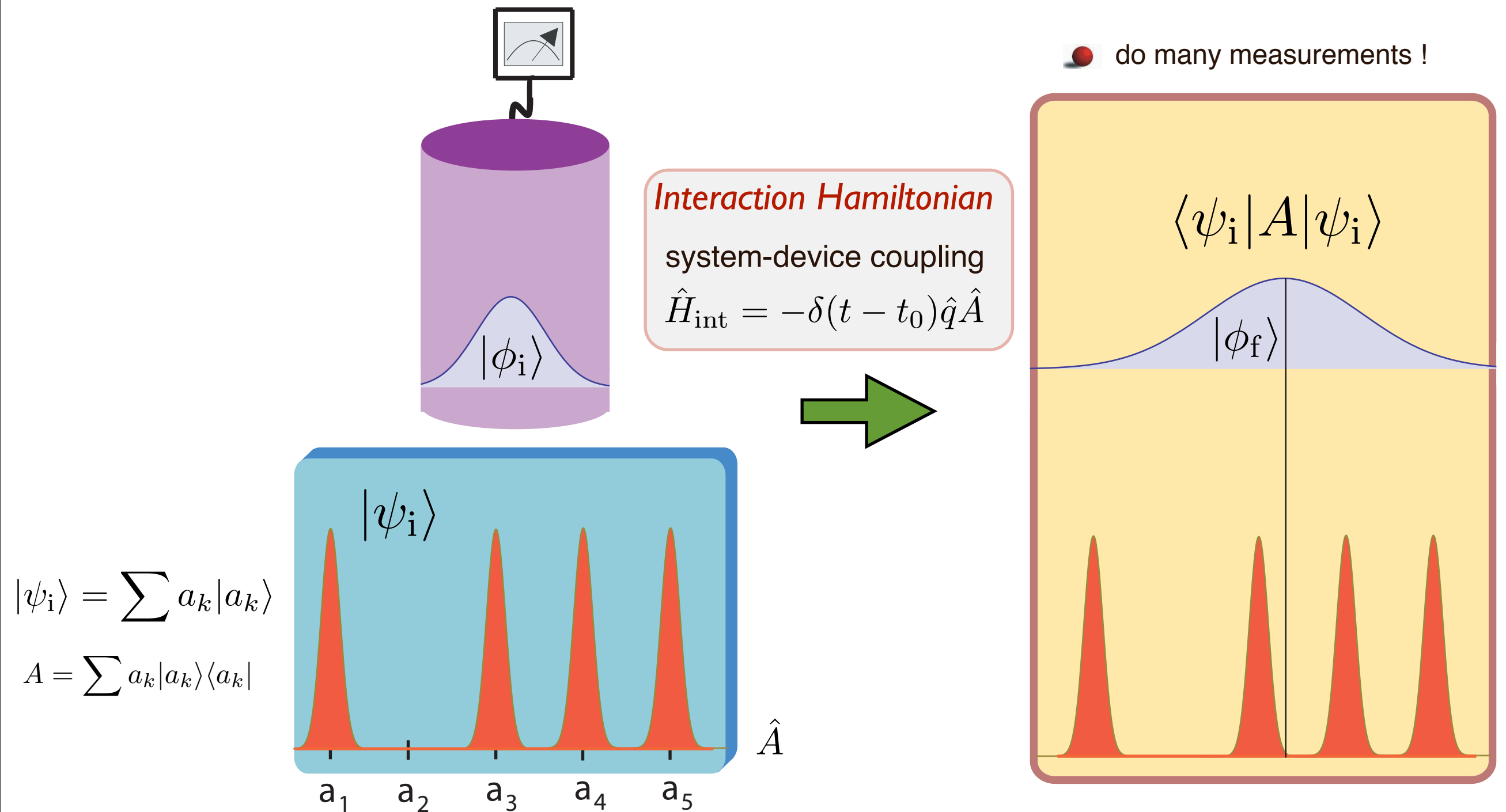
● **Pre-Selection:**

$$|\psi_i\rangle = \sum a_k |a_k\rangle$$

$$A = \sum a_k |a_k\rangle \langle a_k|$$

system whose observable $\hat{A}$ is to be measured

Weak Measurements: ii) Weak Interaction



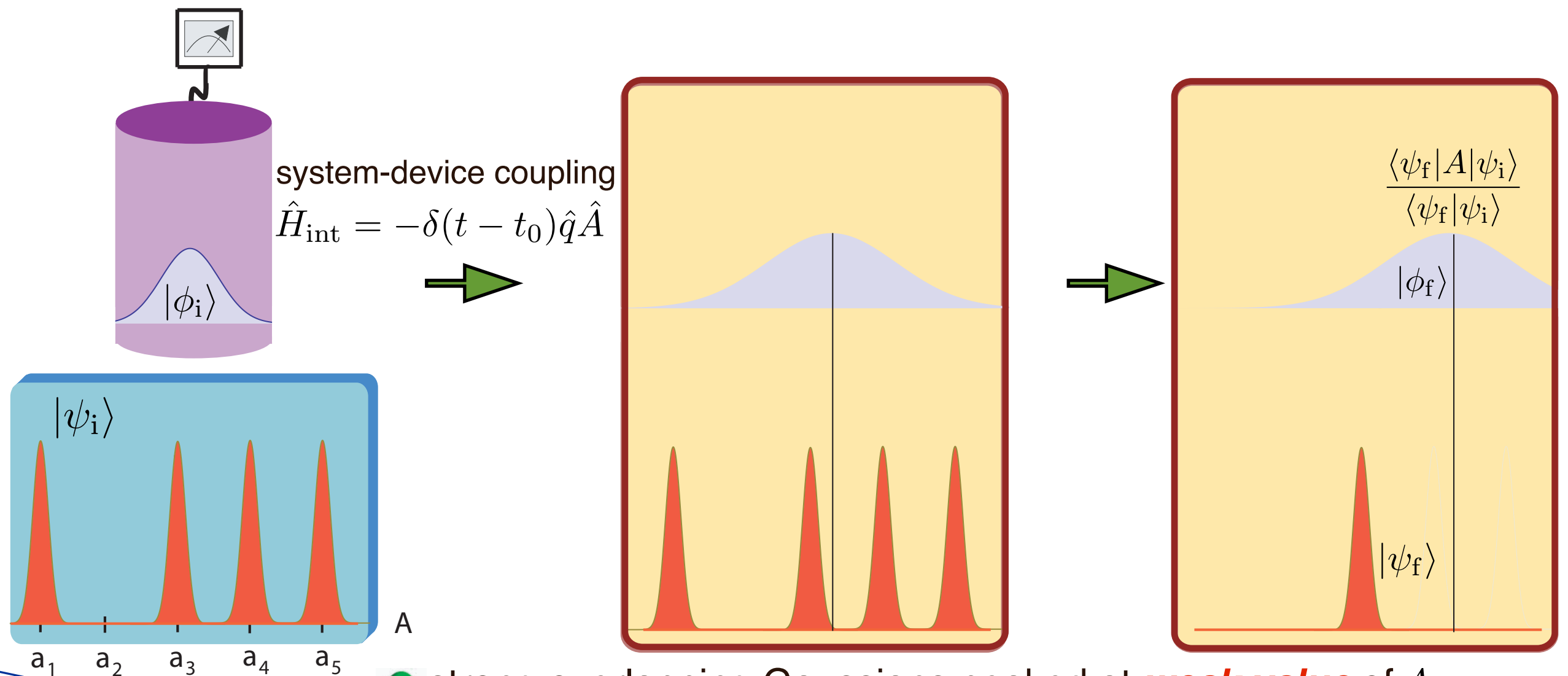
● strong overlapping Gaussians peaked at expectation value of A

Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Weak Measurements: iii) Post-Selection

strong measurement on other observable B on quantum system

● **Post-Selection:** $|\psi_f\rangle = |B = b\rangle = \sum \alpha' |A = a_n\rangle$



● strong overlapping Gaussians peaked at **weak value** of A

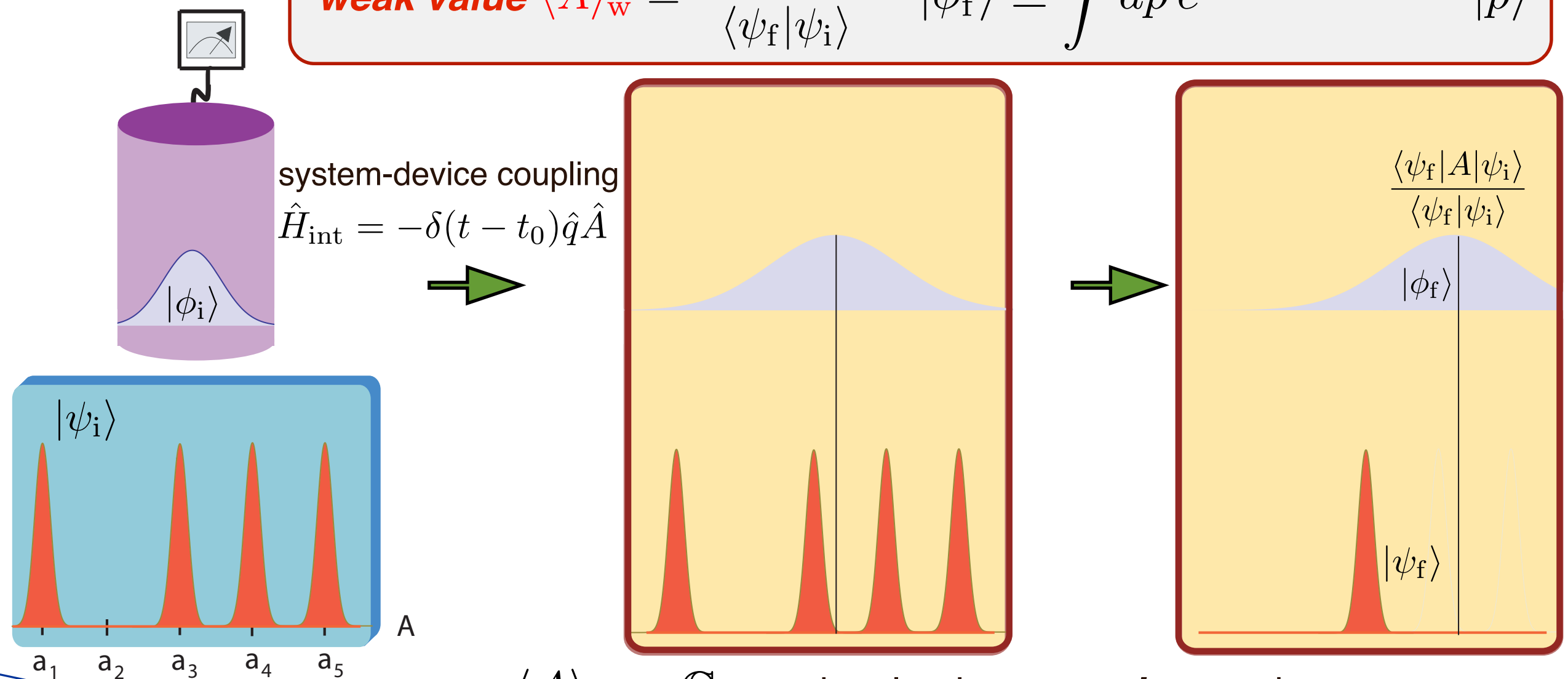
Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Weak Measurements: iii) Post-Selection

strong measurement on other observable B on quantum system

$$|\psi_f\rangle = |B = b\rangle = \sum \alpha' |A = a_n\rangle \quad |\phi_f\rangle = \sum \alpha \alpha'^* \int dq e^{iq a_n} e^{-q^2/(4\Delta^2)} |q\rangle$$

$$\text{weak value } \langle A \rangle_w = \frac{\langle \psi_f | A | \psi_i \rangle}{\langle \psi_f | \psi_i \rangle} \quad |\phi_f\rangle \simeq \int dp e^{-\Delta^2 (p - \langle A \rangle_w)^2} |p\rangle$$

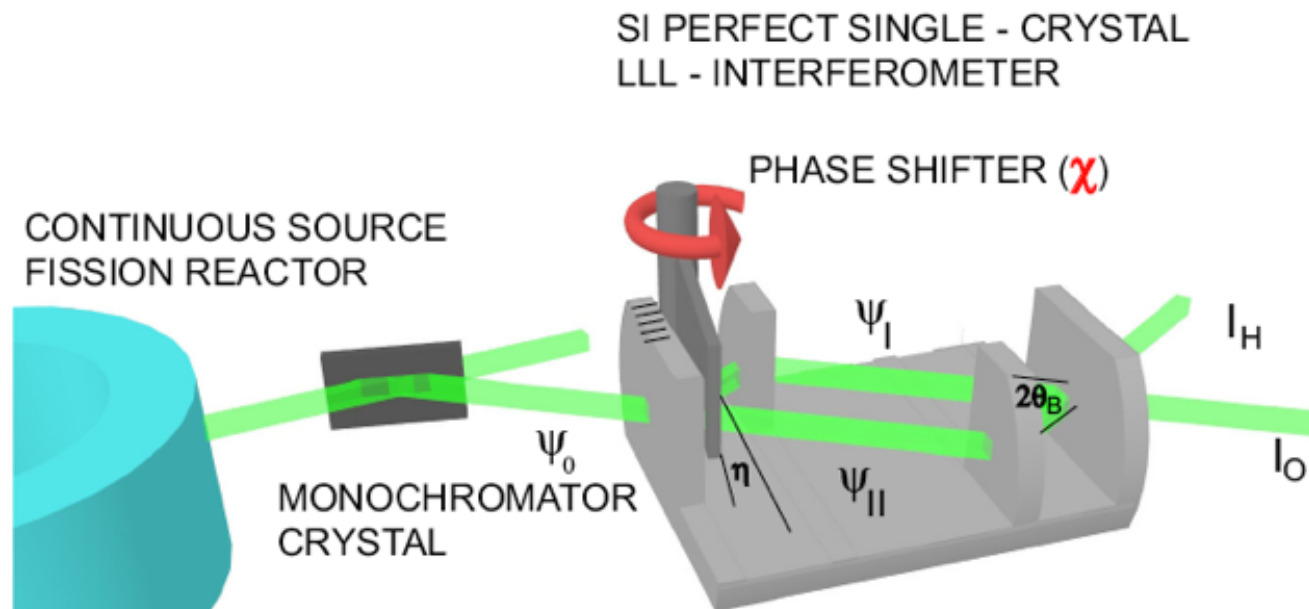


Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Neutron Interferometry

Self interference of massive single-particles

$$\lambda \sim 2\text{\AA}, \quad v \sim 2000 \text{ m/s}, \quad E \sim 20 \text{ meV}$$



$$I_O = \nu(1 + \cos \chi)$$

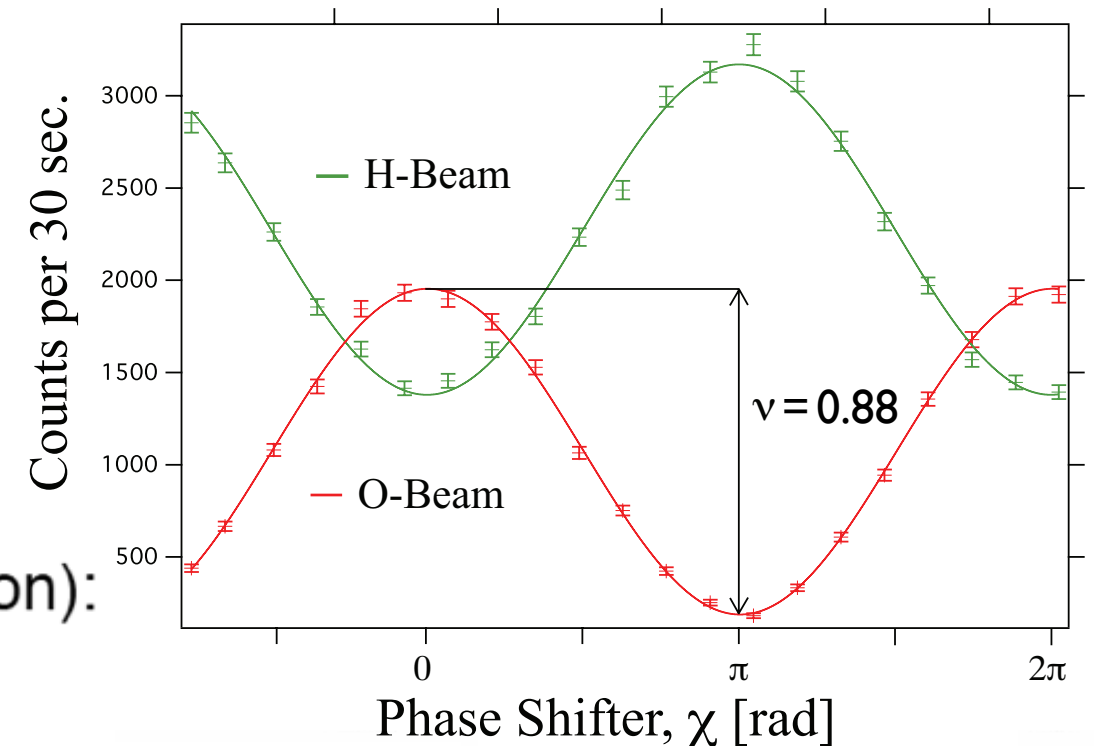
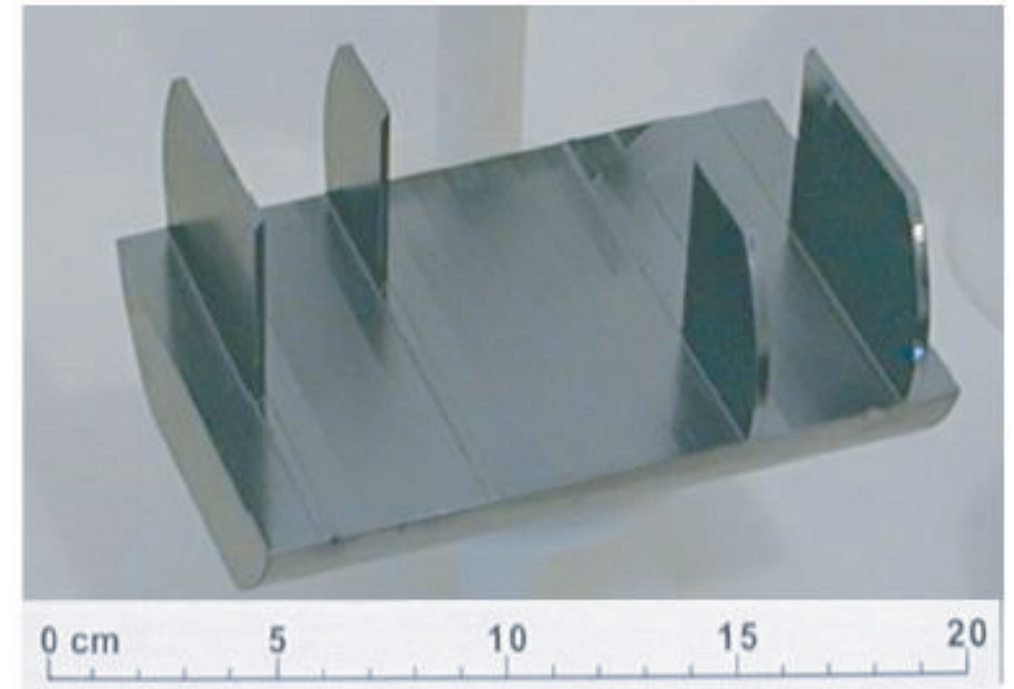
$$I_H = B - \nu \cos \chi$$

Scalar Fermi Potential (Nuclear Interaction):

$$V_{\text{nuc}}(\vec{r}) = \frac{2\pi\hbar^2}{m} b_c N$$

$$\chi = -\lambda N b_c D(\eta)$$

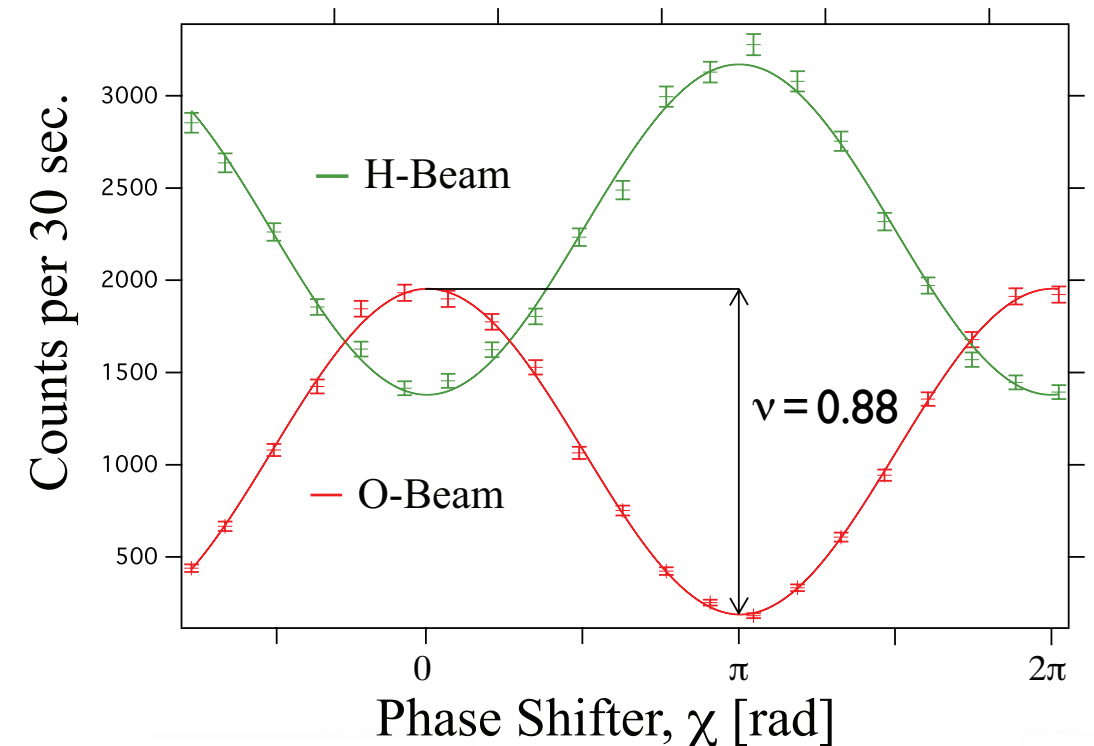
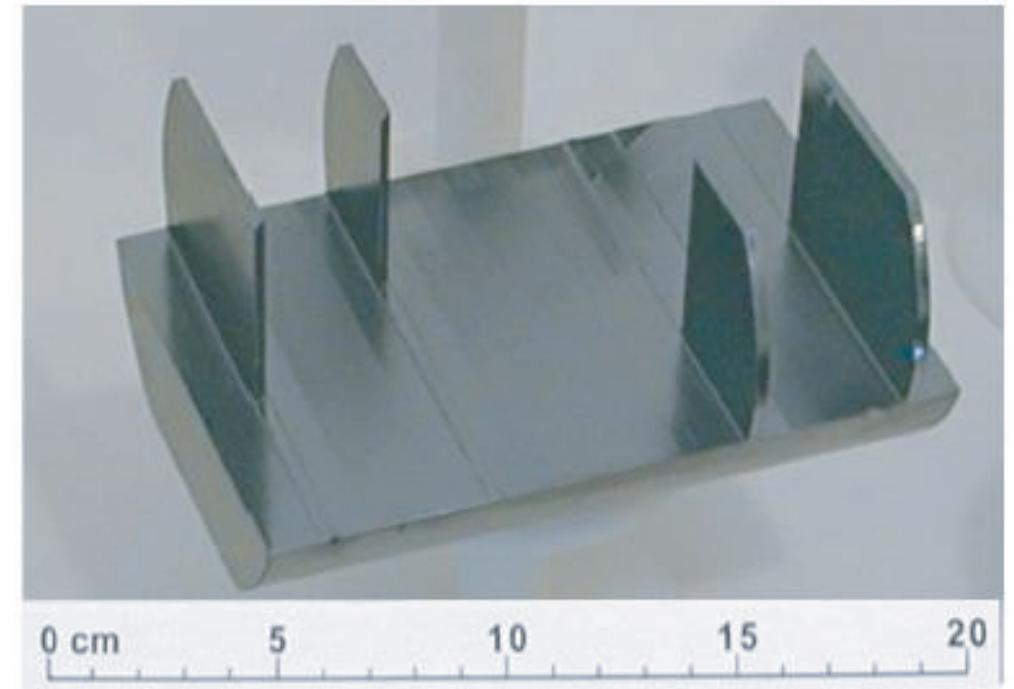
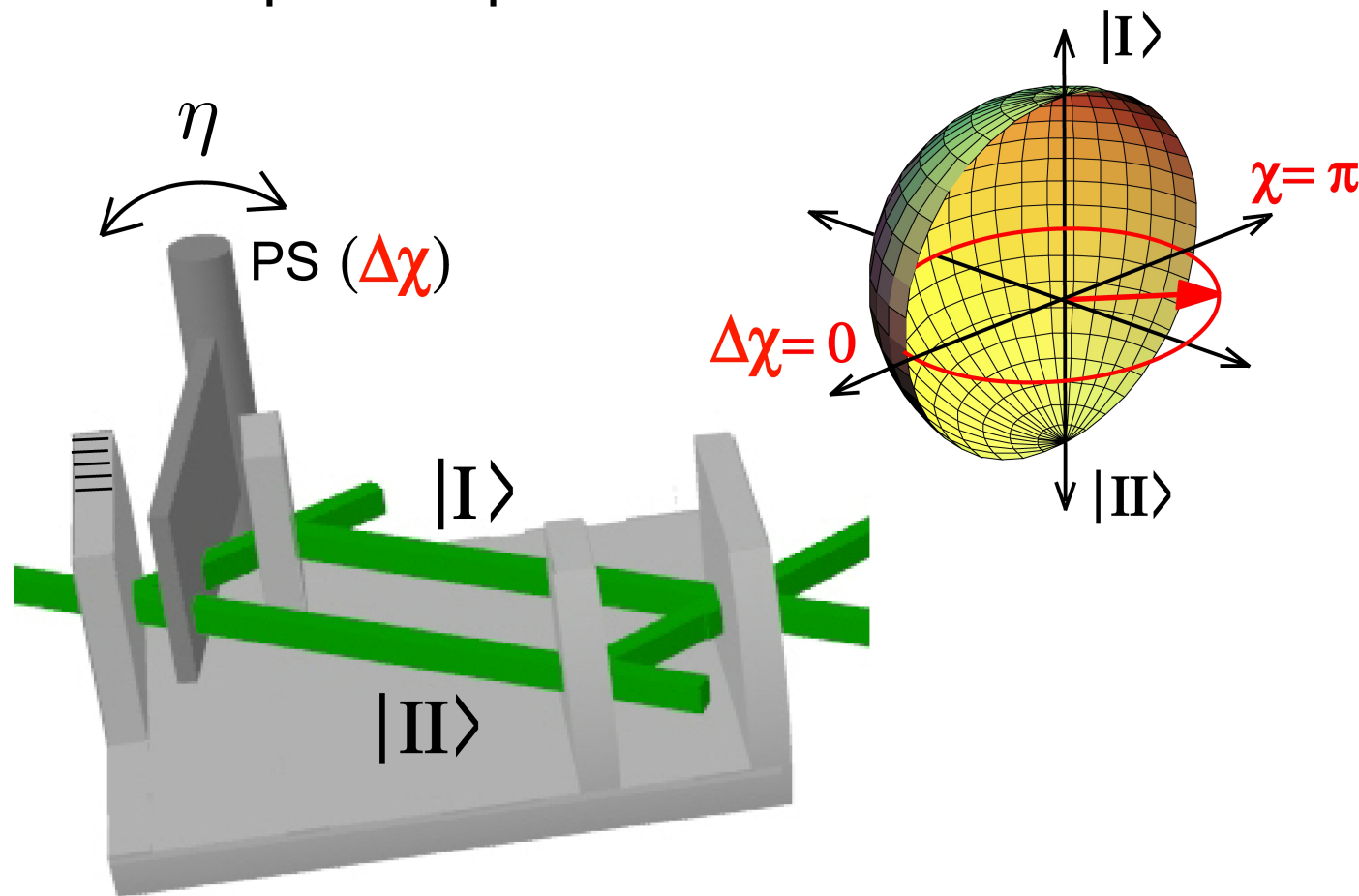
H. Rauch, W. Treimer, and U. Bonse, *PLA.* **47**, 369 (1974).



The Neutron Interferometry - a Qubits-System

- Self interference of massive single-particles

- two-level System: $|I\rangle$, $|II\rangle$
Bloch sphere representation

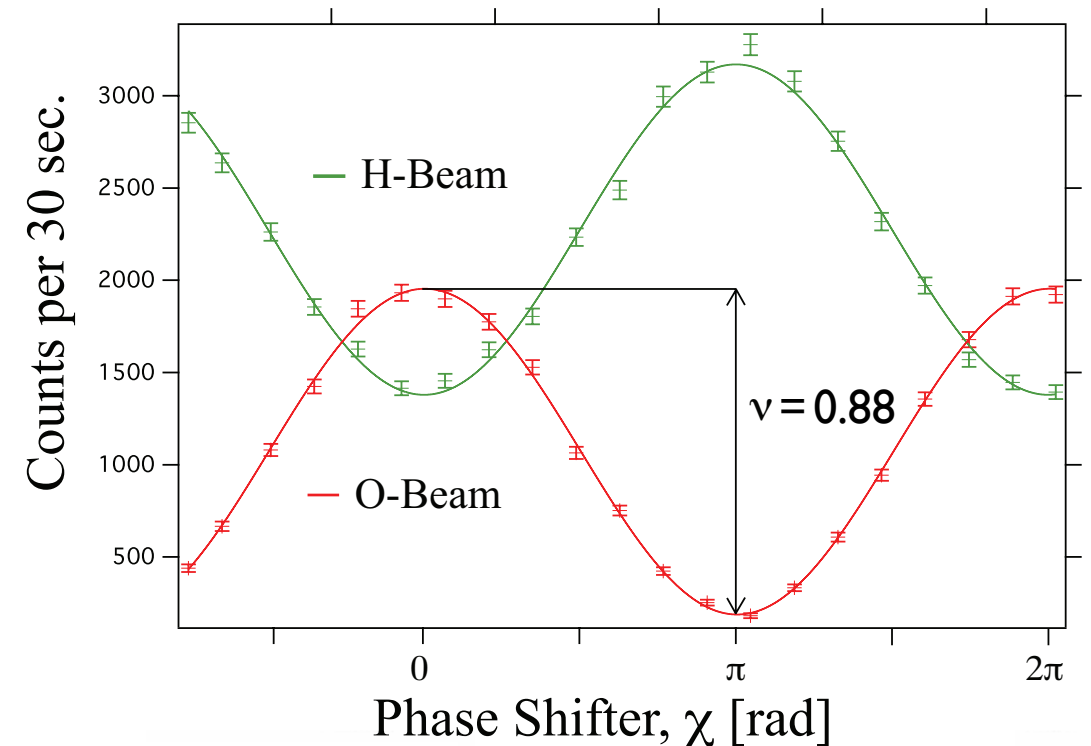
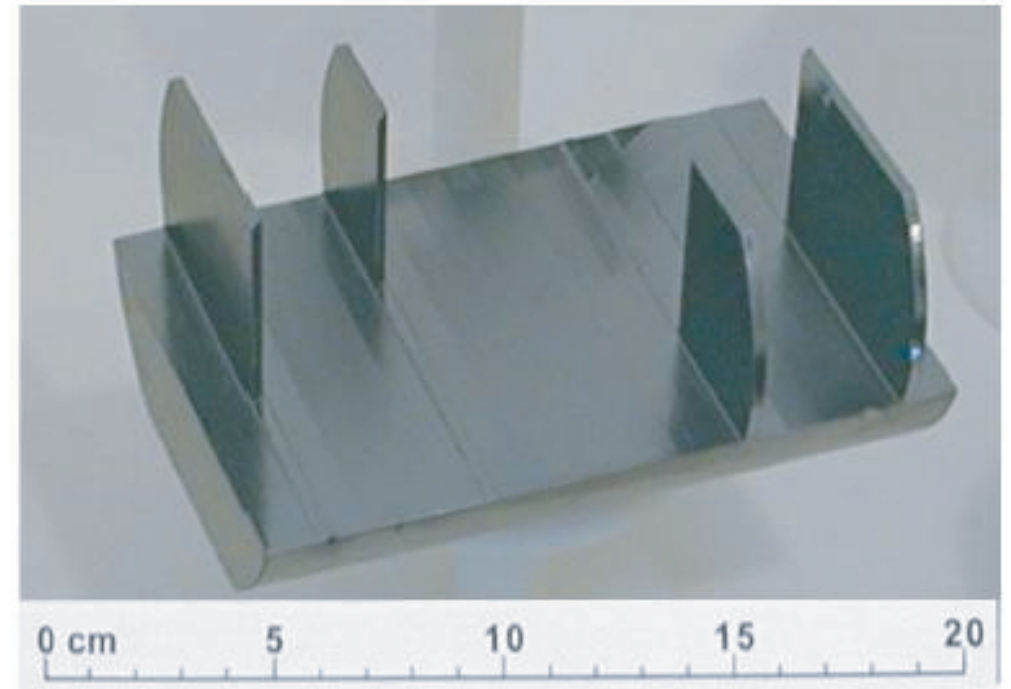
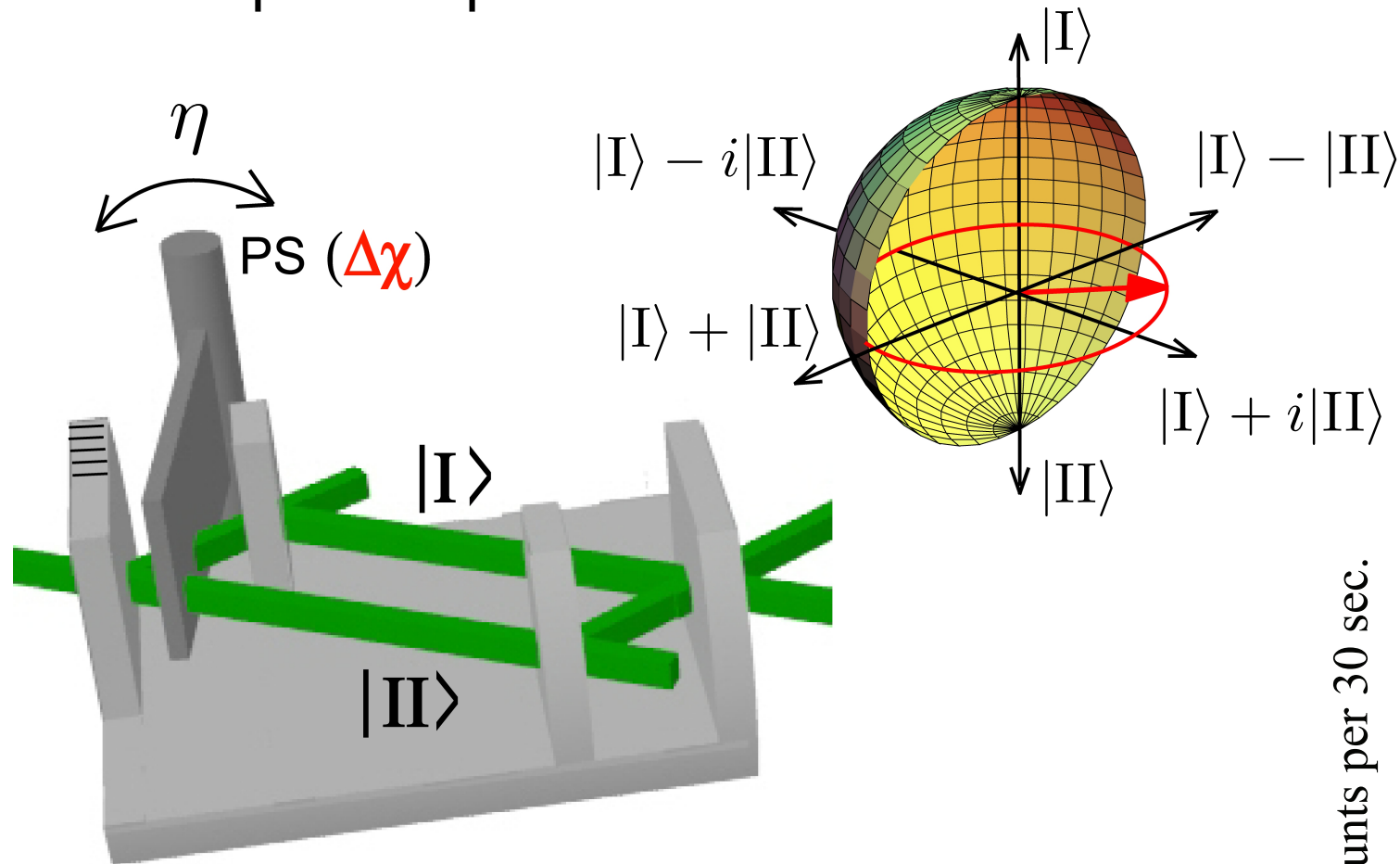


H. Rauch, W. Treimer, and U. Bonse, *PLA*. **47**, 369 (1974).

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- Self interference of massive single-particles

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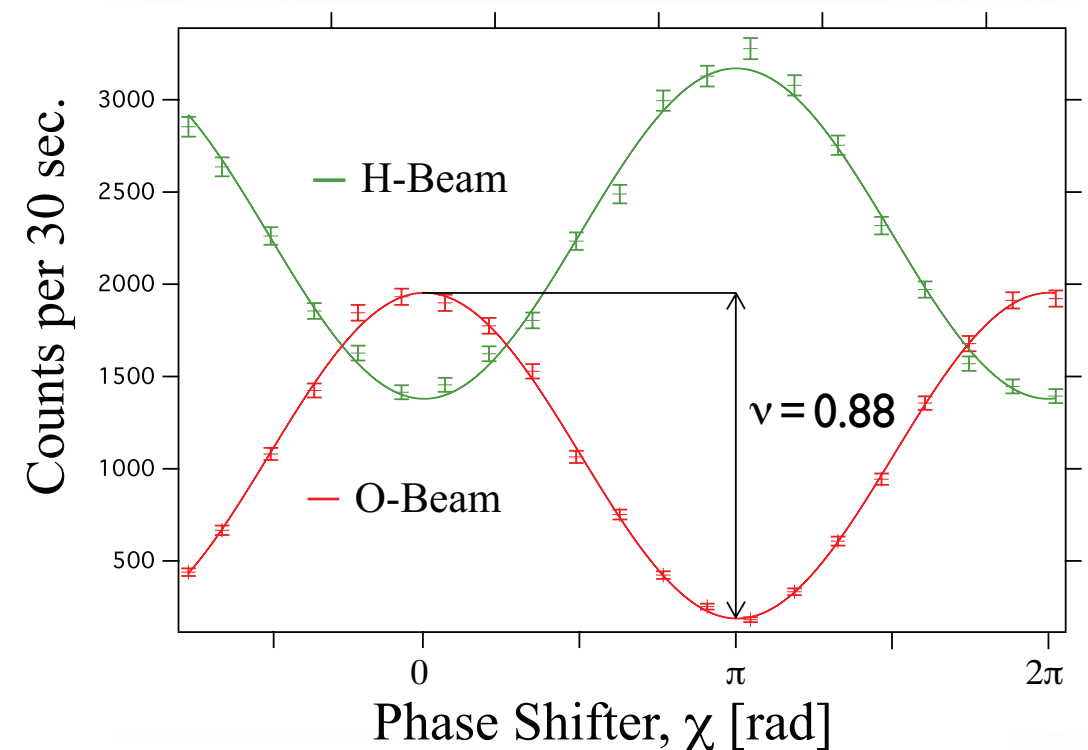
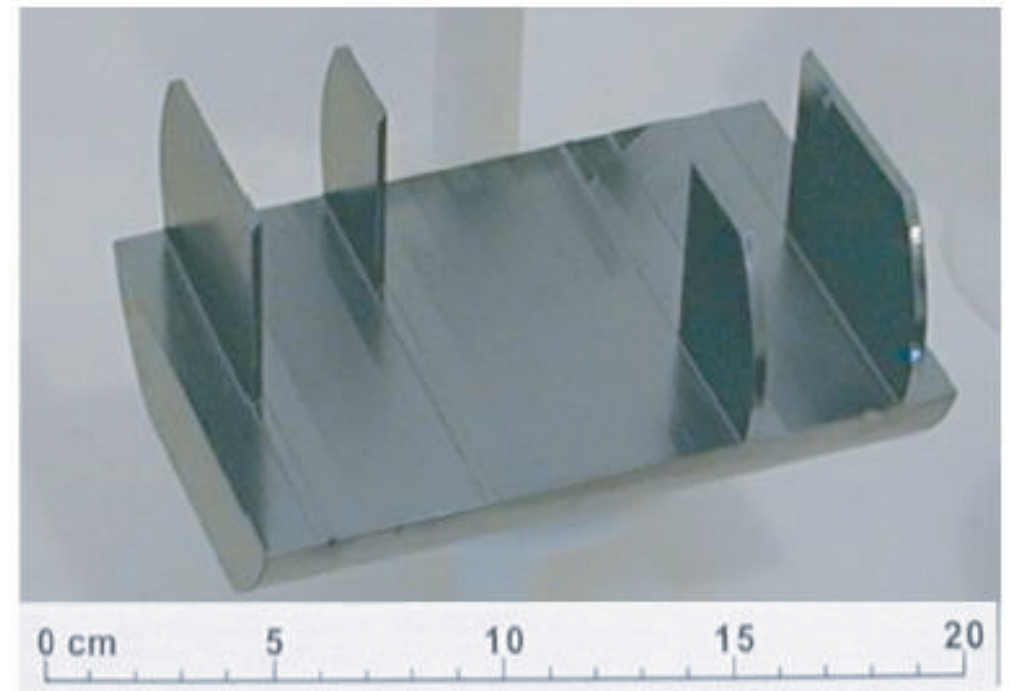
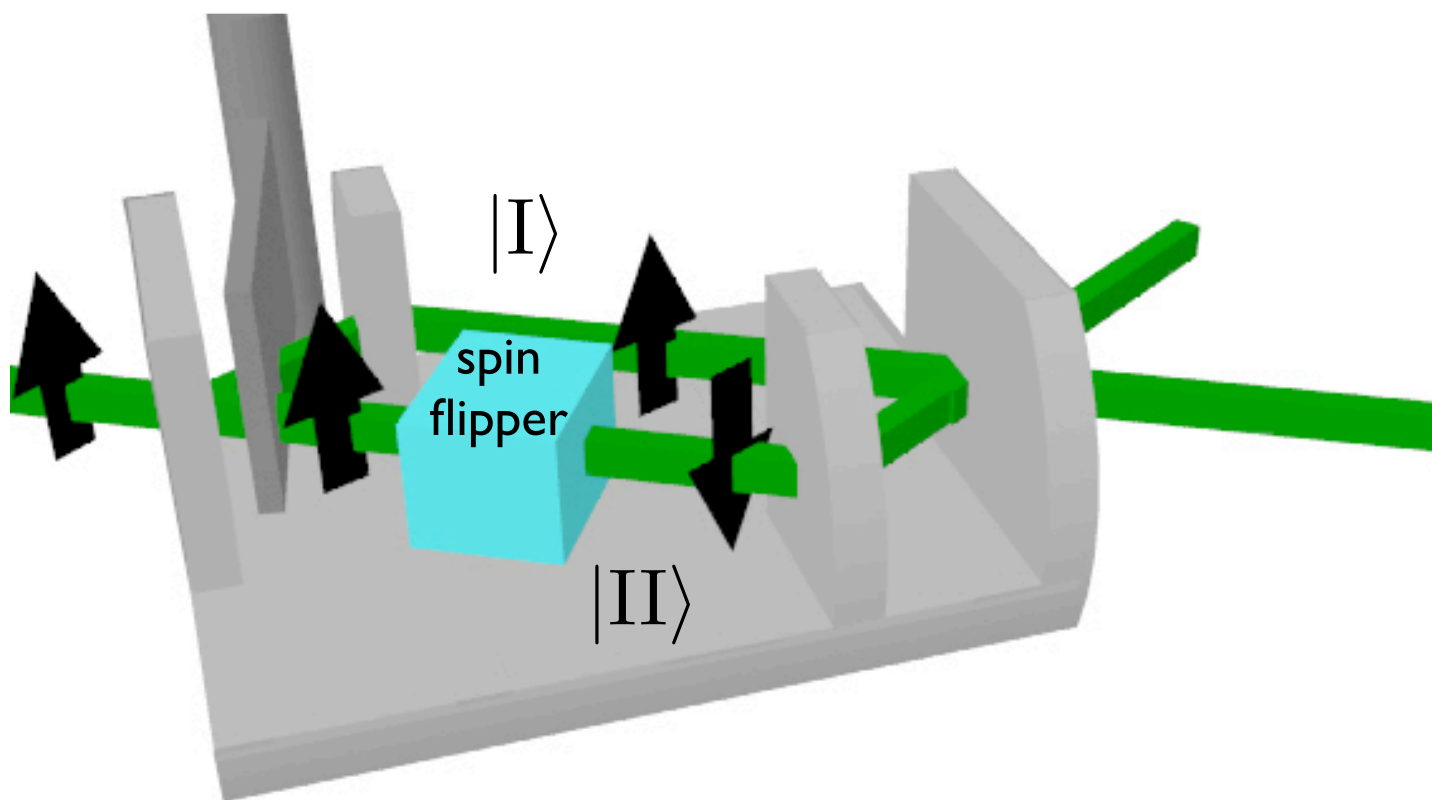
H. Rauch, W. Treimer, and U. Bonse, *PLA*. **47**, 369 (1974).

Neutron Interferometry: Entanglement

- Self interference of massive single-particles

- maximally entangled Bell state :

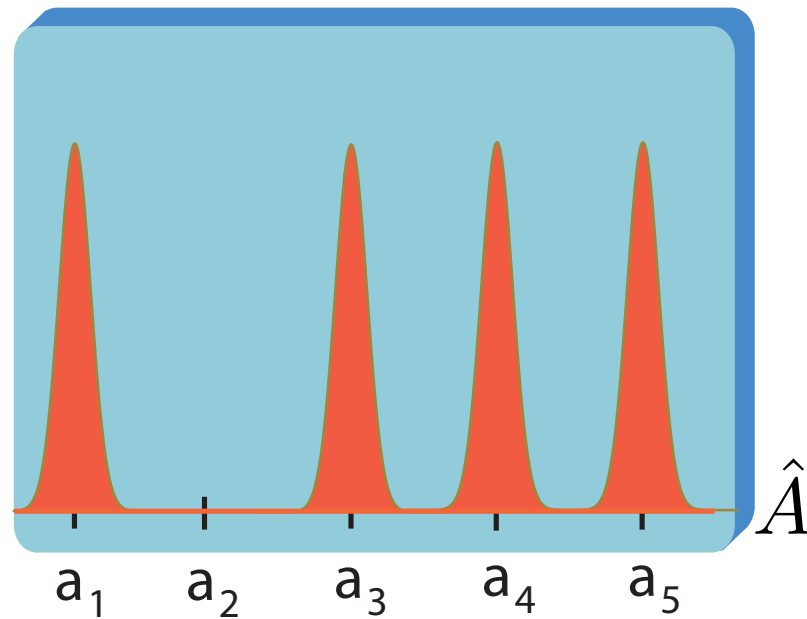
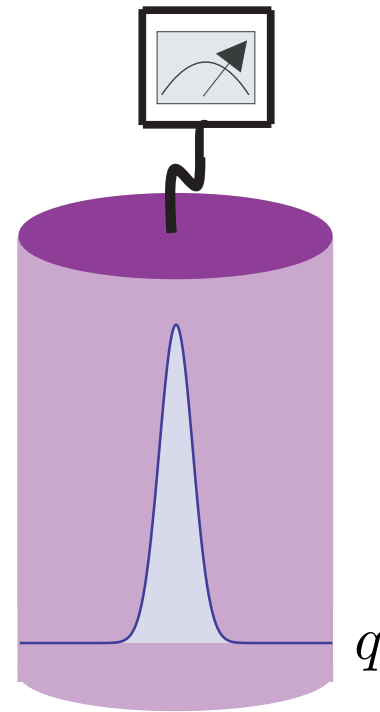
$$|\Psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} \left(|I\rangle \otimes |\uparrow\rangle + |II\rangle \otimes |\downarrow\rangle \right)$$



Y. Hasegawa *et al.*, *Nature* **425**,45 (2003).

Measurement Theory

measurement device
(probe system)



system whose observable $\hat{A}$ is to be measured

J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

12.12.2013

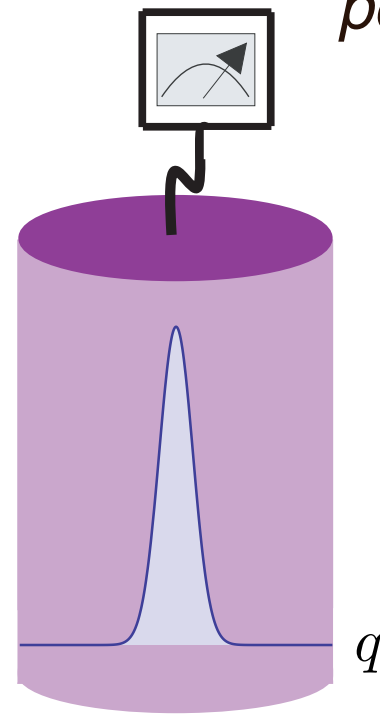
FWF

Neue Arbeiten am Atominstitut

Measurement Theory

initial state of **probe**
(Gaussian) $|\phi_i\rangle$

measurement device
(probe system)



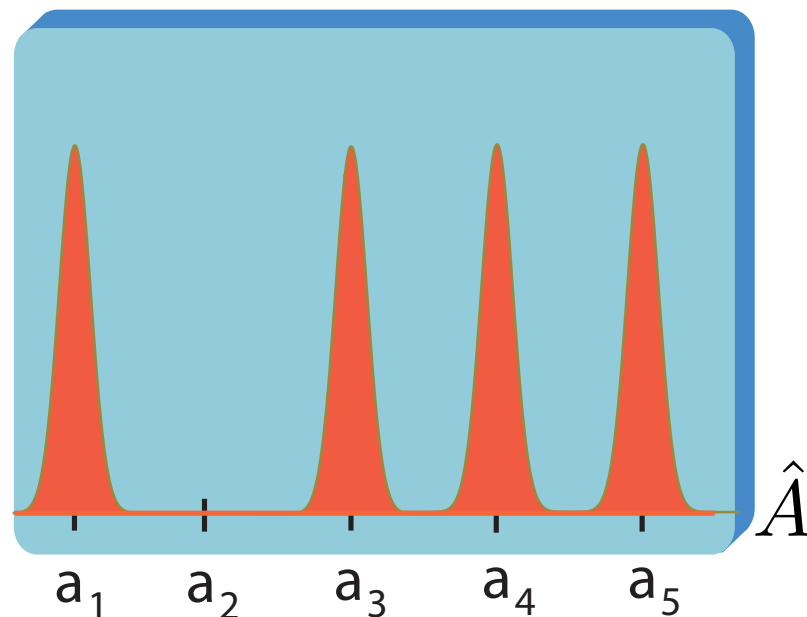
pointer : $p_f - p_i$

$$|\phi_i\rangle_q = \int dq e^{-q^2/(4\Delta^2)} |q\rangle$$

$$|\phi_i\rangle_p = \int dp e^{-\Delta^2 p^2} |p\rangle$$

canonical variable & conjugated momentum $\hat{q}, \hat{p}$

initial spread (width): $\sigma_q = \Delta, \sigma_p = 1/(2\Delta)$



system whose observable $\hat{A}$ is to be measured

J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

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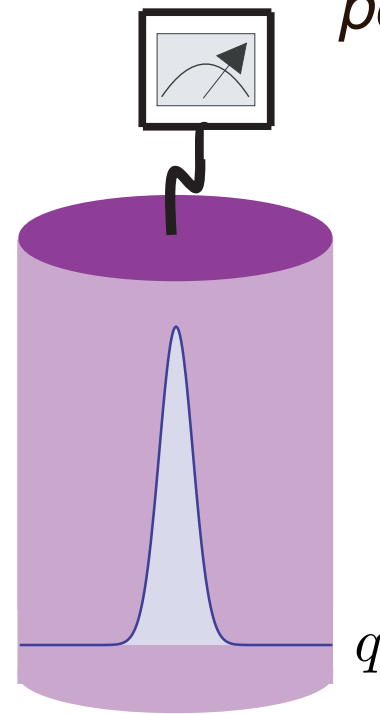
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Neue Arbeiten am Atominstitut

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initial state of **probe**
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measurement device
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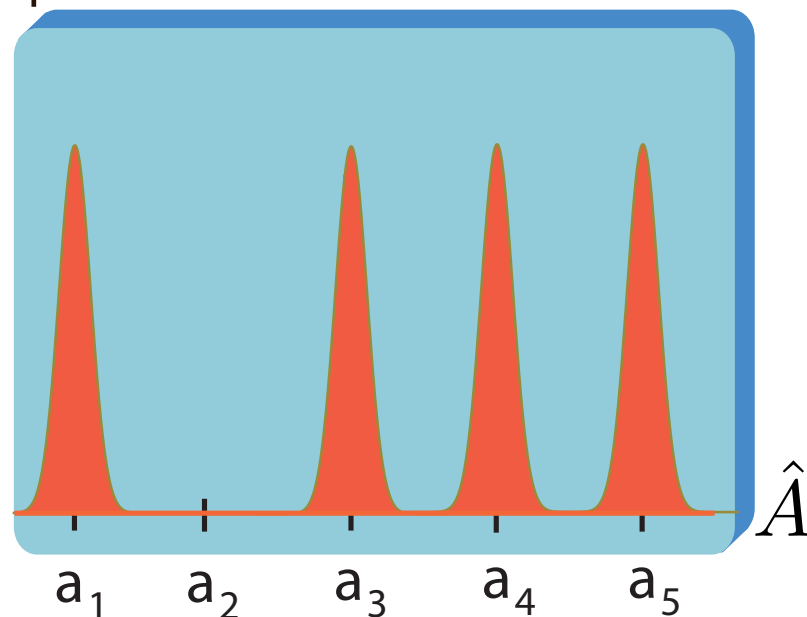
canonical variable & conjugated momentum $\hat{q}, \hat{p}$

initial spread (width): $\sigma_q = \Delta, \sigma_p = 1/(2\Delta)$

state of the **system** expanded
in eigenstates of

$$|\psi_i\rangle = \sum a_k |a_k\rangle$$

$$A = \sum a_k |a_k\rangle \langle a_k|$$



system whose observable $\hat{A}$ is to be measured

J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

12.12.2013

FWF

Neue Arbeiten am Atominstitut

Measurement Theory

initial state of **probe**
(Gaussian) $|\phi_i\rangle$

$$|\phi_i\rangle = \int e^{-q^2/(4\Delta^2)} |q\rangle$$

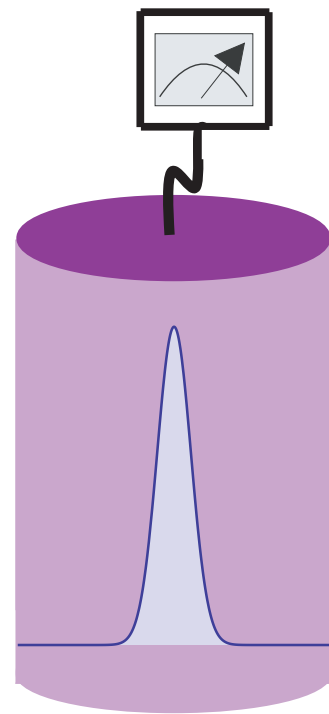
initial spread Δ

$\hat{q}, \hat{p}$ canonical variable
& conjugated momentum

state of the **system** expanded
in eigenstates of $\hat{A}$

$$|\psi_i\rangle = \sum a_k |a_k\rangle$$

$$A = \sum a_k |a_k\rangle \langle a_k|$$

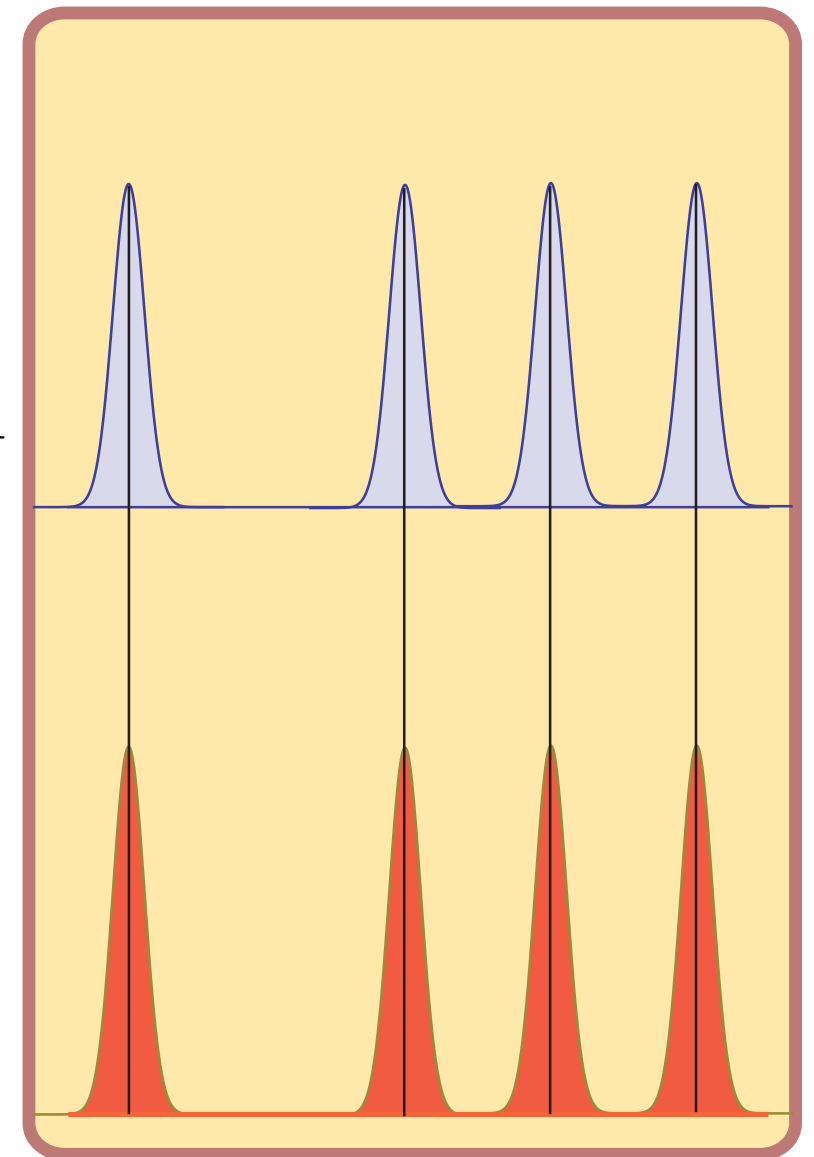
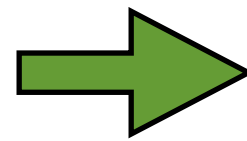
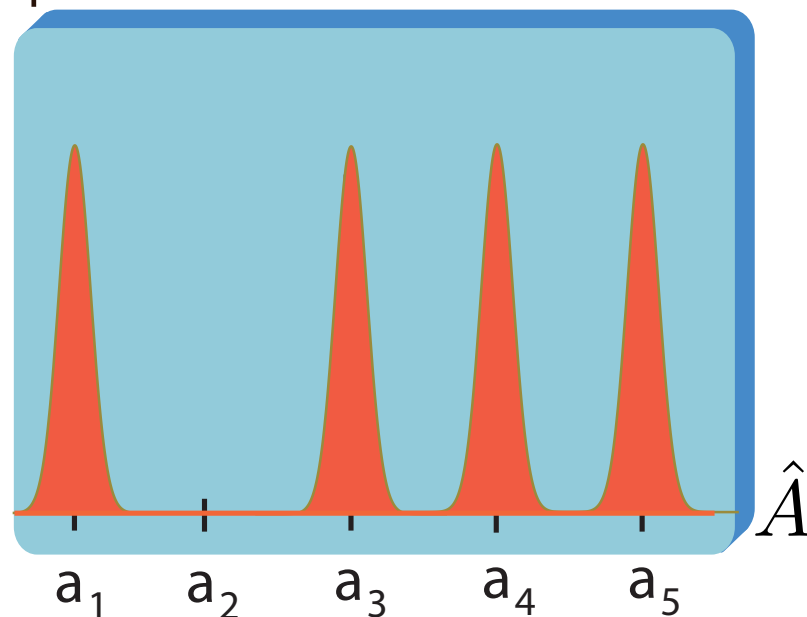


pointer : $p_f - p_i$

system-device coupling

$$\hat{H}_{\text{int}} = -\delta(t - t_0) \hat{q} \hat{A}$$

final state of composite system



creates **entanglement** between system and device

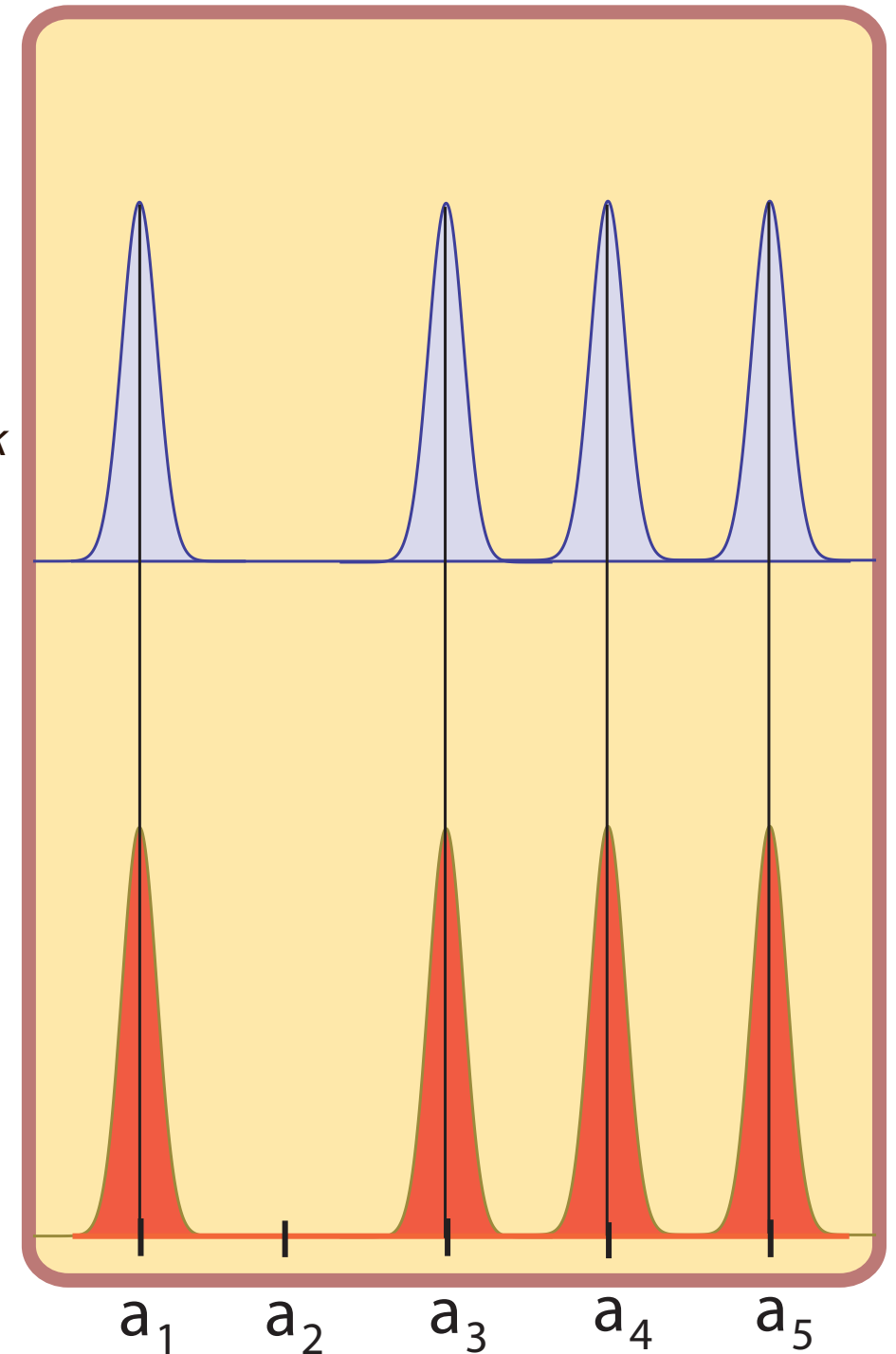
J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

Measurement Theory

- time evolution of composite system

$$\begin{aligned} |\psi_f\rangle |\phi_f\rangle &= e^{-i\int \hat{H}_{\text{int}} dt} |\psi_i\rangle |\phi_i\rangle \\ &= \sum \alpha_k \int dq e^{-(q-a_k)^2/(4\Delta^2)} |a_k\rangle |q\rangle \end{aligned}$$

superposition of Gaussians of width Δ centered at a_k



J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

Measurement Theory

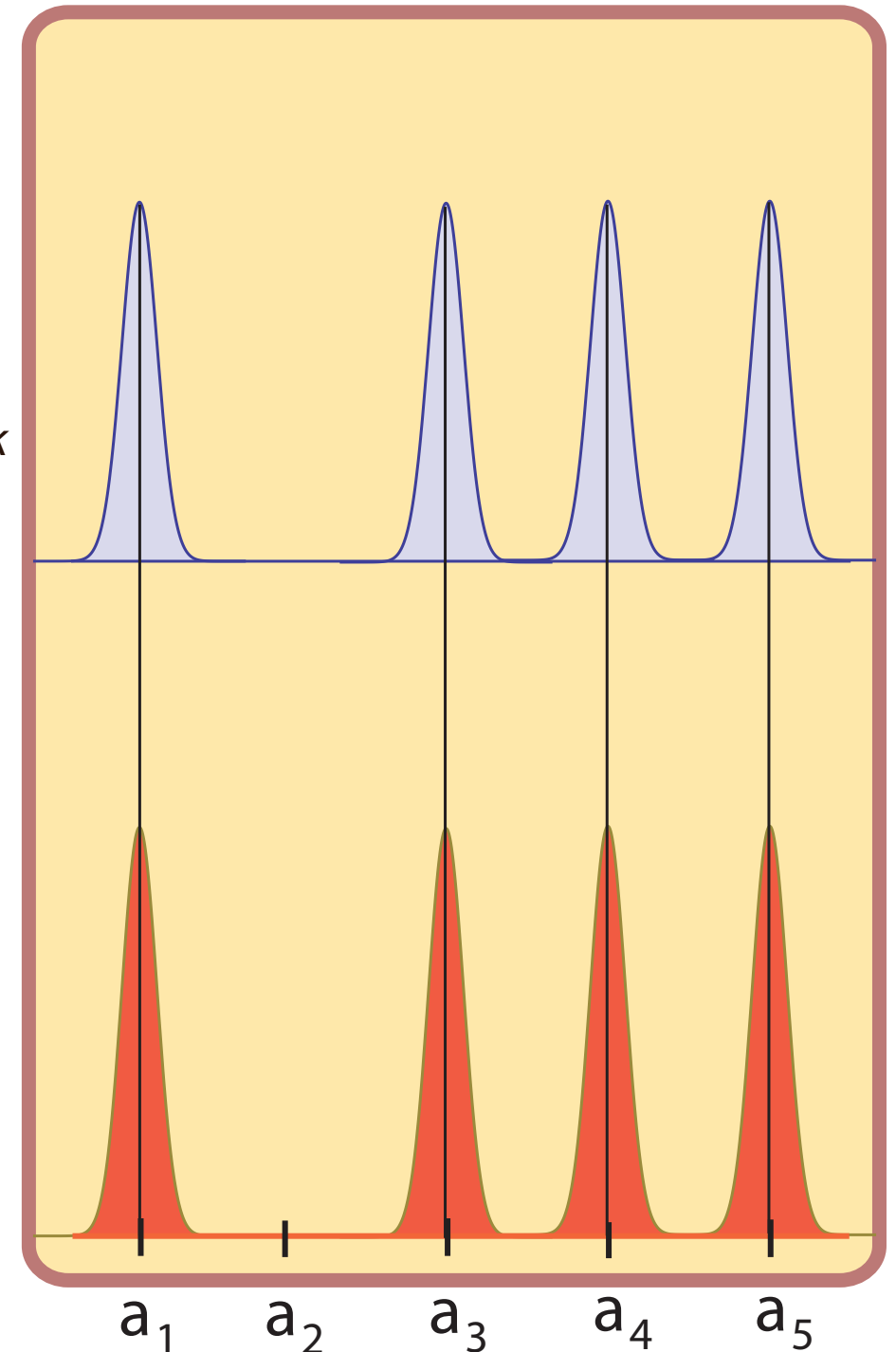
- time evolution of composite system

$$|\psi_f\rangle|\phi_f\rangle = e^{-i\int \hat{H}_{\text{int}} dt} |\psi_i\rangle|\phi_i\rangle$$
$$= \sum \alpha_k \int dq e^{-(q-a_k)^2/(4\Delta^2)} |a_k\rangle|q\rangle$$

superposition of Gaussians of width Δ centered at a_k

- probability distribution of the pointer states

$$Prop(p) = \sum |\alpha_k|^2 e^{-2\Delta^2(p-a_k)^2}$$



J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

Measurement Theory

- time evolution of composite system

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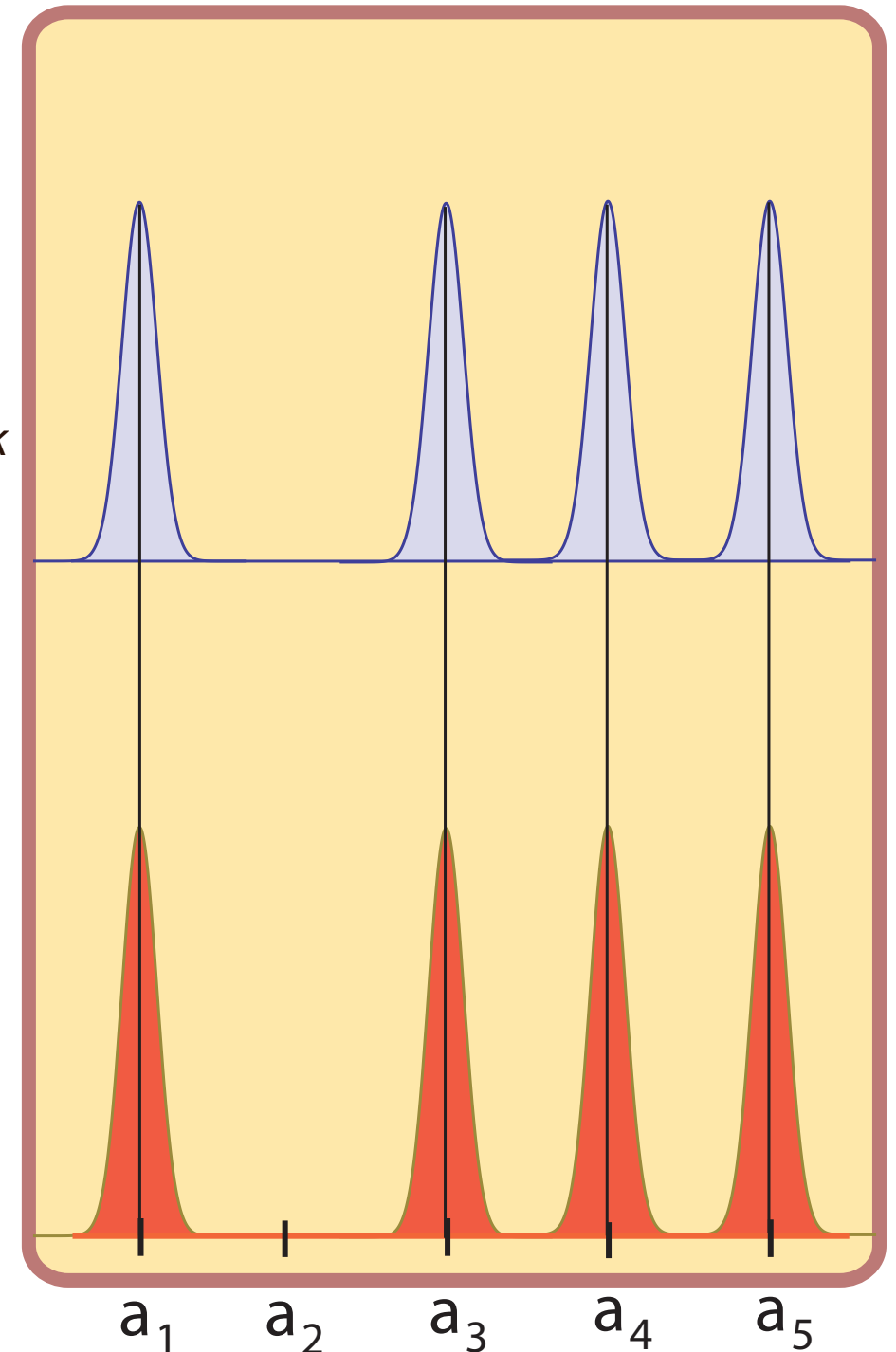
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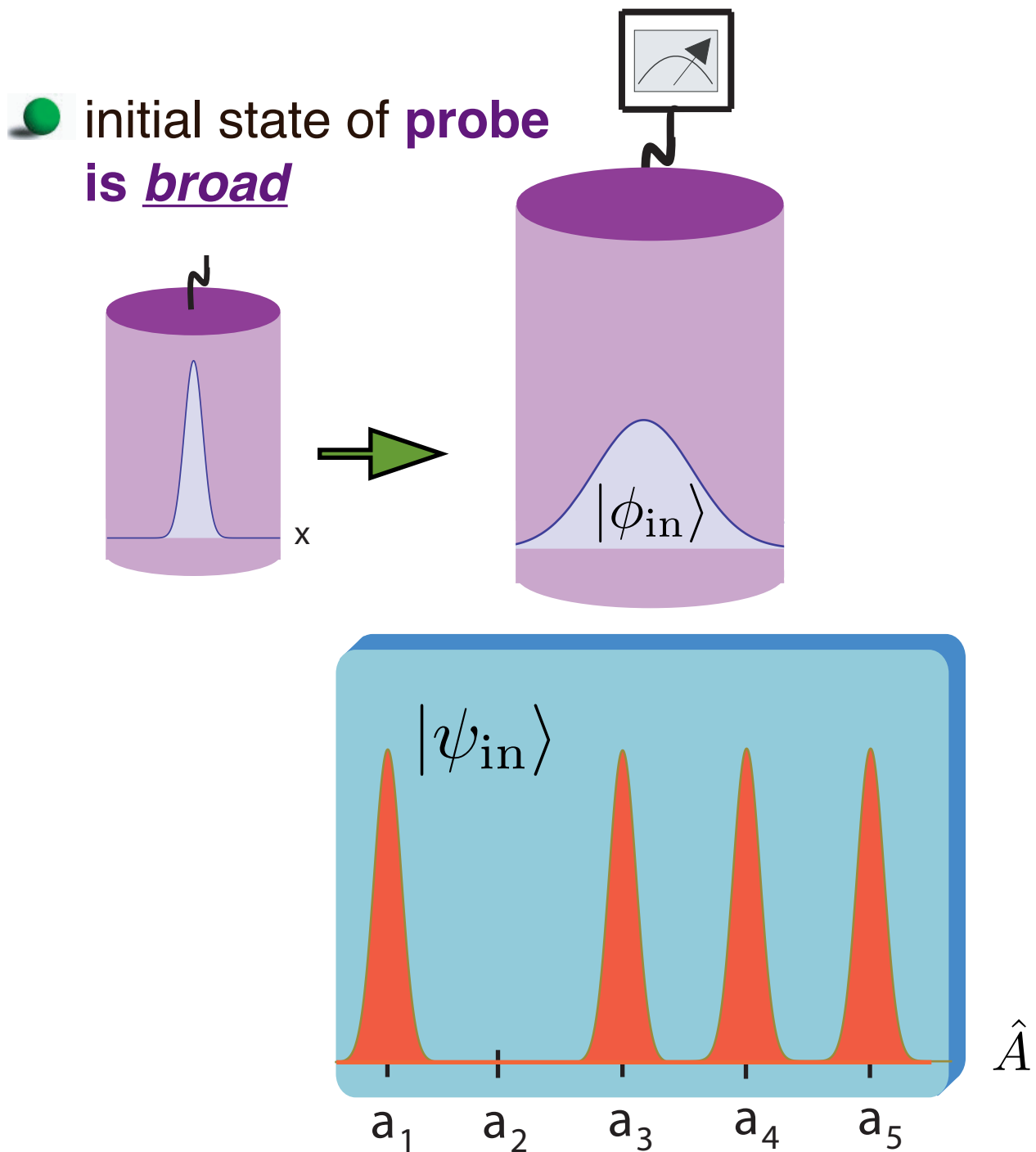
- $\lim \Delta \rightarrow 0$ usual strong measurement

- only possible results are eigenvalues a_k
- probability of outcome a_k is $|\alpha_k|^2$
- if measurement yields a_k system left in eigenstate $|A = a_k\rangle$



J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, Univ. Press. Princeton, (1955).

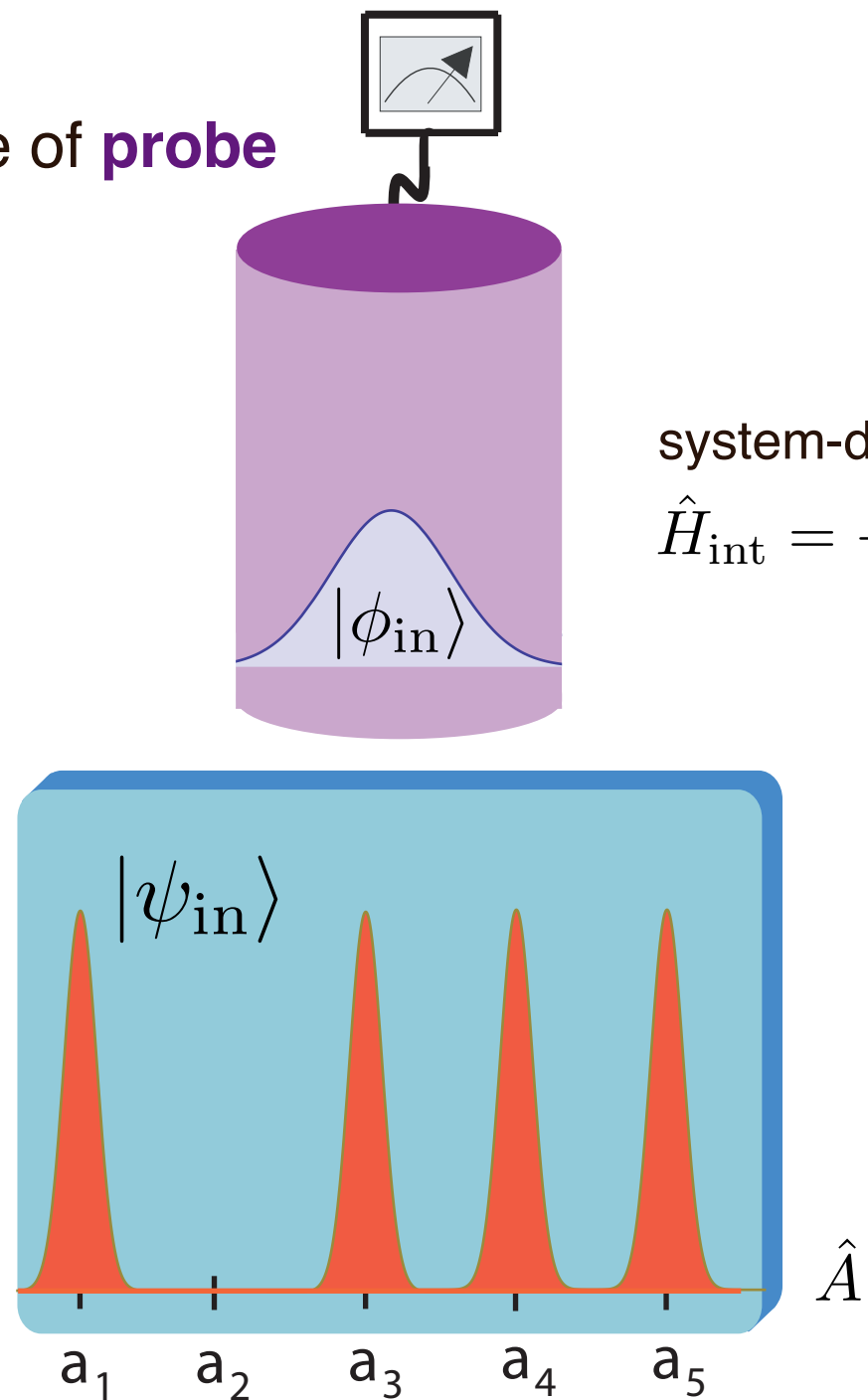
Weak Measurements



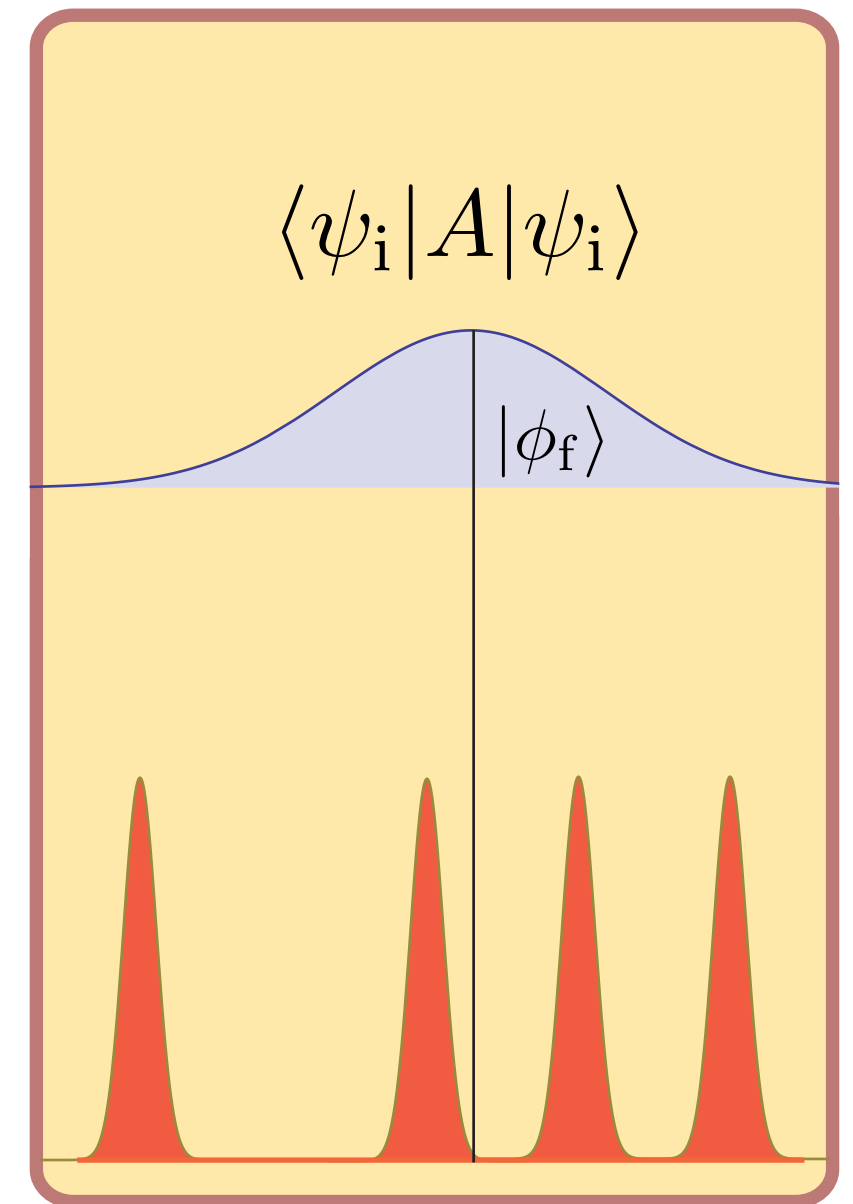
Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Weak Measurements

- initial state of **probe** is broad



- do many measurements !



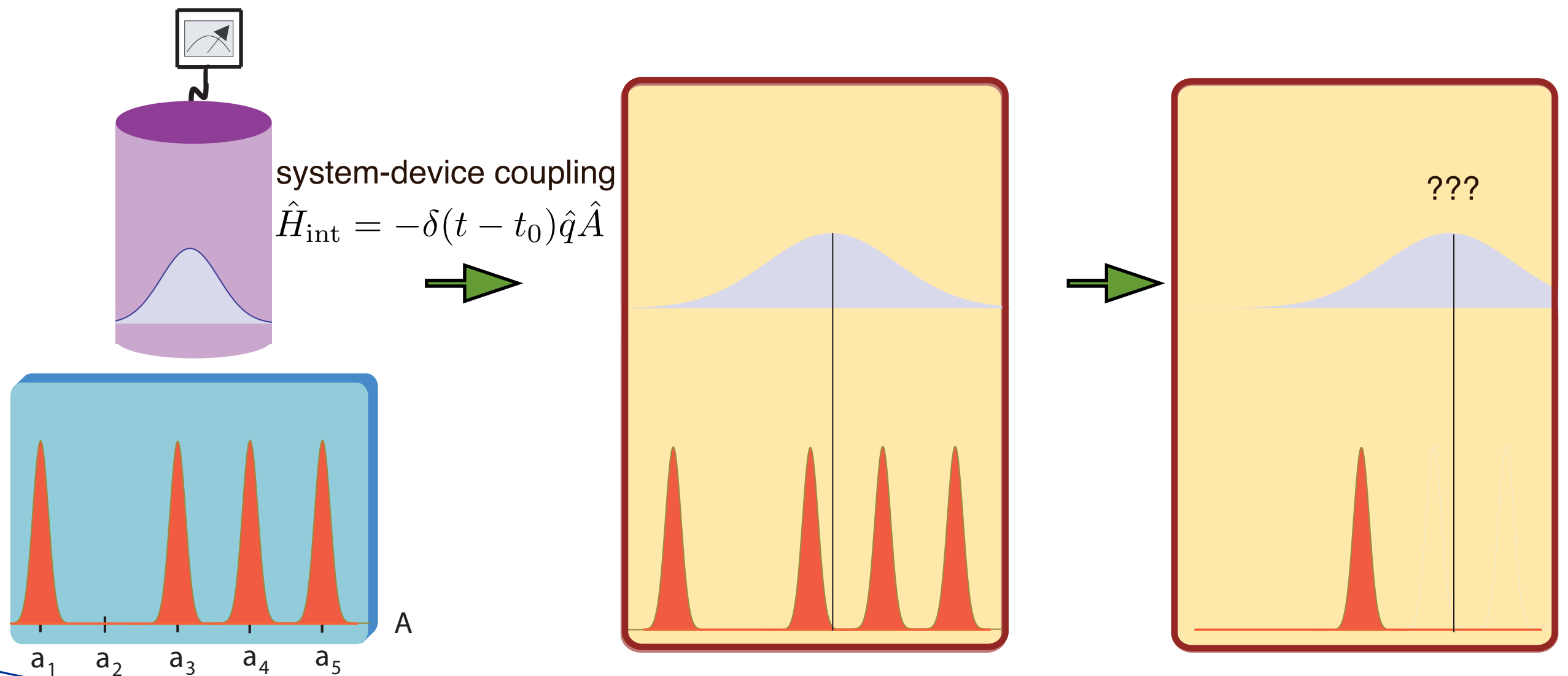
- strong overlapping Gaussians peaked at expectation value of A

Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Weak Measurements including Post-Selection

strong measurement on other observable B on quantum system

$$|\psi_f\rangle = |B = b\rangle = \sum \alpha' |A = a_n\rangle \quad |\phi_f\rangle = \langle\psi_f| e^{-i \int H_{\text{int}} dt} |\psi_i\rangle |\phi_i\rangle$$

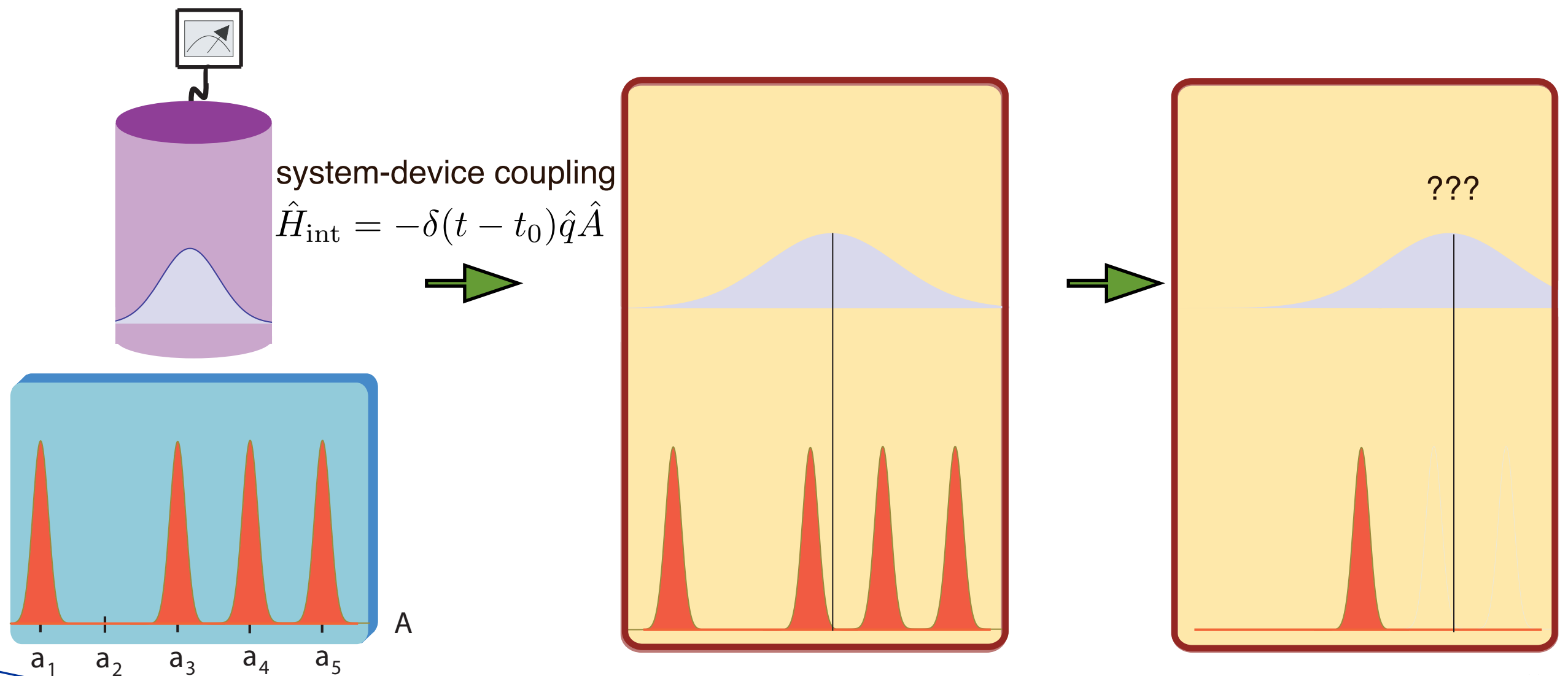


● strong overlapping Gaussians peaked **not** a expectation value of A
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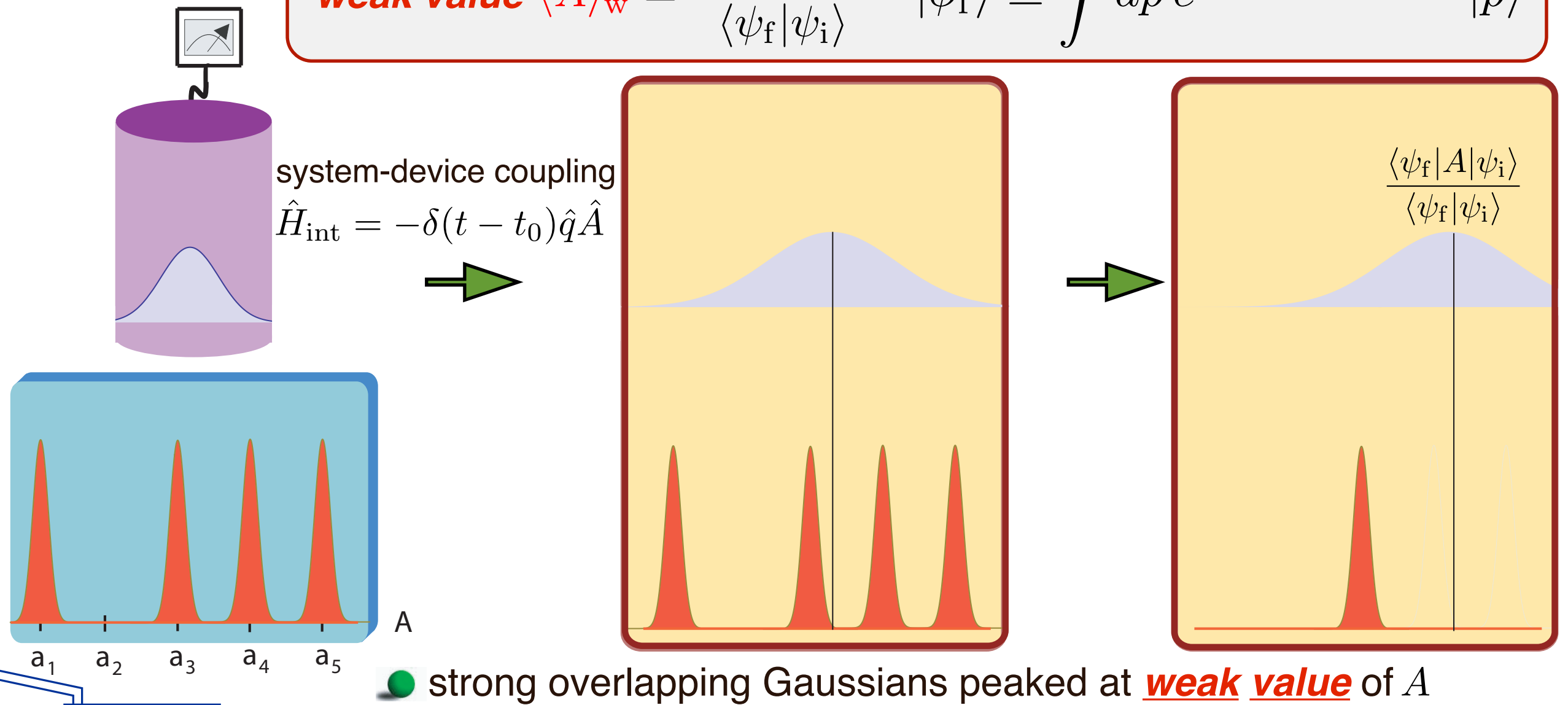
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Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

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● Properties of the **weak value**:

● $\langle A \rangle_w \in \mathbb{C}$ weak value is a **complex** number

Weak Measurements including Post-Selection

strong measurement on other observable B on quantum system

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- $\langle A \rangle_w \in \mathbb{C}$ weak value is a **complex** number
- $\Re \langle A \rangle_w$ real part: shift of pointer **value** of observable (shift of center of Gaussian in p -space)

Weak Measurements including Post-Selection

strong measurement on other observable B on quantum system

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- $\Re \langle A \rangle_w$ real part: shift of pointer **value** of observable (shift of center of Gaussian in p -space)
- $\Im \langle A \rangle_w$ imaginary part: information on **back-action** (shift of center of Gaussian in q -space; q coupled via $\hat{H}_{\text{int}}$)

Weak Measurements including Post-Selection

strong measurement on other observable B on quantum system

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- Meaning of the **weak measurements**
 - describes the **effect** a quantum system has on a measuring device (probe system) when weakly interacting (no projection)

Weak Measurements including Post-Selection

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- value an observable A would have on **average (during the evolution)** for the **given** pre- and post-selected ensemble

Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, *Phys. Rev.* **135B**, 1410 (1964).

Y. Aharonov, D. Albert, and L. Vaidman, *Phys. Rev. Lett.* **60**, 1351 (1988).

Weak Measurements including Post-Selection

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● Meaning of the **weak measurements**

- describes the **effect** a quantum system has on a measuring device (probe system) when weakly interacting
- value an observable A would have on **average (during the evolution)** for the **given** pre- and post-selected ensemble
- **time symmetric formulation of QM:** pre- and post-selected state travel in both directions of time, interfering at measurement

B. Reznik and Y. Aharonov, *Phys. Rev. A* **52**, 2538 (1995).

Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, *Phys. Rev.* **135B**, 1410 (1964).

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● Meaning of the **weak measurements**

- describes the **effect** a quantum system has on a measuring device (probe system) when weakly interacting
- value an observable A would have on **average (during the evolution)** for the **given** pre- and post-selected ensemble

Weak Measurements including Post-Selection

strong measurement on other observable B on quantum system

$$|\psi_f\rangle = |B = b\rangle = \sum \alpha' |A = a_n\rangle \quad |\phi_f\rangle = \sum \alpha \alpha'^* \int dq e^{iq a_n} e^{-q^2/(4\Delta^2)} |q\rangle$$

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B. Reznik and Y. Aharonov, *Phys. Rev. A* **52**, 2538 (1995).

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● Meaning of the **weak measurements**

● **effect** on a measuring device

Outside standard Quantum Mechanics !!!

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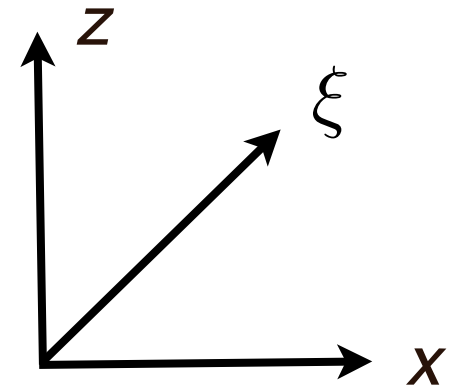
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Example: Weak Measurements of Spin

- initial state $|\psi_i\rangle = |\uparrow_x\rangle$ (*pre-selection*)
- final state $|\psi_f\rangle = |\uparrow_z\rangle$ $|\uparrow_x\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle + |\downarrow_z\rangle)$
- observable $A = \sigma_\xi = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$

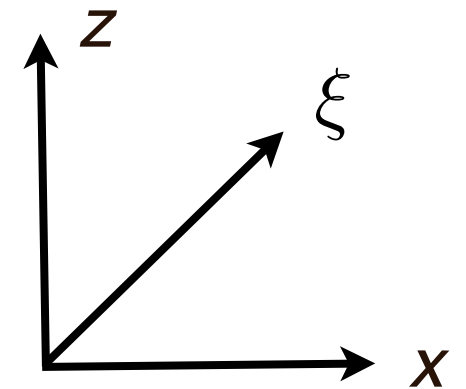


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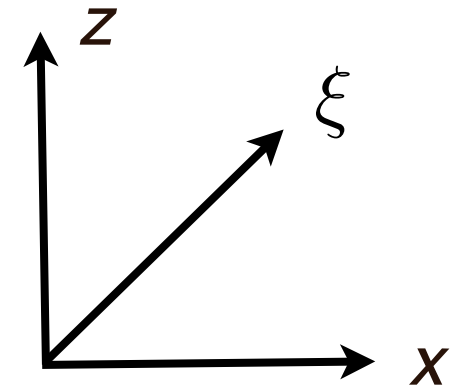
● weak value $\langle\sigma_\xi\rangle_w = \frac{\langle\psi_f|\sigma_\xi|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} = \sqrt{2}$ ● expectation value $\langle\sigma_\xi\rangle = \frac{1}{\sqrt{2}}$

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$\hat{H}_{\text{int}} = -\delta(t - t_0)\hat{q}\hat{A}$

$|\uparrow_x\rangle = \frac{1}{2}\left(\cos\frac{\pi}{4}|\uparrow_\xi\rangle + \sin\frac{\pi}{4}|\downarrow_\xi\rangle\right)$

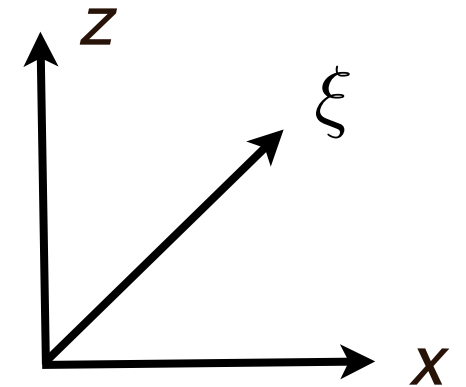
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$$\text{Prob}(q) = \left(\cos^2\frac{\pi}{4} e^{-(q-1)^2/(2\Delta^2)} + \sin^2\frac{\pi}{4} e^{-(q-1)^2/(2\Delta^2)} \right)$$

Example: Weak Measurements of Spin

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for $\Delta \ll 1$ strong measurement, peaks at eigenvalues

for $\Delta \gg 1$ without post-selection, peaks at expectation value

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for $\Delta \ll 1$ strong measurement, peaks at eigenvalues

for $\Delta \gg 1$ **without post-selection**, peaks at expectation value

● post-selection on final state $|\psi_f\rangle = |\uparrow_z\rangle = \frac{1}{2} \left(\sin \frac{\pi}{4} |\uparrow_\xi\rangle - \cos \frac{\pi}{4} |\downarrow_\xi\rangle \right)$

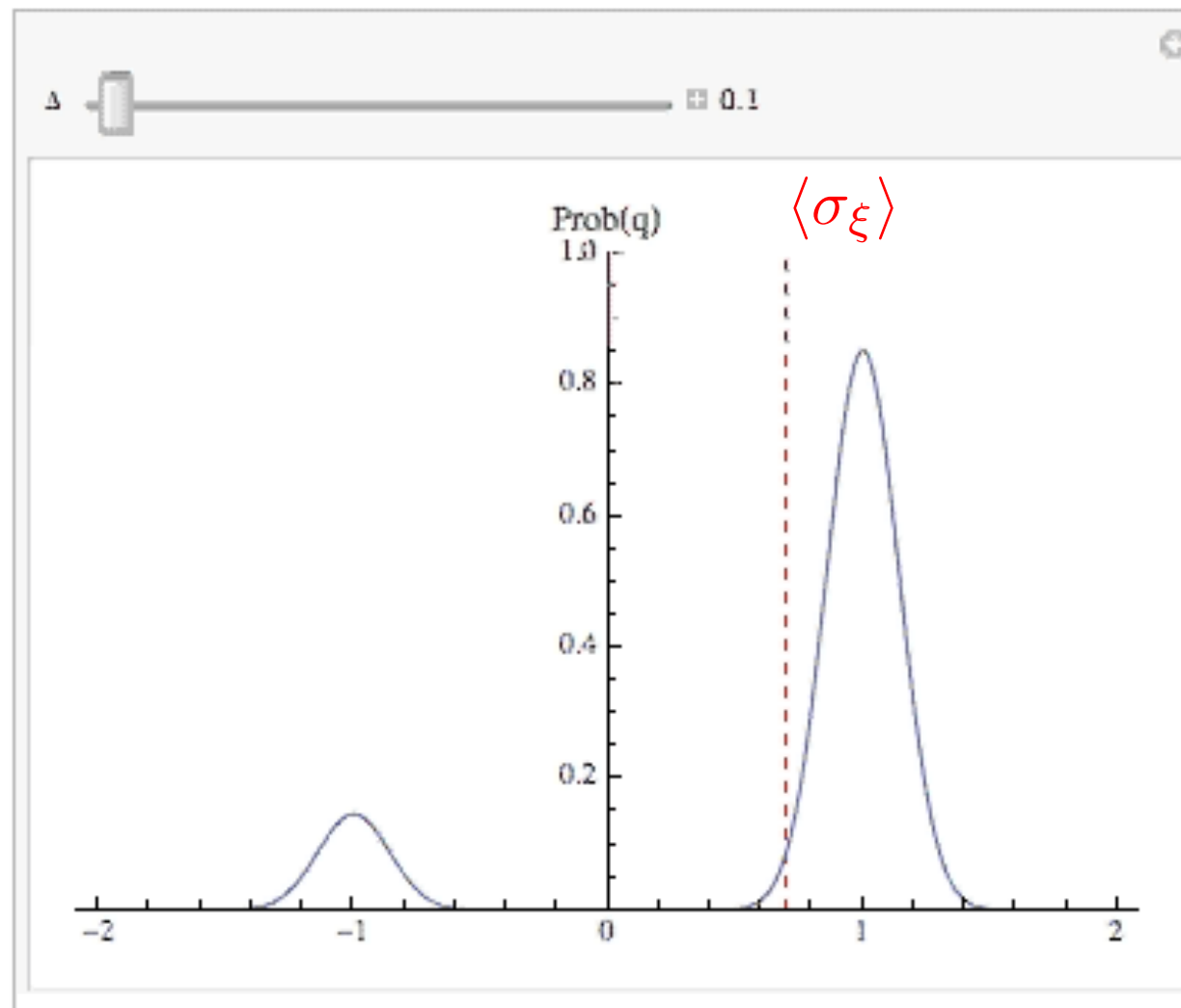
$$|\phi_f\rangle = \int dq \left(\cos \frac{\pi}{4} e^{-(q-1)^2/(4\Delta^2)} - \sin \frac{\pi}{4} e^{-(q-1)^2/(4\Delta^2)} \right) |q\rangle$$
$$Prob(q) = \left(\cos \frac{\pi}{4} e^{-(q-1)^2/(4\Delta^2)} - \sin \frac{\pi}{4} e^{-(q-1)^2/(4\Delta^2)} \right)^2$$

for $\Delta \ll 1$ strong measurement, peaks at eigenvalues

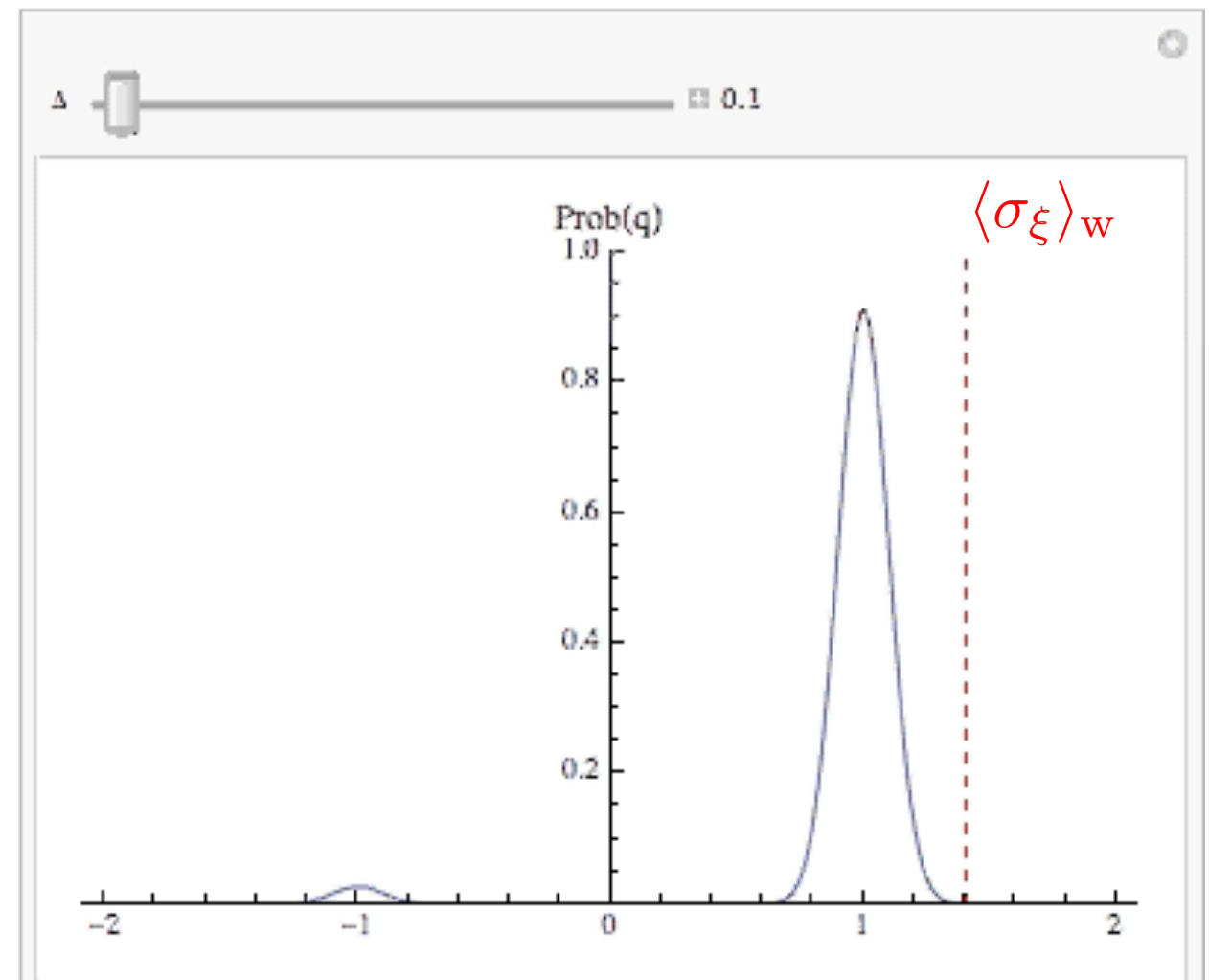
for $\Delta \gg 1$ **with post-selection**, peaks at **weak value**

Example: Weak Measurements of Spin

● without *post-selection*



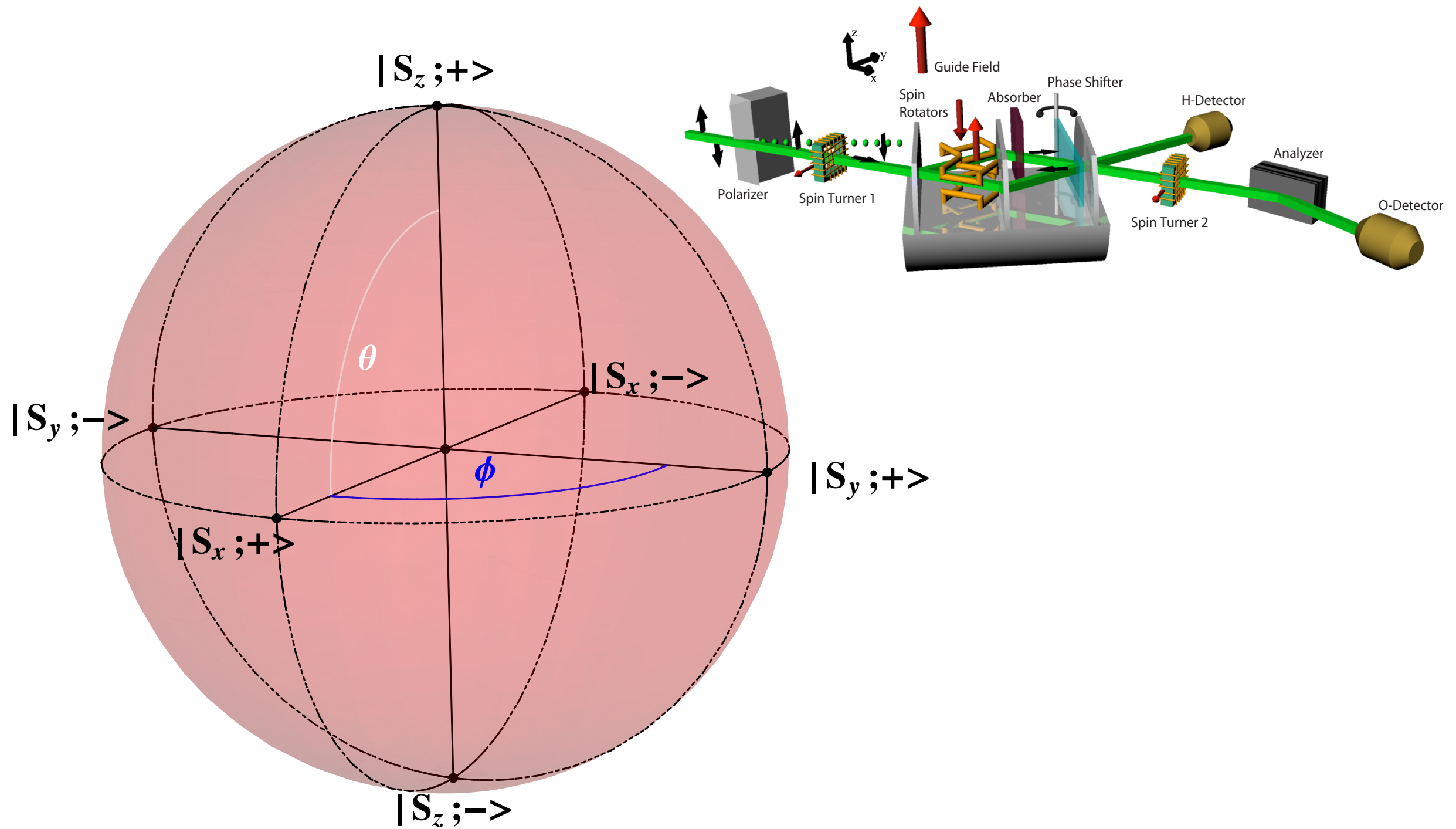
● with *post-selection*



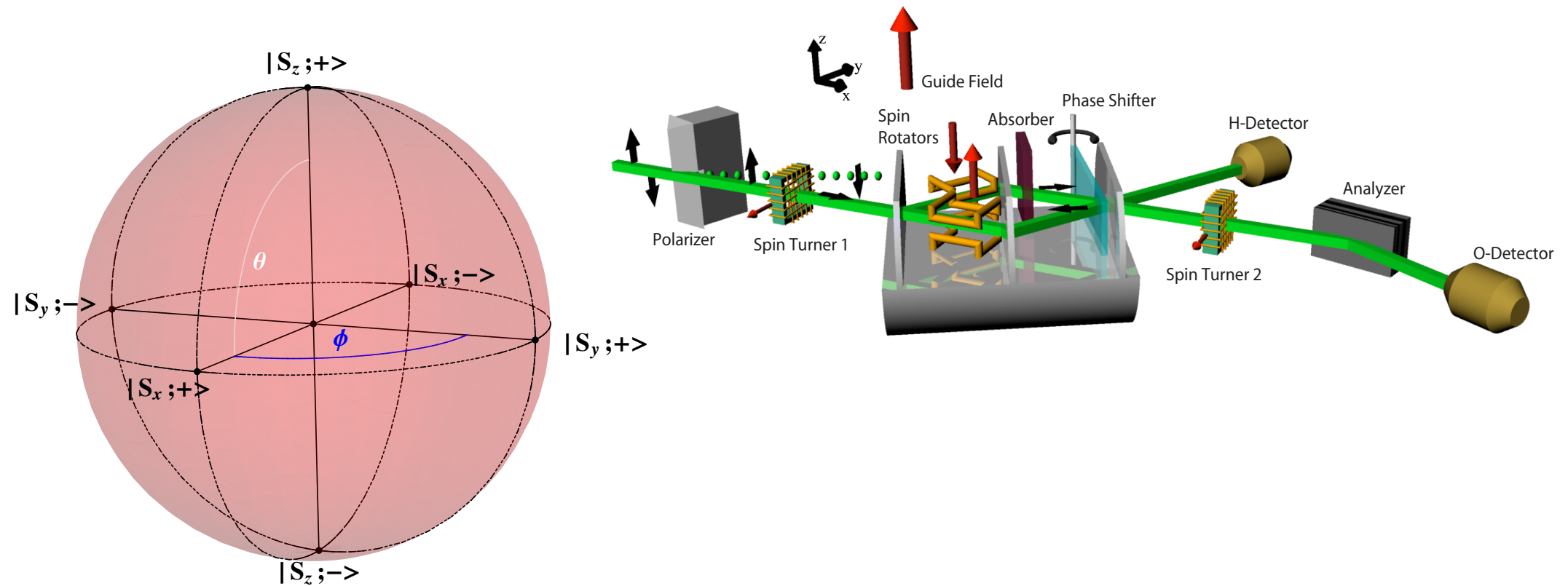
..... expectation value $\langle \sigma_\xi \rangle = 1/\sqrt{2}$

..... weak value $\langle \sigma_\xi \rangle_w = \sqrt{2}$

Experimental Setup: Spin a Two-Level System

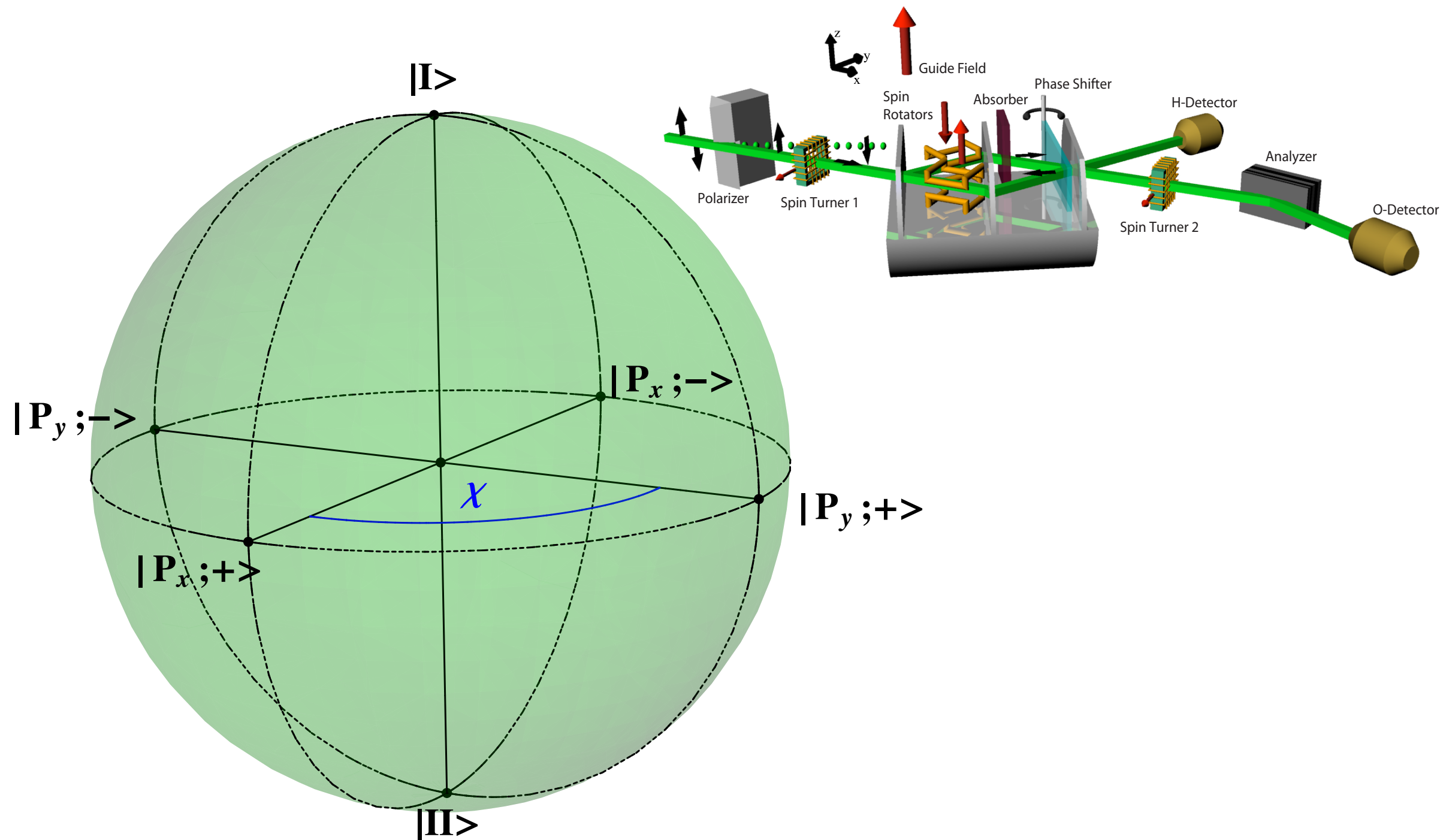


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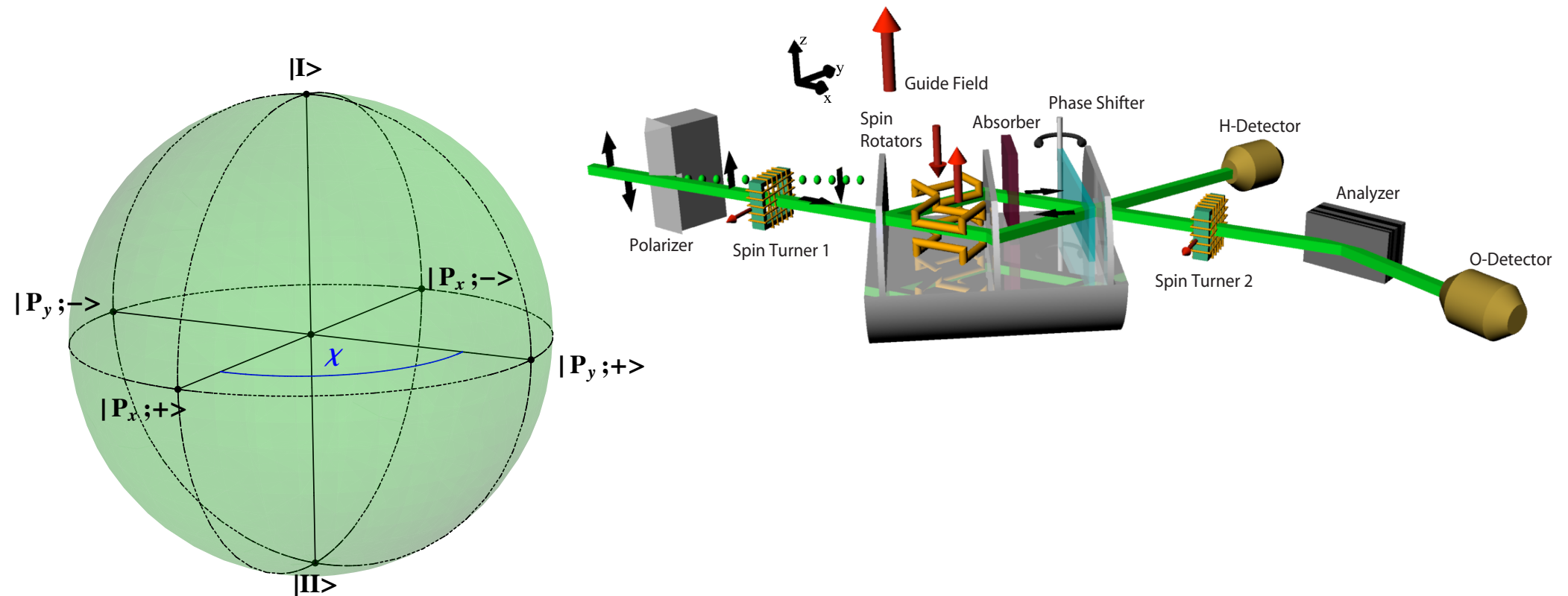


- $|S_z; \pm\rangle$ denotes the spin state in $\pm z$ -direction
- $|S_z; \pm\rangle$ forms a basis, so that $|S_x; \pm\rangle = \frac{1}{\sqrt{2}} (|S_z; +\rangle \pm |S_z; -\rangle)$
- ϕ is the azimuth angle on the Bloch Sphere, θ is the polar angle
- $\langle \hat{\sigma}_z^s \rangle$ denotes the Pauli matrix for the spin

Experimental Setup: Path another Two-Level System

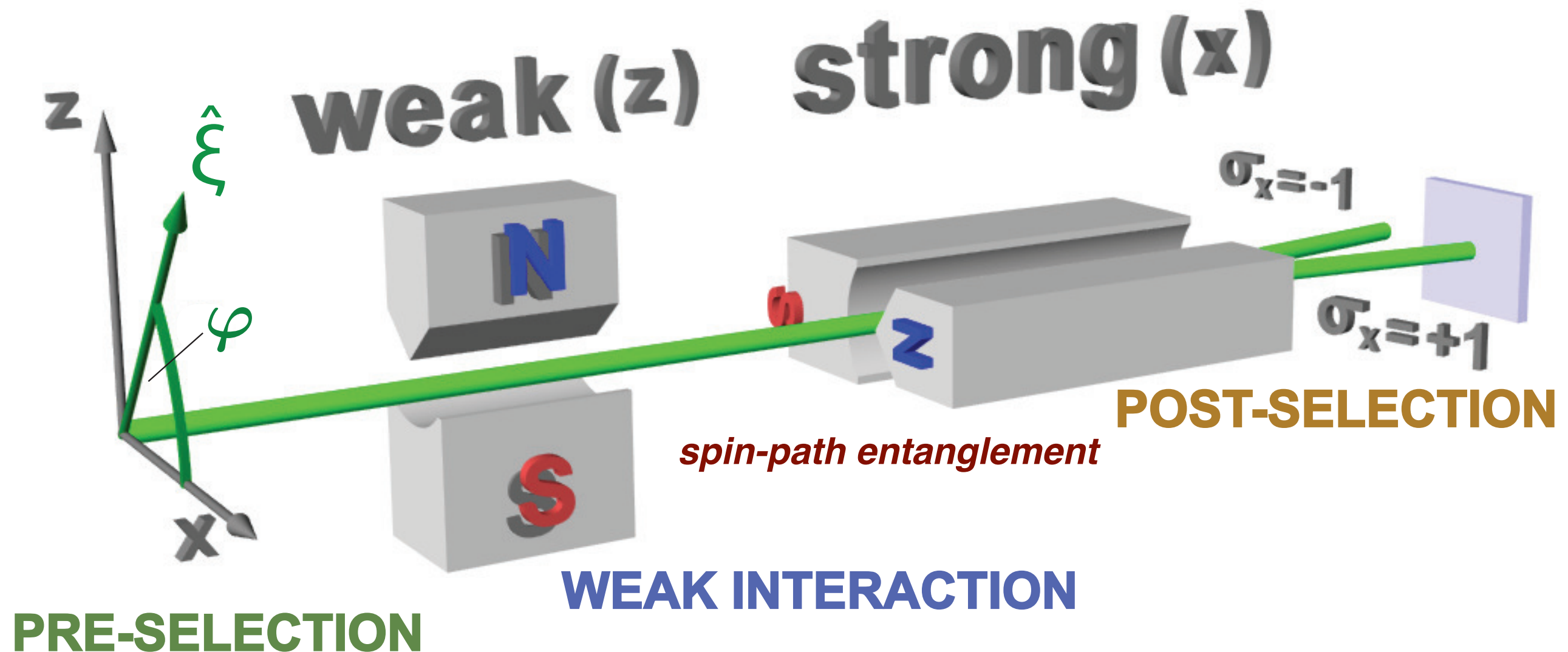


Experimental Setup: Path another Two-Level System



- The interferometer is a two level system!
- Its basis is given by the two path states $|I\rangle$ and $|II\rangle$
- The phase shifter φ scans the xy-plane of the Bloch Sphere
- $\langle \hat{\sigma}_z^p \rangle$ denotes the Pauli matrix for the path

Proposed Setup by AAV (for massive Particles)

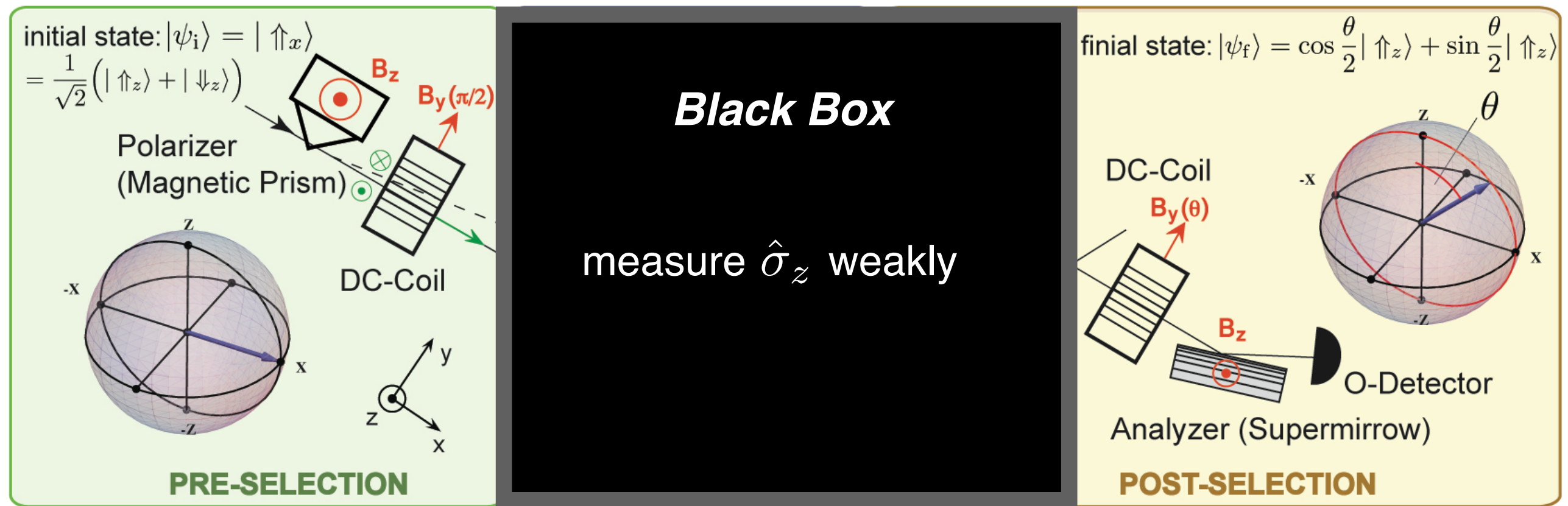


Yakir Aharonov, *Phys. Rev. Lett.* **60**, 1351 (1988).

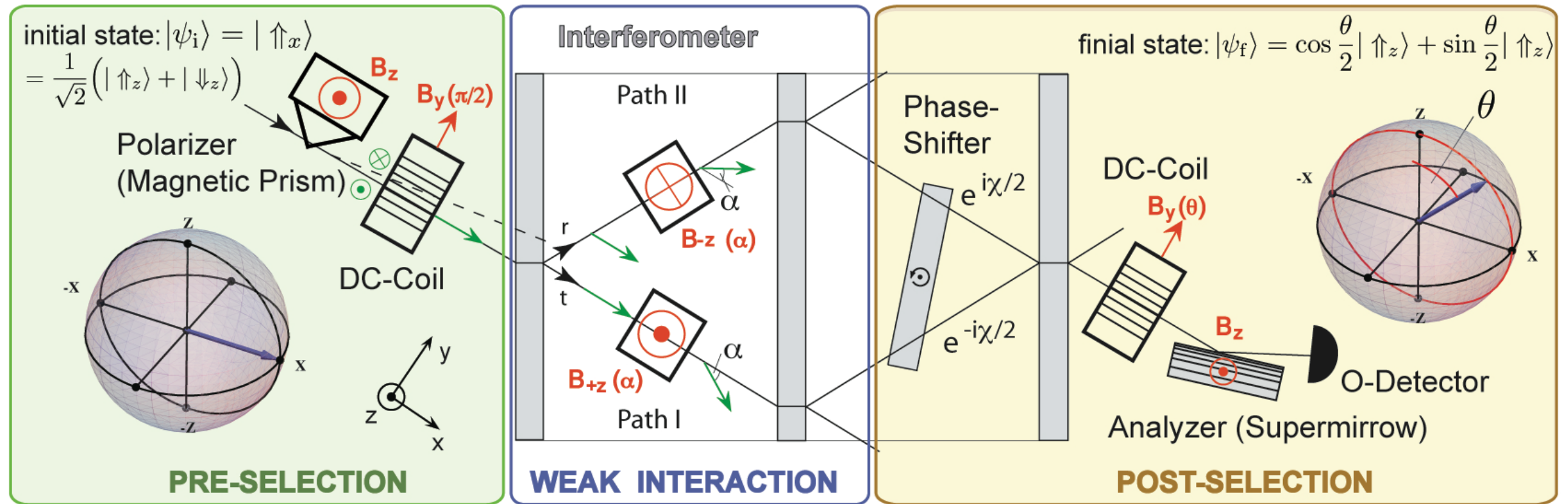
04.03.2014

FWF

Neutron optical scheme @ S18 (ILL)

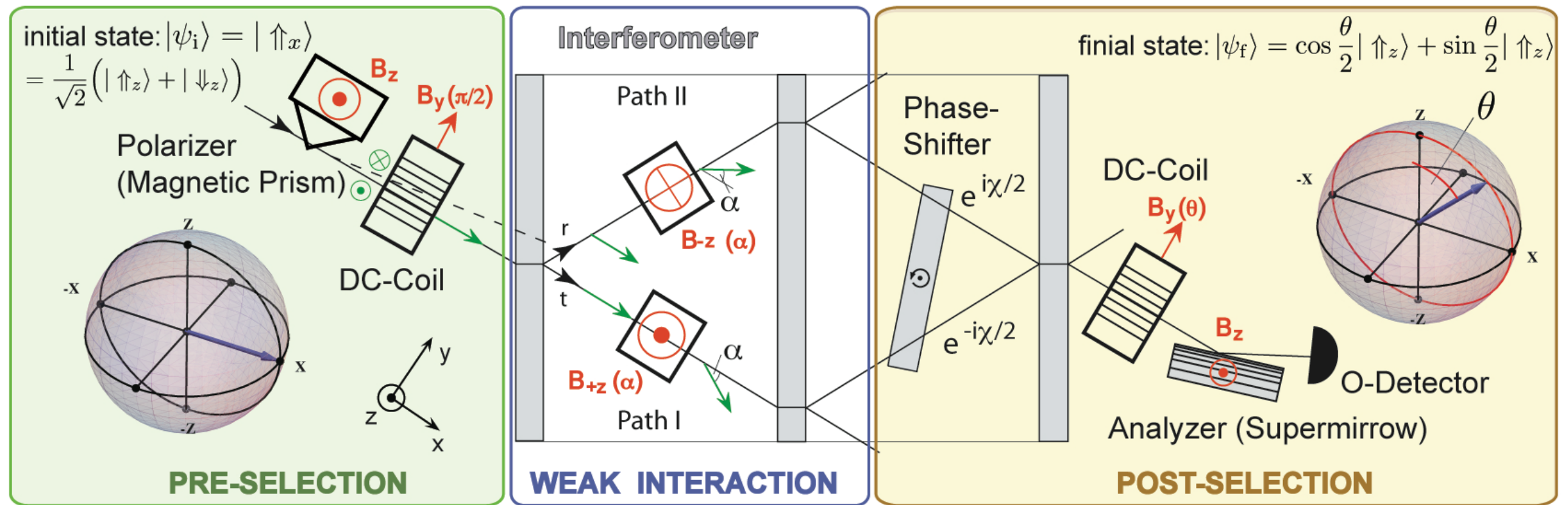


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● probe system states not **Gaussians** !
two-level system: path I and II

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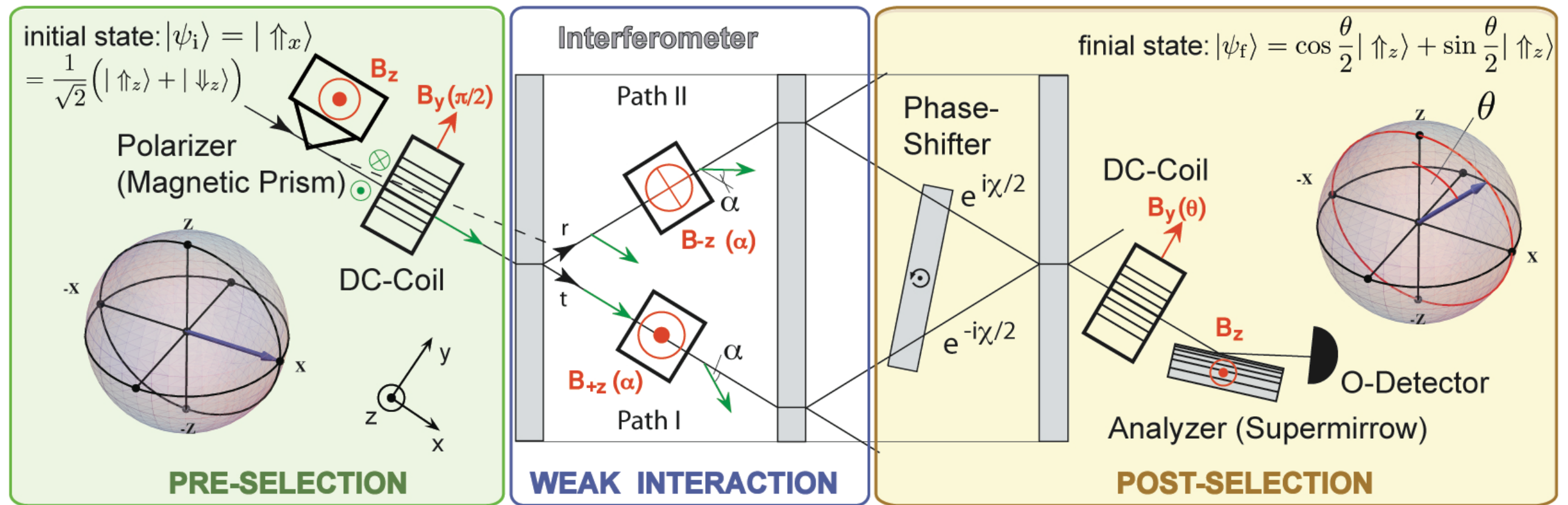
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- interaction Hamiltonian

$$\hat{H}_{\text{int}} = -\vec{\mu}\vec{B}(\hat{\Pi}_I - \hat{\Pi}_{II}) \equiv \frac{\alpha\hat{\sigma}_z^s\hat{\sigma}_z^p}{2}$$

Sponar, Denkmayr, Geppert, Lemmel, Matzkin, Hasegawa *submitted to Phys. Rev. Lett.*

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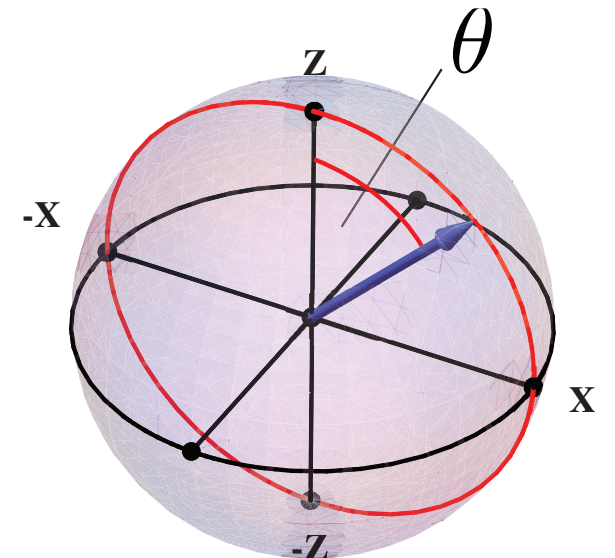
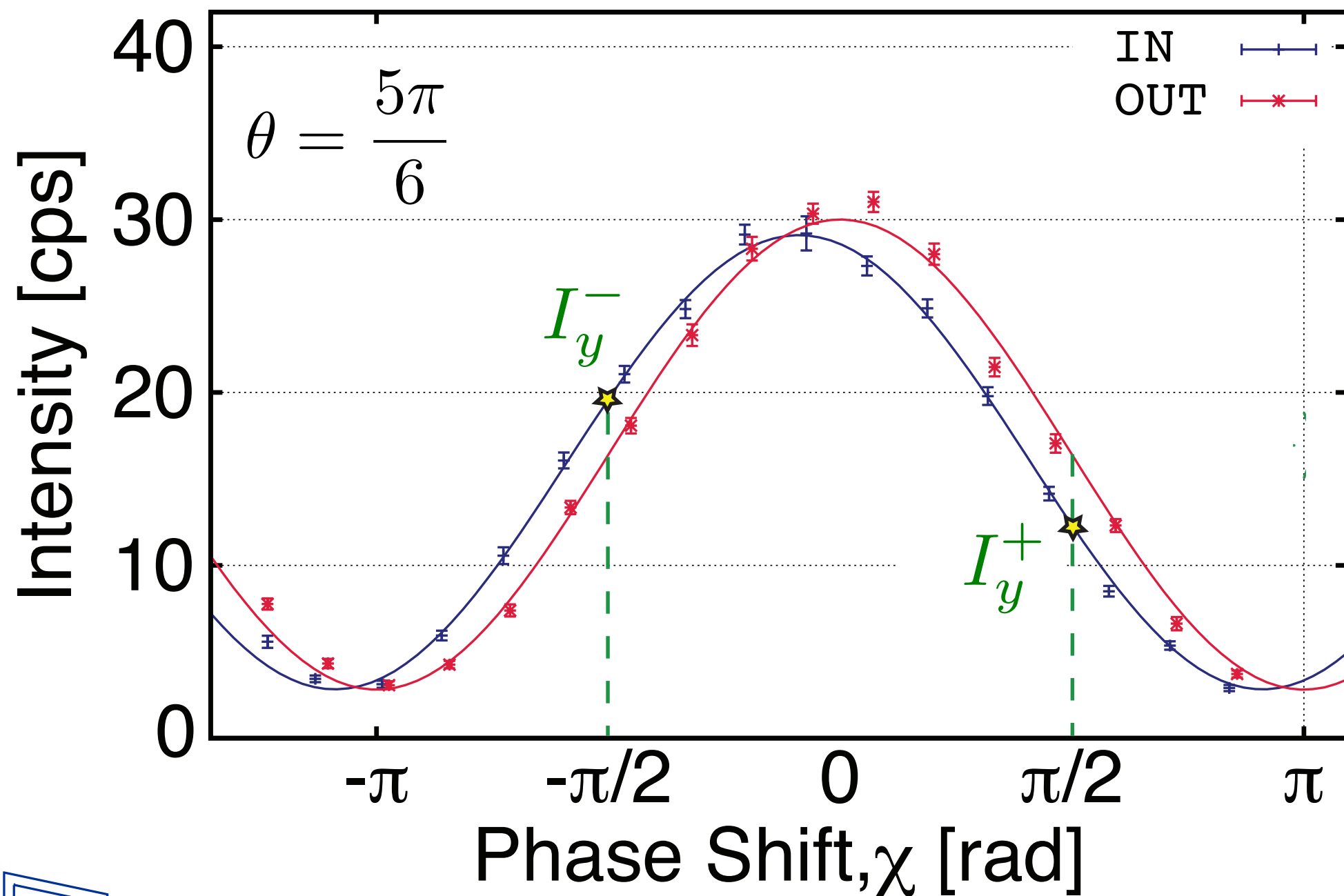
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real part of weak from count rates

$$\Re\langle\hat{\sigma}_z^S\rangle_w = \frac{1}{\alpha} \arcsin\left(\frac{I_y^+ - I_y^-}{I_y^+ + I_y^-}\right)$$

Results

● final state: $|\psi_f\rangle = \cos \frac{\theta}{2} |\uparrow_z\rangle + \sin \frac{\theta}{2} |\uparrow_x\rangle$



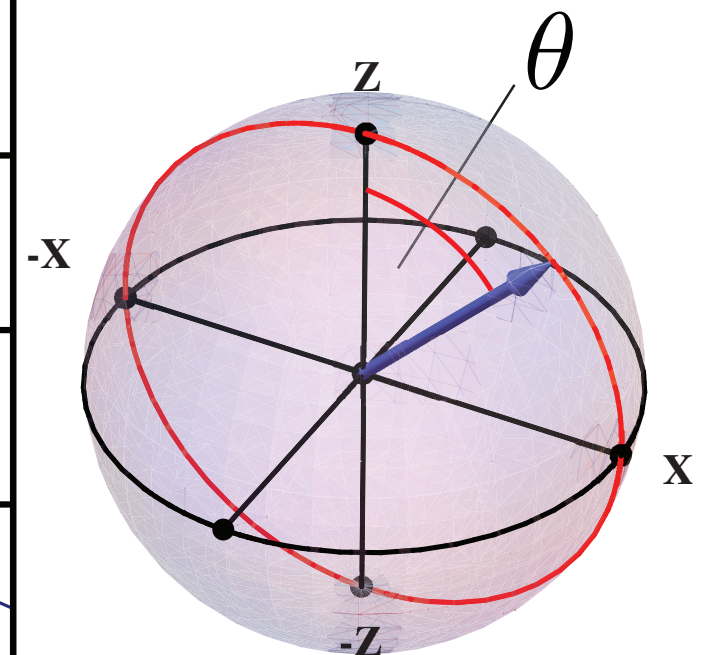
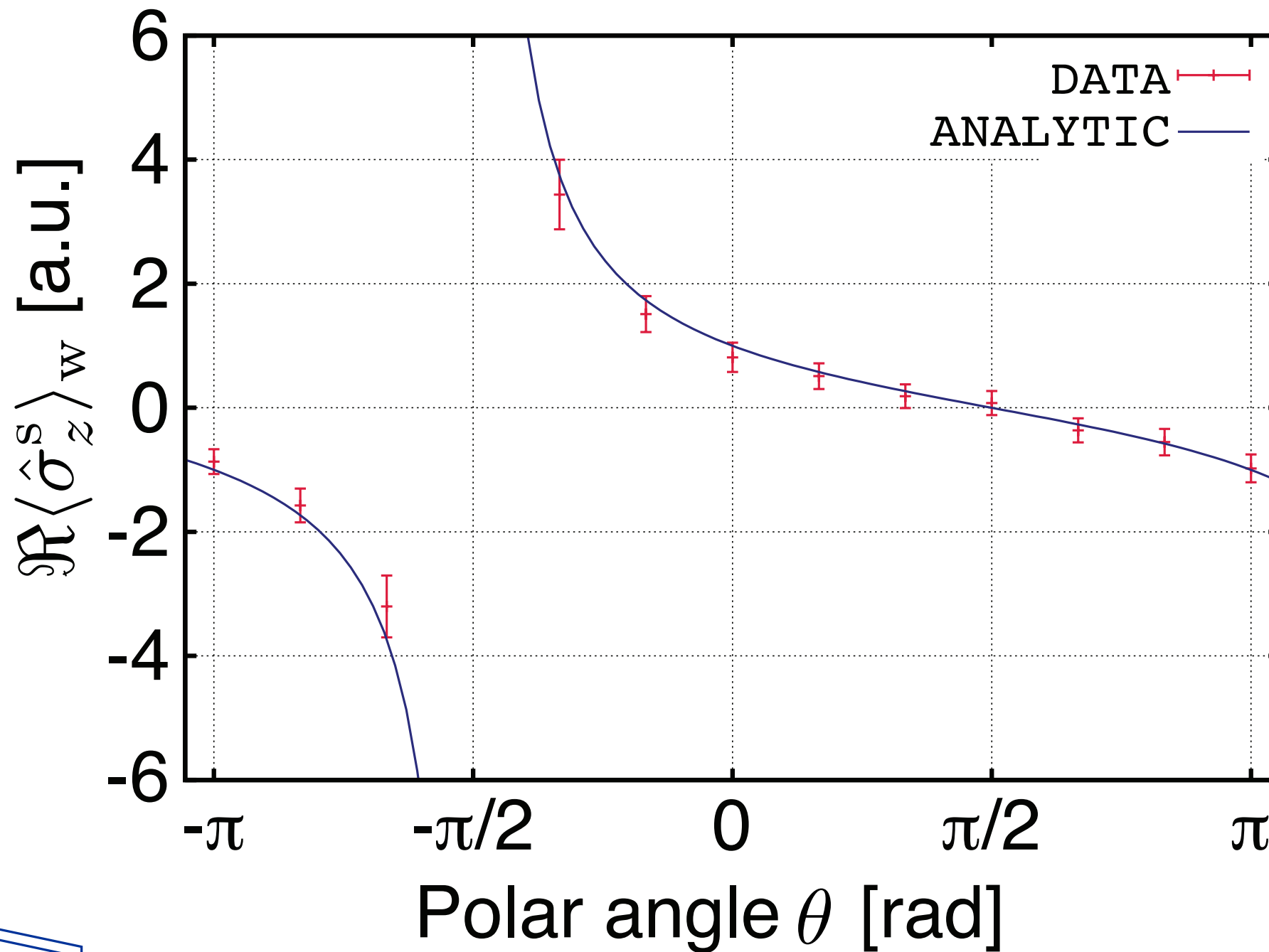
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FWF

● **Part I: Violation Error-Disturbance Uncertainty Relation**

● **Data Treatment**

Error and Disturbance for Projection Operators

● **Error:** **measurement operators:** $O_\lambda = \langle \lambda | U | \xi \rangle$ acting on obj.-space $\mathcal{H}^{\text{obj}}$ (U on $\mathcal{H}^{\text{obj}} \otimes \mathcal{H}^{\text{app}}$)

$$\epsilon(A)^2 = \left\| \sum_{\lambda} O_{\lambda} (\lambda - A) |\psi\rangle \right\|^2 \quad \text{meter observable } M \text{ has basis } |\lambda\rangle : M = \sum_{\lambda} \lambda |\lambda\rangle \langle \lambda|$$

If the O_λ are mutually orthogonal projection operators sum and norm can be exchanged

$$\epsilon(A)^2 = \left\| (O_A - A) |\psi\rangle \right\|^2 \quad \text{output operator: } O_A = \sum_{\lambda} \lambda O_{\lambda}$$

both experimentally inaccessible (operator different times) !!!

● **Disturbance:** $\eta(B)^2 = \sum_{\lambda} \left\| [O_{\lambda}, B] |\psi\rangle \right\|^2$

$$\| \dots \| = \| X |\psi\rangle \| = \langle \psi | X^\dagger X | \psi \rangle^{\frac{1}{2}}$$

M. Ozawa, *Ann. Phys.* **311**, 350 (2004).

Measurement Scheme

O_A – Measurement

$$O_A = \sum_{\lambda=+, -} \lambda O_\lambda = O_+ - O_-$$

B – Measurement

$$B = \sum_{\lambda=+, -} \lambda B_\lambda = B_+ - B_-$$

$$\begin{aligned} O_+ &= | + o_a \rangle \langle + o_a | \\ O_- &= | - o_a \rangle \langle - o_a | \end{aligned}$$

$$\begin{aligned} B_+ &= | + b \rangle \langle + b | \\ B_- &= | - b \rangle \langle - b | \end{aligned}$$

$$\begin{aligned} B_+ &= | + b \rangle \langle + b | \\ B_- &= | - b \rangle \langle - b | \end{aligned}$$

I_{++}

I_{+-}

I_{-+}

I_{--}

Intensities:

$$I_{++} \propto \langle \psi | O_+^\dagger B_+ O_+ | \psi \rangle$$

$$I_{+-} \propto \langle \psi | O_+^\dagger B_- O_+ | \psi \rangle$$

$$I_{-+} \propto \langle \psi | O_-^\dagger B_+ O_- | \psi \rangle$$

$$I_{--} \propto \langle \psi | O_-^\dagger B_- O_- | \psi \rangle$$

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$|\psi\rangle$

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$$\frac{(I_{++} + I_{-+}) - (I_{+-} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi | \overbrace{\sum_{\lambda=+,-} O_\lambda^\dagger B O_\lambda}^{X_B \text{ modified output operator:}} | \psi \rangle = \langle \psi | X_B | \psi \rangle \Rightarrow \text{disturbance } \eta(B)$$

$$\frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi | O_A | \psi \rangle \Rightarrow \text{error } \epsilon(A)$$

Standard Deviations for Spin-1/2 Measurements

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

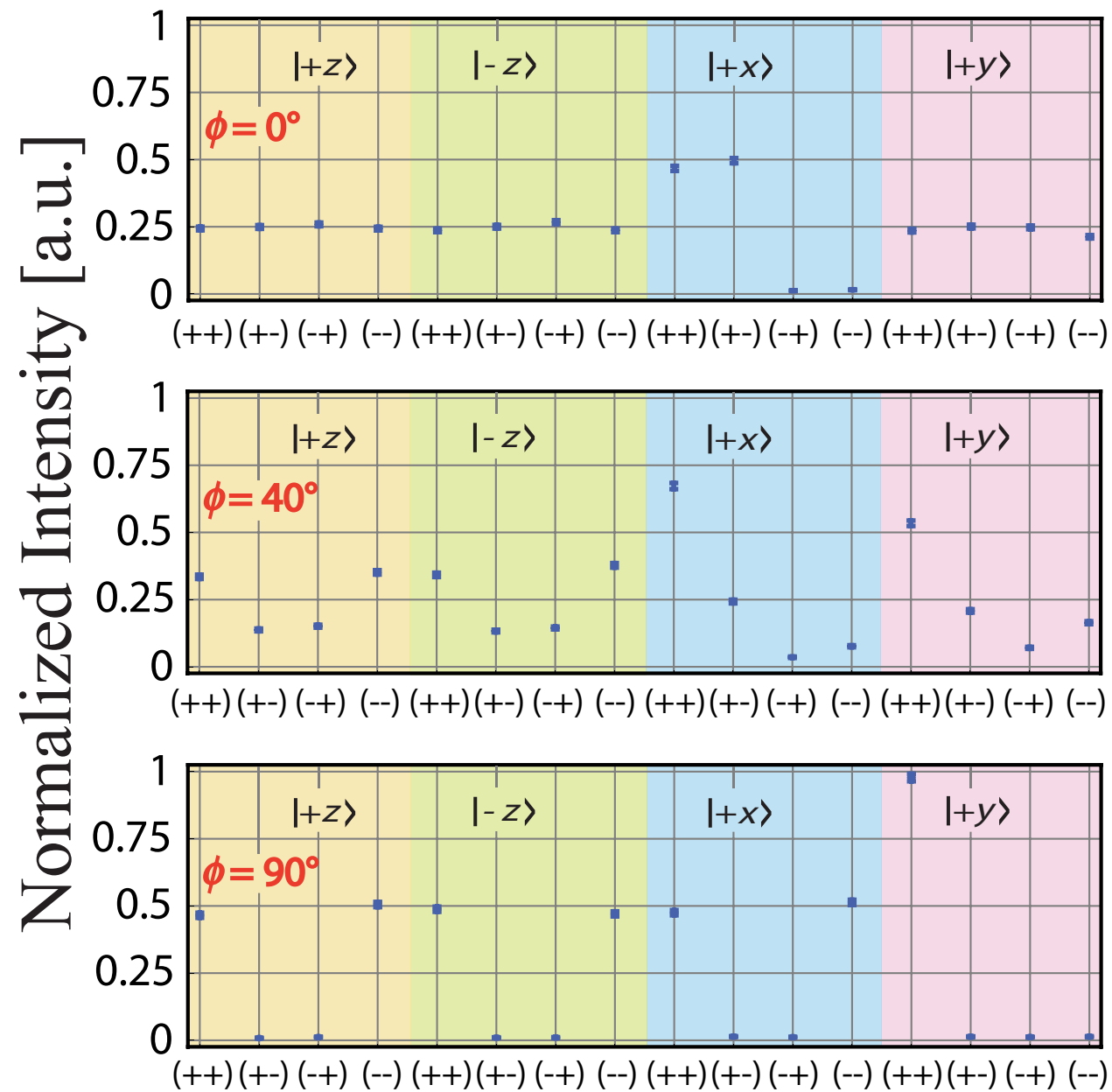
- **standard deviations** (measured separately):

$$\sigma(\hat{\sigma}_j)^2 = \underbrace{\langle \psi | \hat{\sigma}_j^2 | \psi \rangle}_1 - \langle \psi | \hat{\sigma}_j | \psi \rangle^2 = 1 - (\langle \psi | \hat{\sigma}_{j+} | \psi \rangle - \langle \psi | \hat{\sigma}_{j-} | \psi \rangle)^2 = 1 - \left(\frac{I_+ - I_-}{I_+ + I_-} \right)^2$$

$$A = \sigma_x, \quad B = \sigma_y, \quad \text{and} \quad |\psi\rangle = | + z \rangle$$

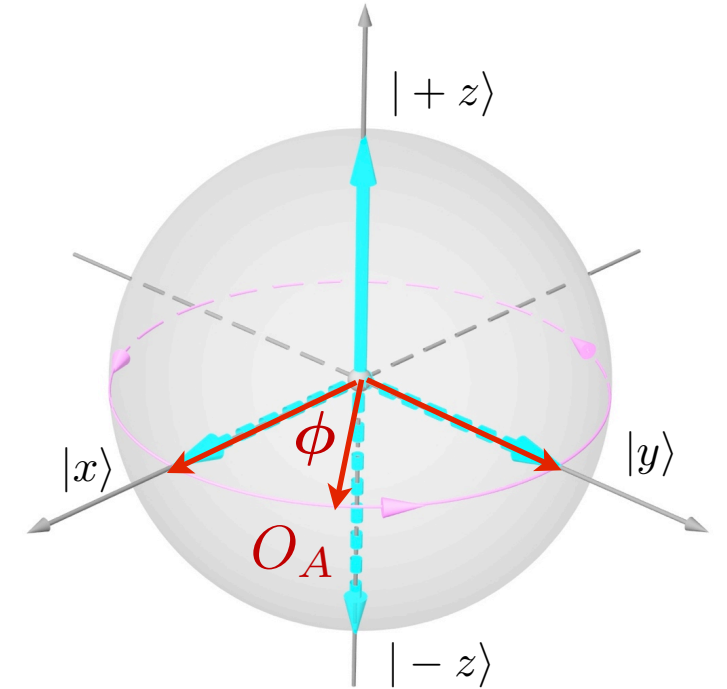
$$\sigma(\hat{\sigma}_x)^2 = \underbrace{\langle +z | \hat{\sigma}_x^2 | +z \rangle}_1 - \langle +z | \hat{\sigma}_x | +z \rangle^2 = 1 = \sigma(\hat{\sigma}_y)^2$$

Experimental Data



$$A = \sigma_x$$

$$B = \sigma_y$$



$$O_A = \sigma_\phi = \sigma_x \sin \phi + \sigma_y \cos \phi$$

$$\frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi_i | O_A | \psi_i \rangle$$

$$\frac{(I_{++} + I_{-+}) - (I_{+-} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi_i | X_B | \psi_i \rangle$$

$$|\psi_i\rangle = | + z \rangle, | - z \rangle, | x \rangle, | y \rangle$$

$$\epsilon(A)^2 = 2 + \langle +z | \sigma_\phi | +z \rangle + \langle -z | \sigma_\phi | -z \rangle - \langle x | \sigma_\phi | x \rangle$$

$$\eta(B)^2 = 2 + \langle +z | X_B | +z \rangle + \langle -z | X_B | -z \rangle - \langle y | X_B | y \rangle$$

Standard Deviations for Spin-1/2 Measurements

$$\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B) \geq \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle|$$

● standard deviations:

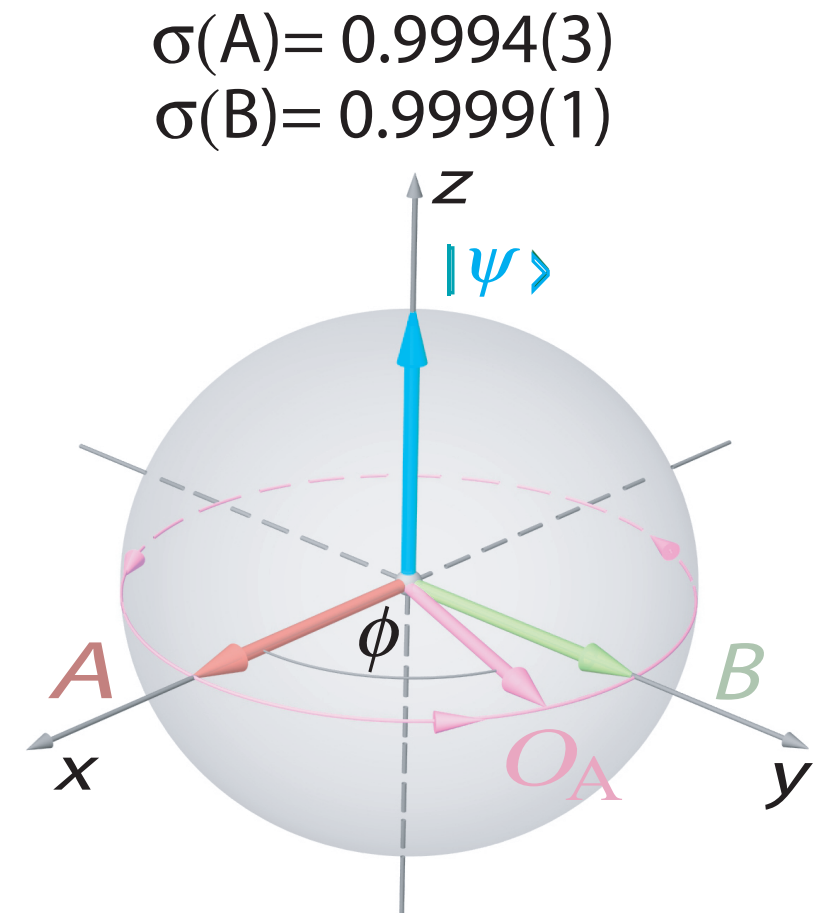
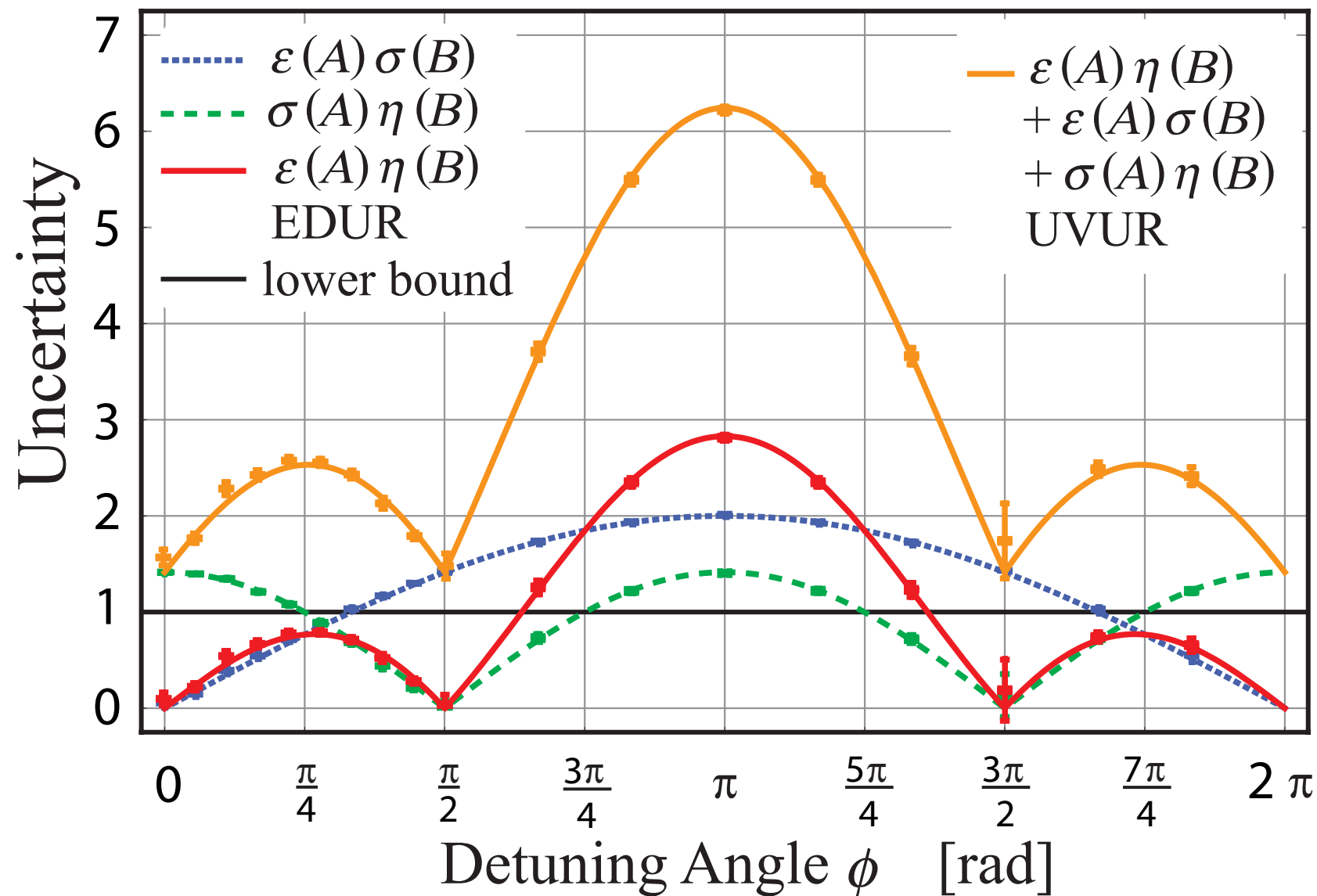
$$\sigma(\hat{\sigma}_j)^2 = \underbrace{\langle \psi | \hat{\sigma}_j^2 | \psi \rangle}_1 - \langle \psi | \hat{\sigma}_j | \psi \rangle^2 = 1 - (\langle \psi | \hat{\sigma}_{j+} | \psi \rangle - \langle \psi | \hat{\sigma}_{j-} | \psi \rangle)^2 = 1 - \left(\frac{I_+ - I_-}{I_+ + I_-} \right)^2$$

$$A = \sigma_x, \quad B = \sigma_y, \quad \text{and} \quad |\psi\rangle = | + z \rangle$$

$$\sigma(\hat{\sigma}_x)^2 = \underbrace{\langle +z | \hat{\sigma}_x^2 | +z \rangle}_1 - \langle +z | \hat{\sigma}_x | +z \rangle^2 = 1 = \sigma(\hat{\sigma}_y)^2$$

$$\bullet \text{ limit: } \frac{1}{2} |\langle \psi | [A, B] | \psi \rangle| = 1$$

Experimental Results $\phi \in [0, 2\pi]$



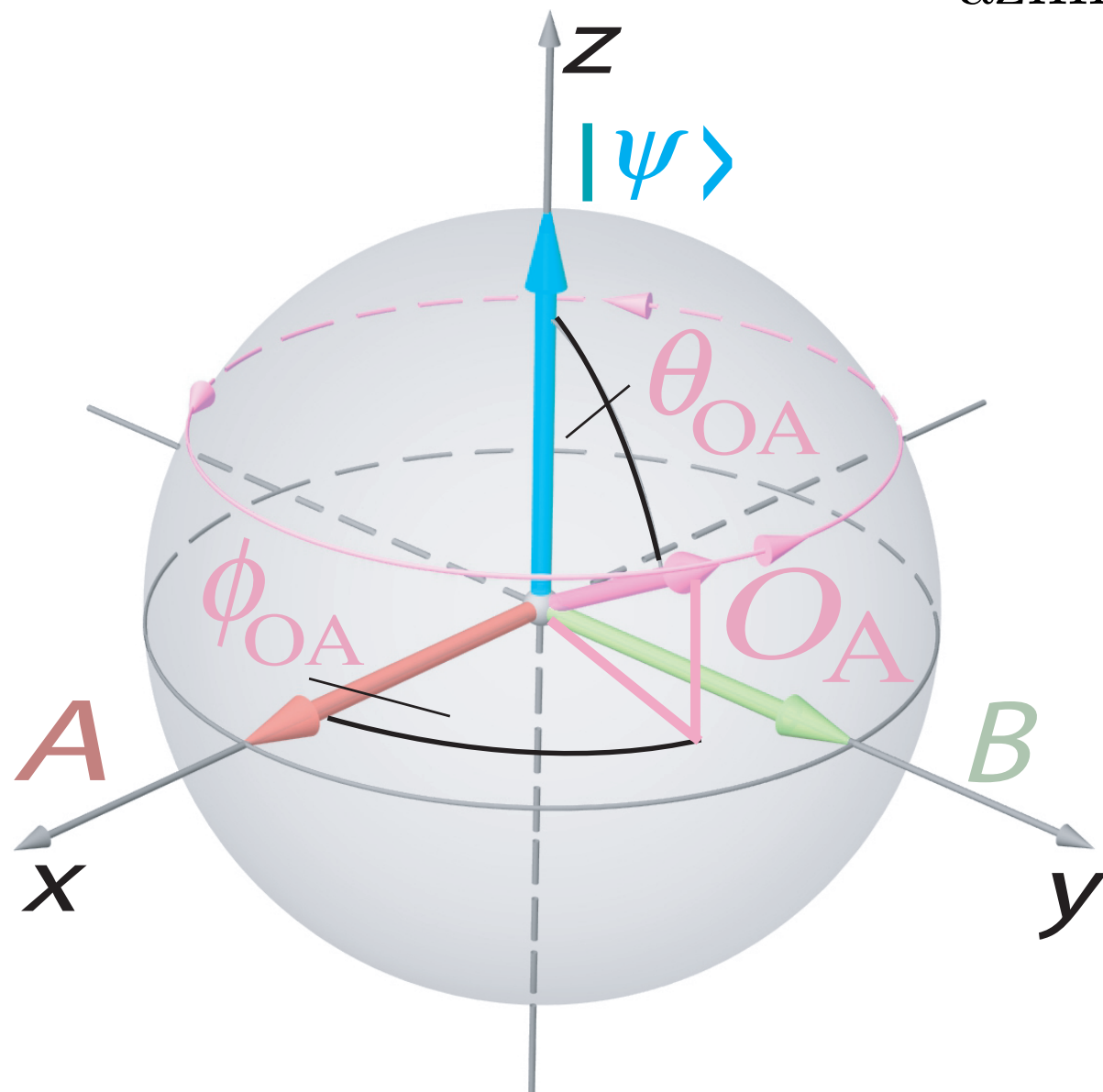
error, disturbance: trade - off not for all values of ϕ
 Heisenberg's EDUR also above limit

● **Part I: Violation Error-Disturbance Uncertainty Relation**

● Further Experimental Results

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

azimuthal angle OA : ϕ_{OA}
polar angle OA : θ_{OA}



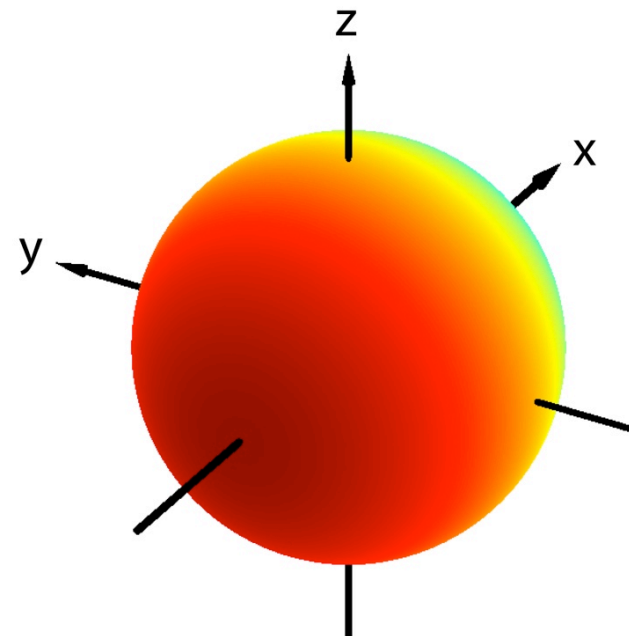
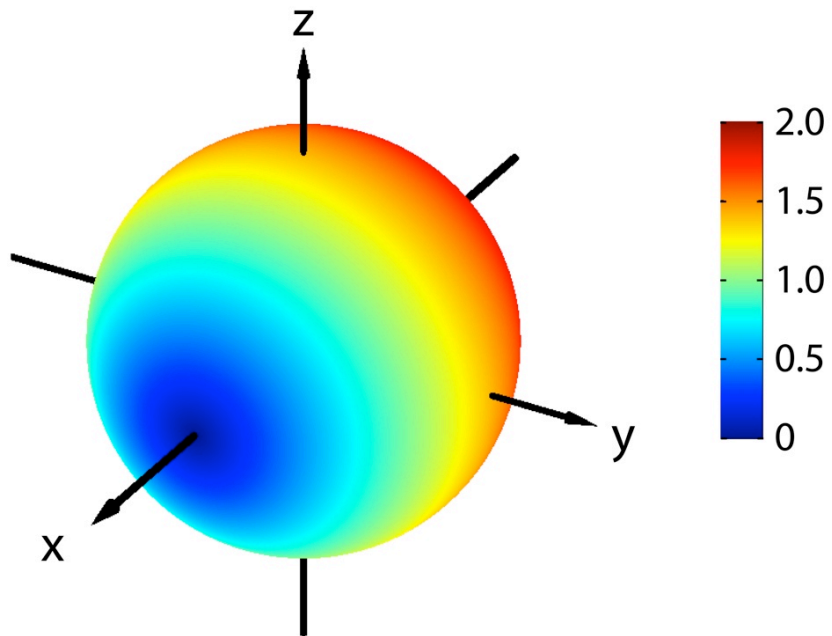
$$\epsilon(A) = \sqrt{2 - 2 \cos \phi_{OA} \sin \theta_{OA}}$$

$$\eta(B) = \sqrt{2 - 2 \sin^2 \phi_{OA} \sin^2 \theta_{OA}}$$

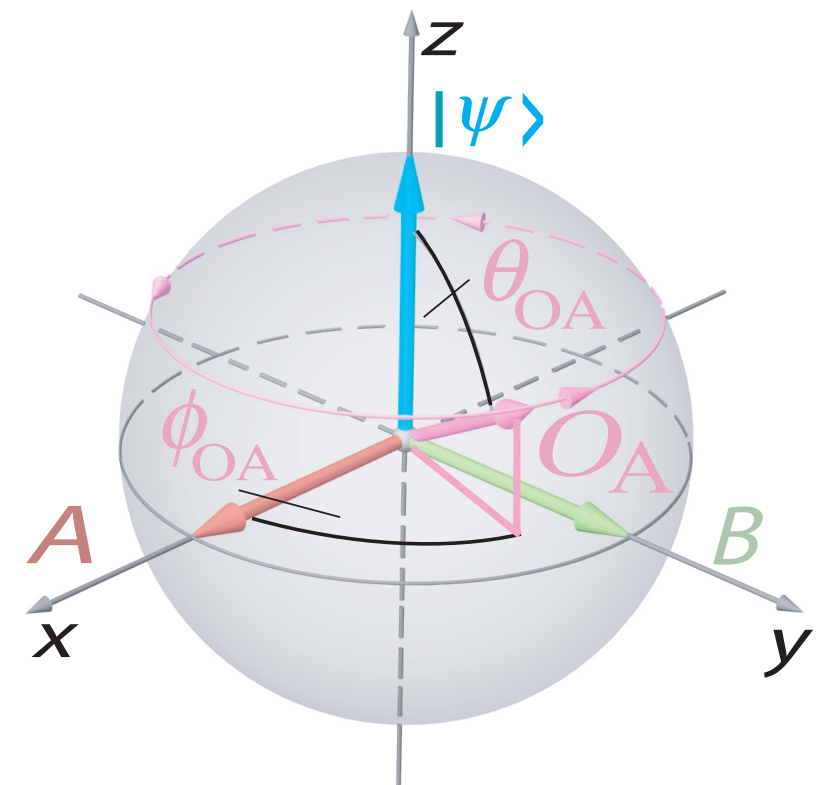
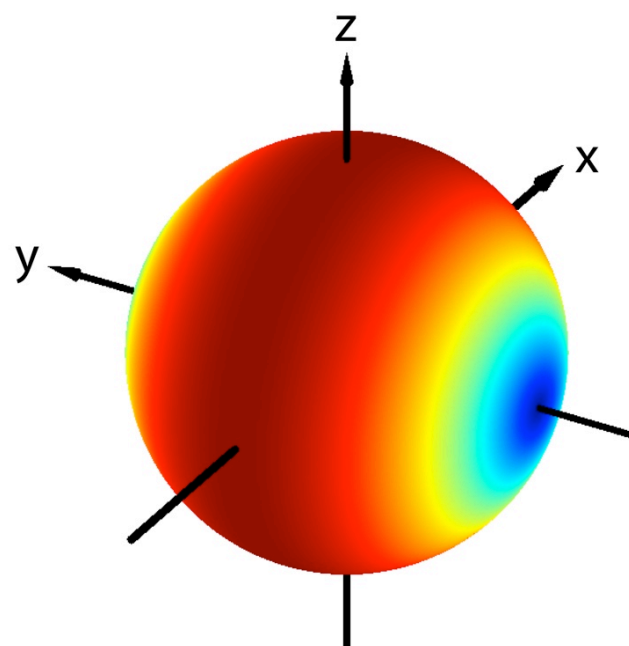
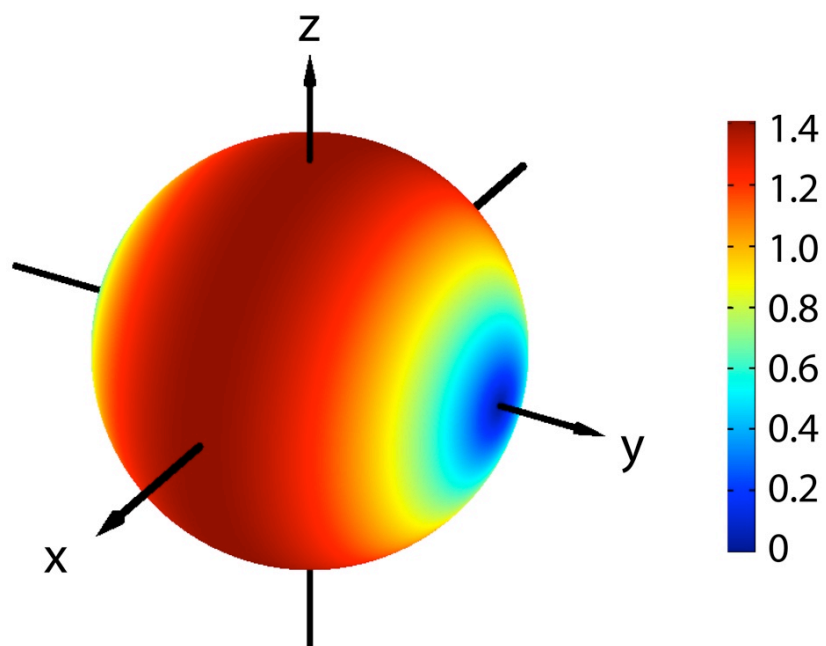
$$\sigma(A) = 1, \sigma(B) = 1$$

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

● **Error: $\epsilon(A)$**



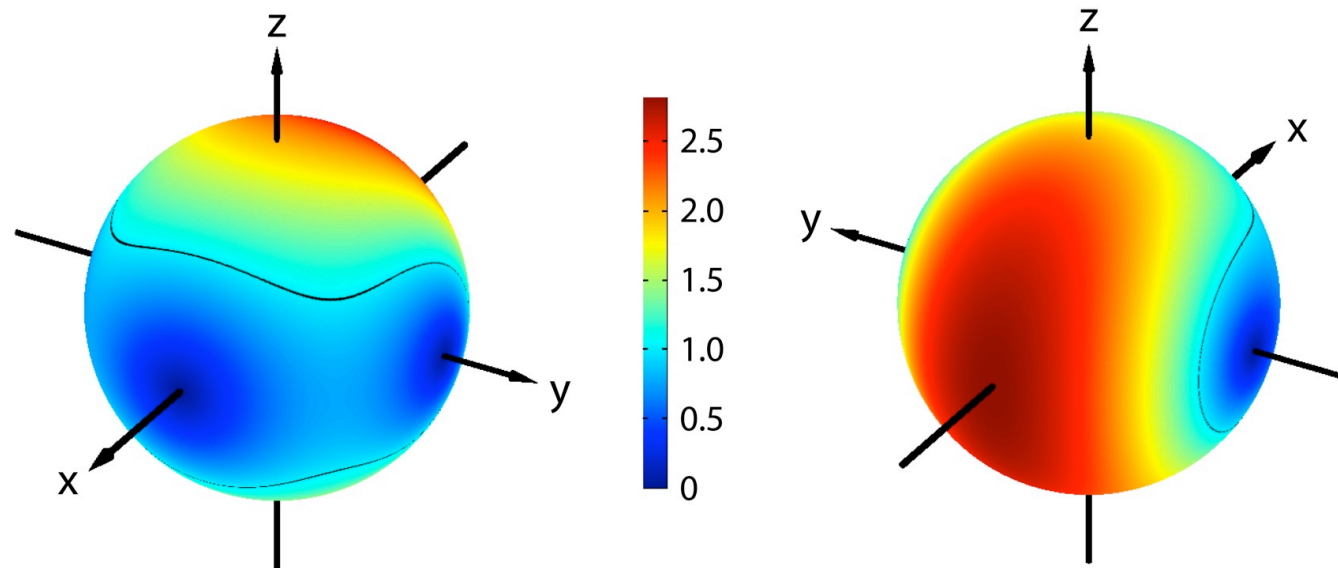
● **Disturbance: $\eta(B)$**



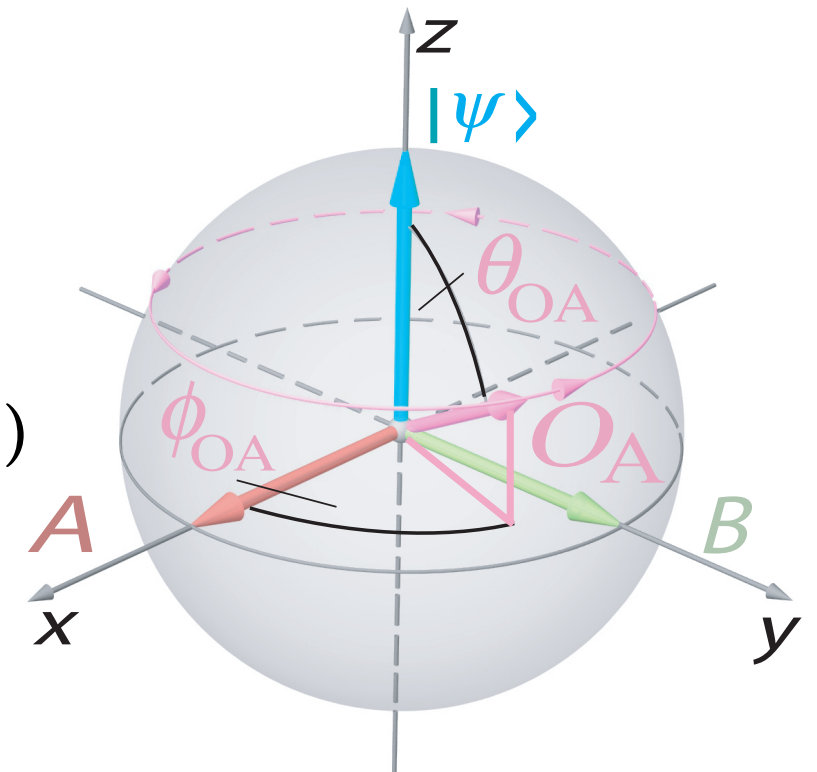
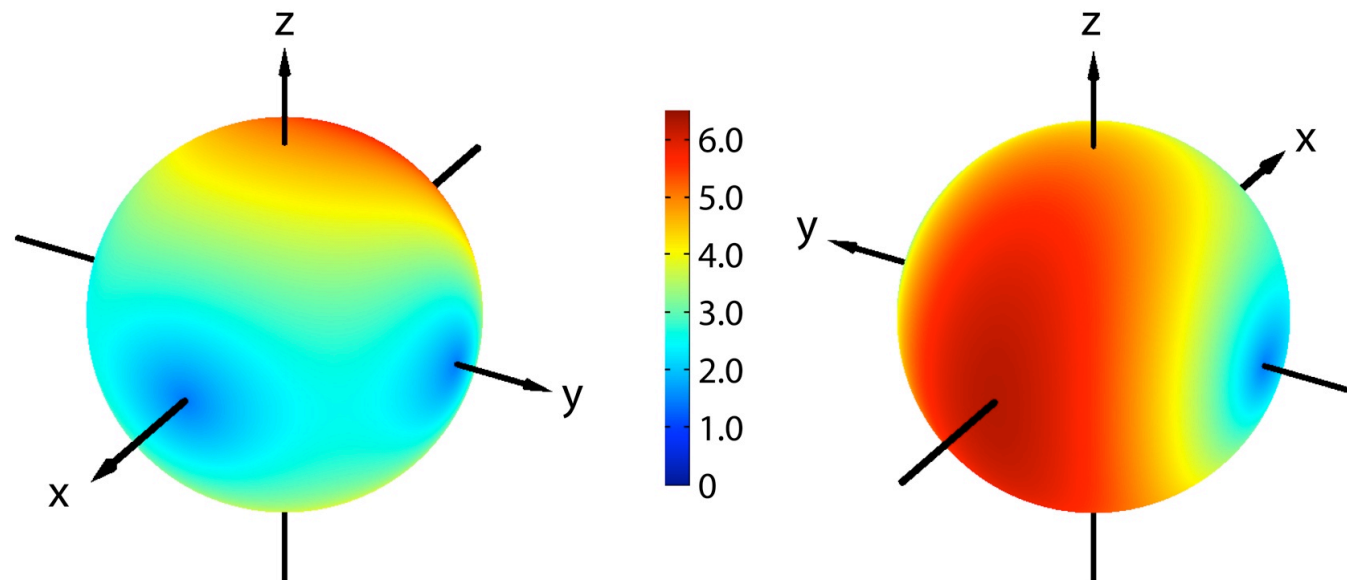
G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparation*

O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

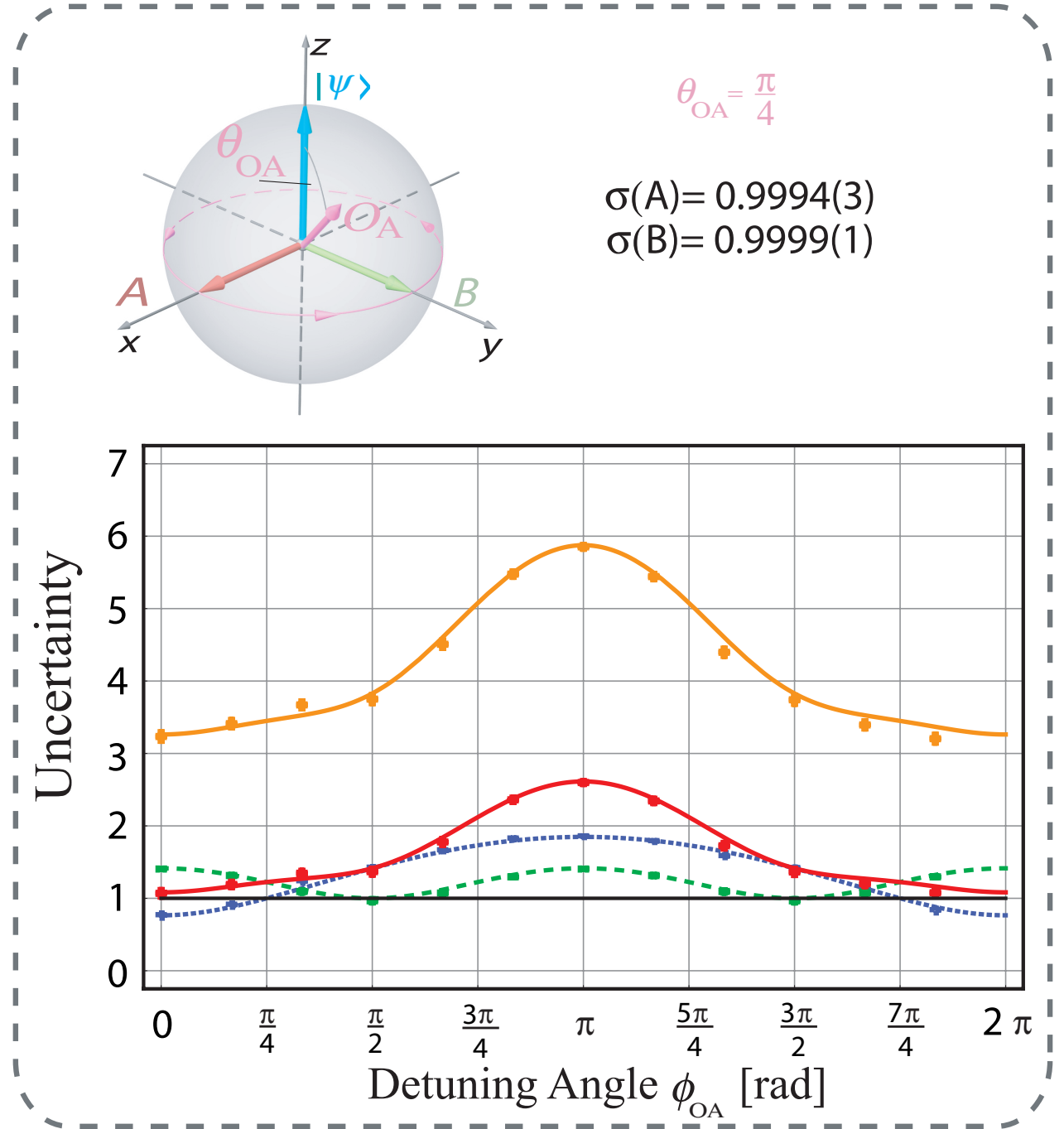
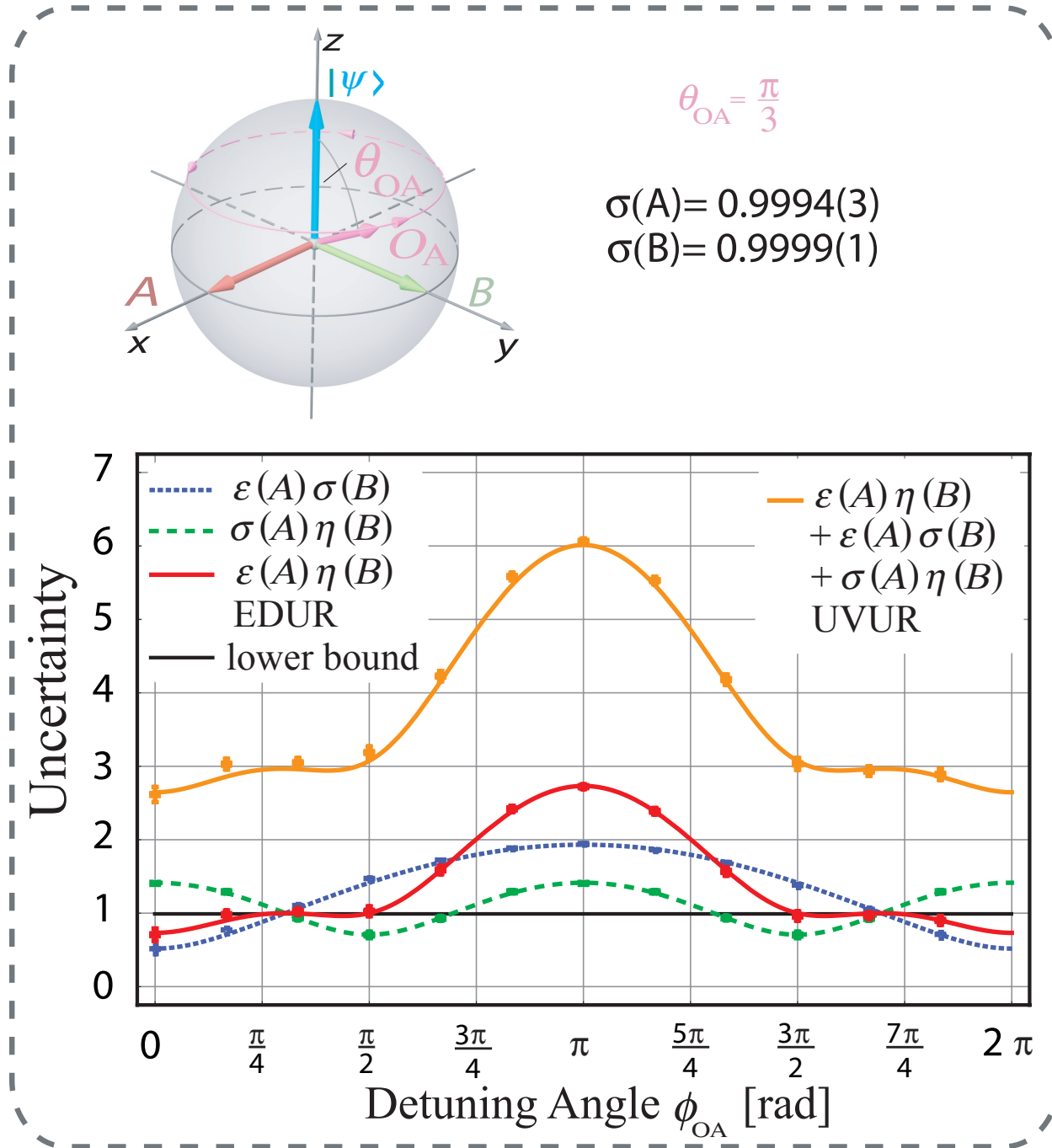
● **Heisenbergs EDUR:** $\epsilon(A)\eta(B)$



● **Ozawa's UVUR:** $\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B)$



Experimental Results $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

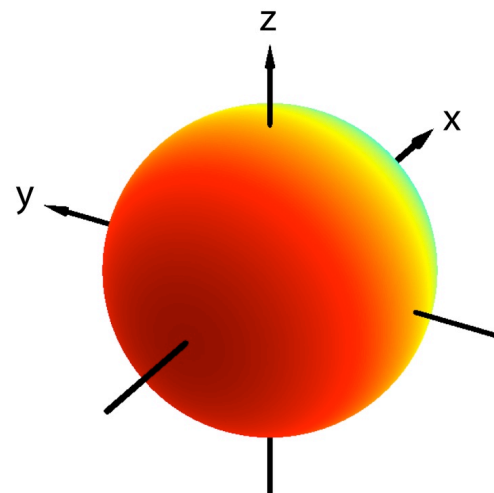
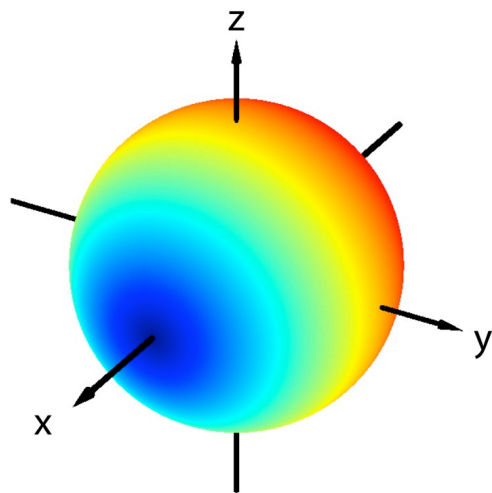


$\epsilon(A) \& \eta(B) \neq 0 \quad \forall \phi_{O_A}$ unless $\theta_{O_A} = \pi/2$, limit remains unchanged

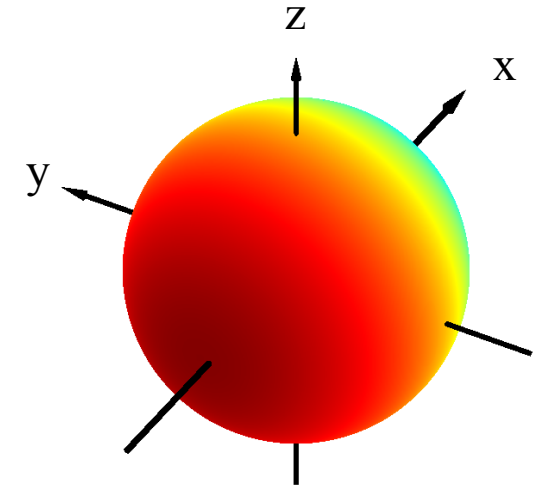
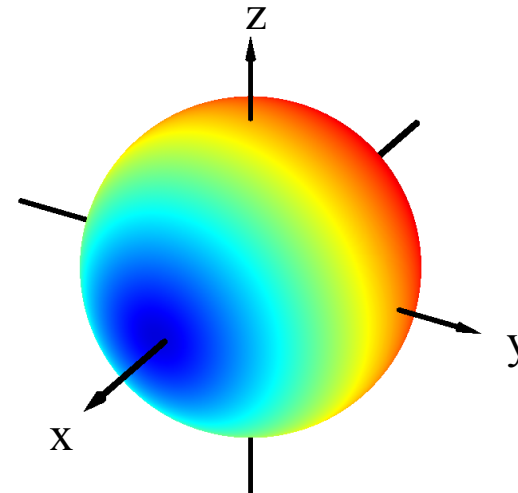
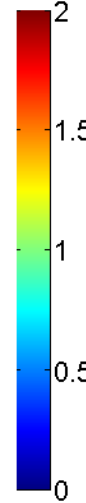
G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparation*

Experimental Results $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

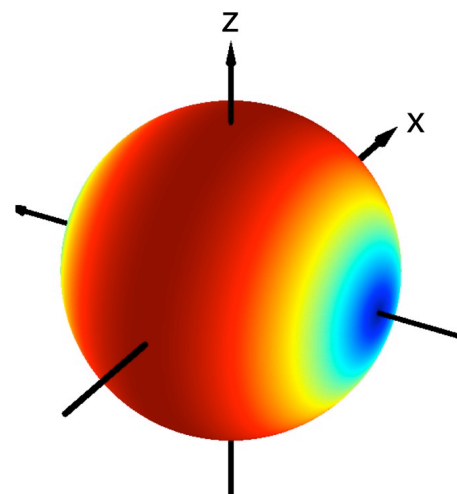
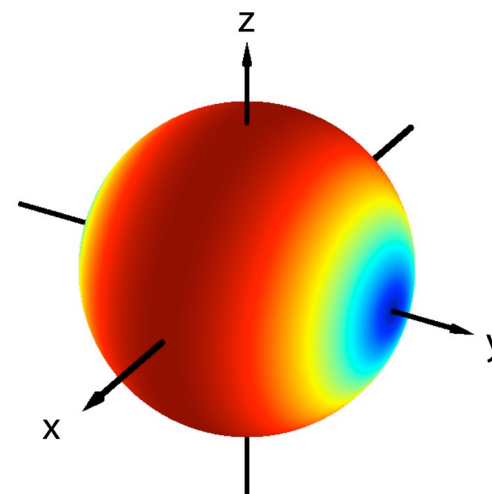
Theoretical Error:



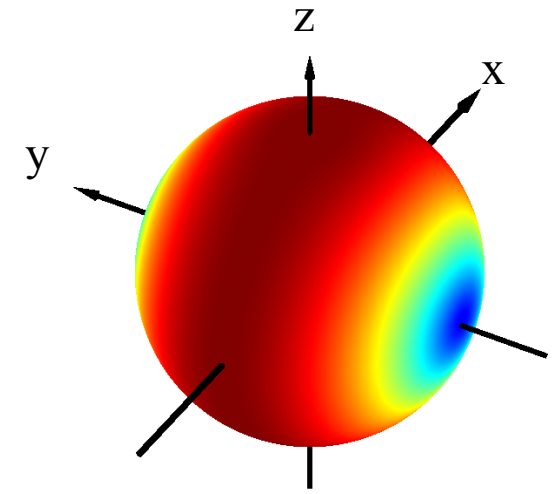
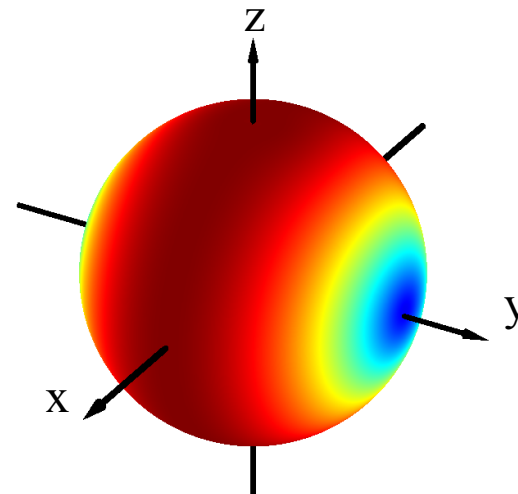
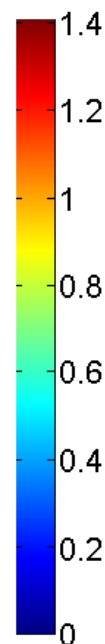
Measured Error:



Theoretical Disturbance:



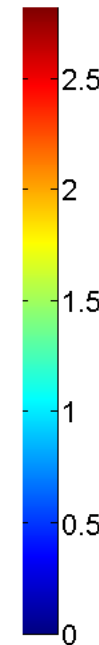
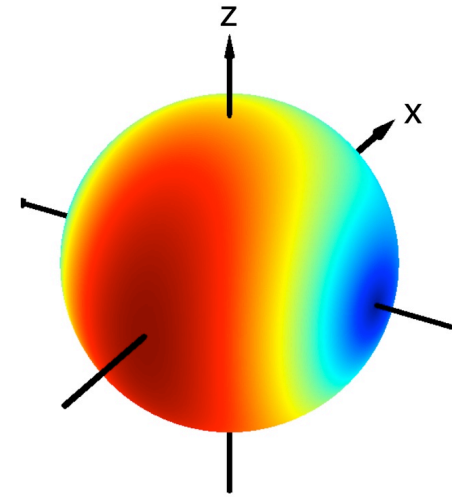
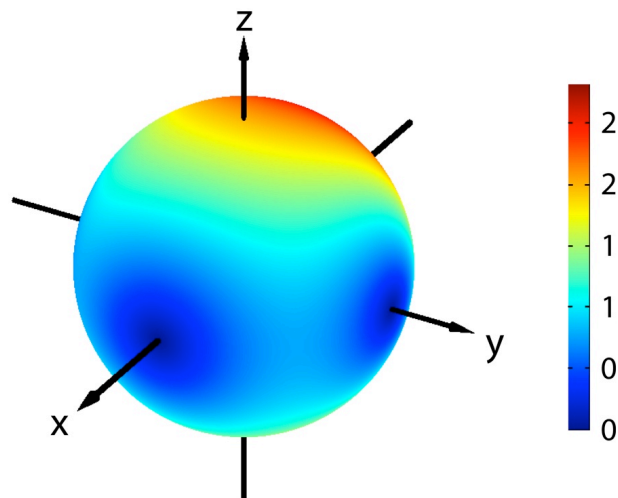
Measured Disturbance:



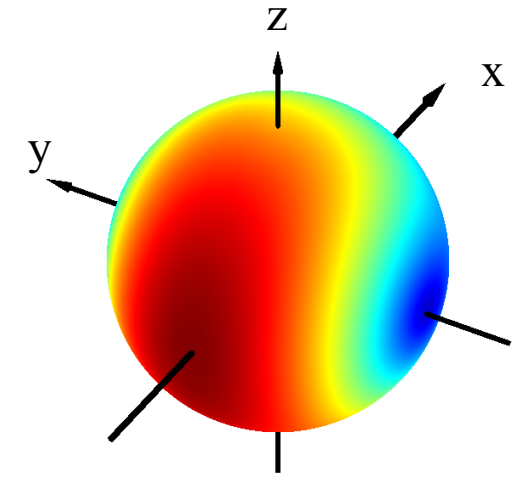
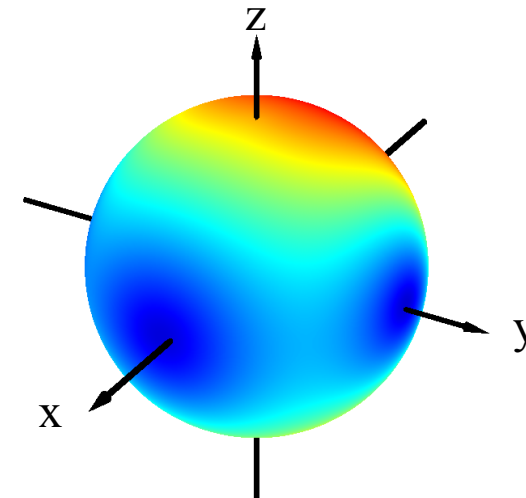
O_A – Variation : $\phi_{O_A} \in [0, 2\pi]$, and $\theta_{O_A} \in [0, \pi]$

Heisenbergs EDUR : $\epsilon(A)\eta(B)$

● Theory:

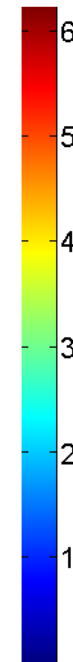
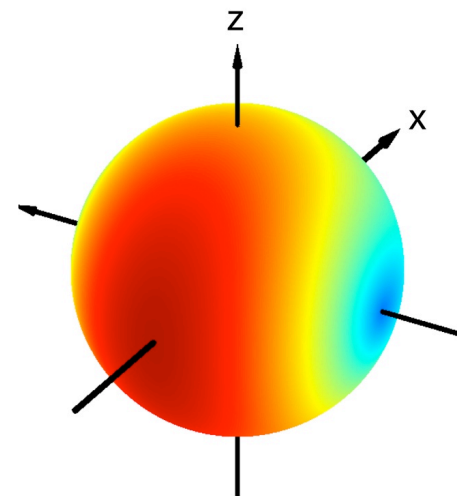
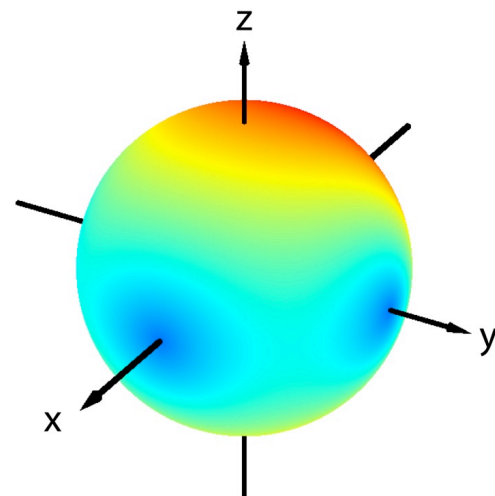


● Measurement:

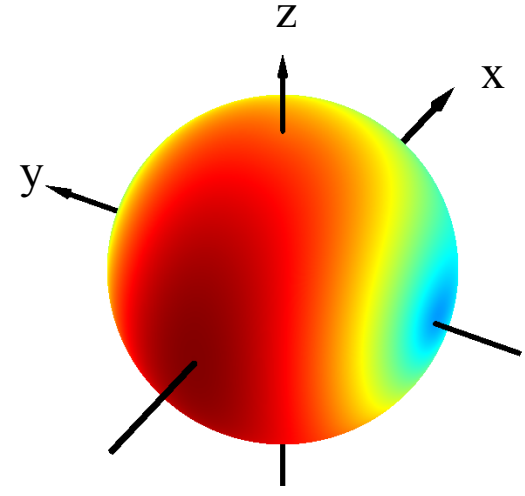
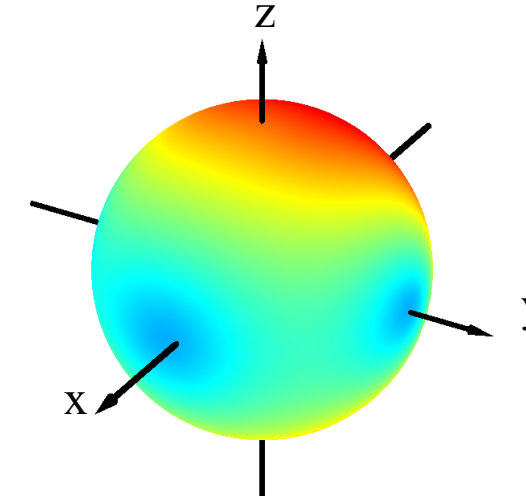


Ozawa's UVUR : $\epsilon(A)\eta(B) + \epsilon(A)\sigma(B) + \sigma(A)\eta(B)$

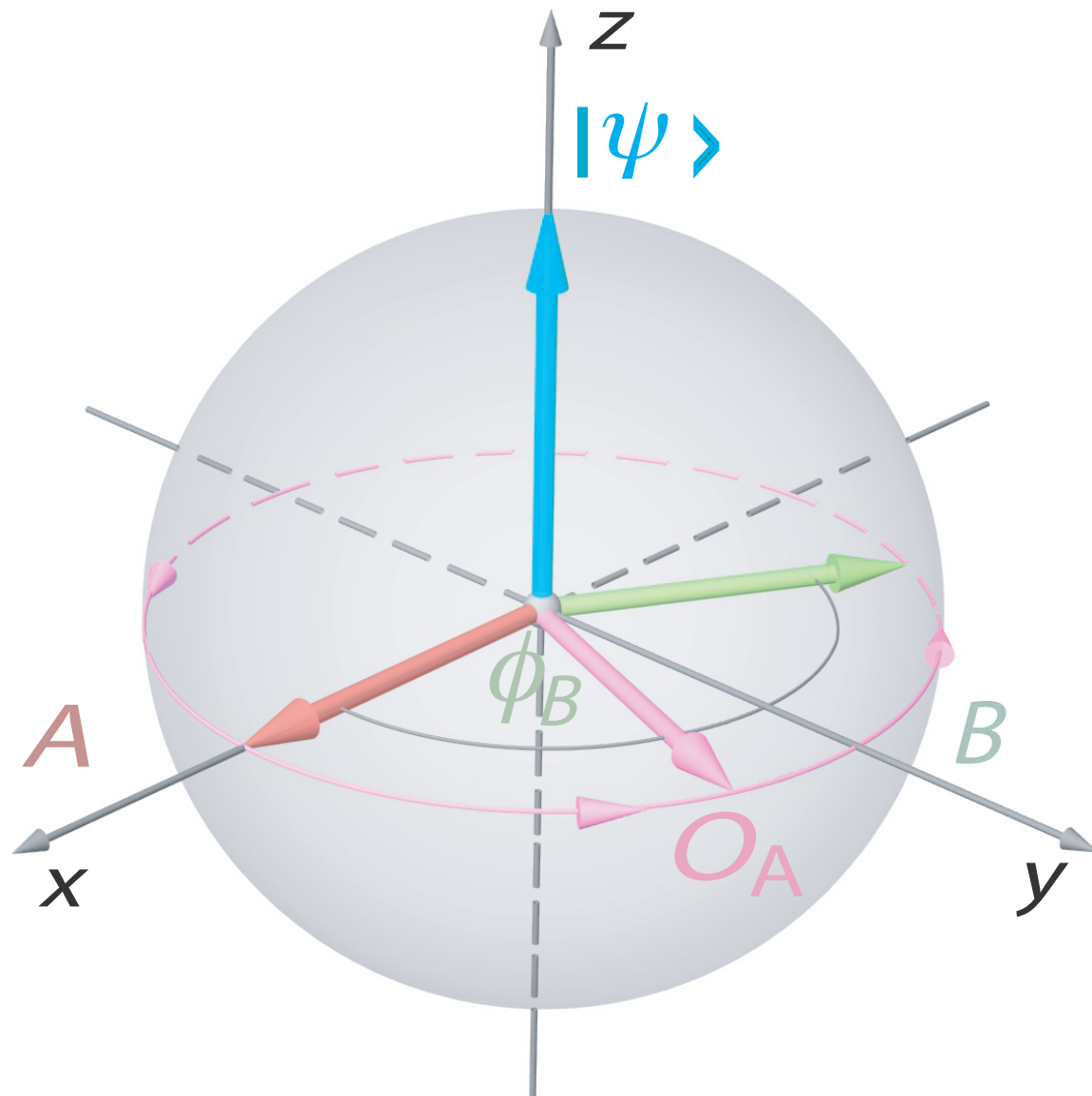
● Theory:



● Measurement:



B – Variation : Experimental Results $\phi_B \in [0, 2\pi]$



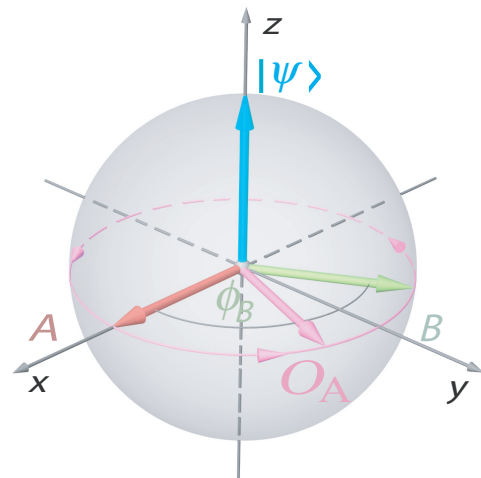
$$\epsilon(A) = 2 \sin \frac{\phi_{OA}}{2}$$

$$\eta(B) = \sqrt{2} |\sin(\phi_{OA} - \phi_B)|$$

$$\sigma(A) = 1, \sigma(B) = 1$$

$$\frac{1}{2} \langle \psi | [A, B] | \psi \rangle = |\sin \phi_B|$$

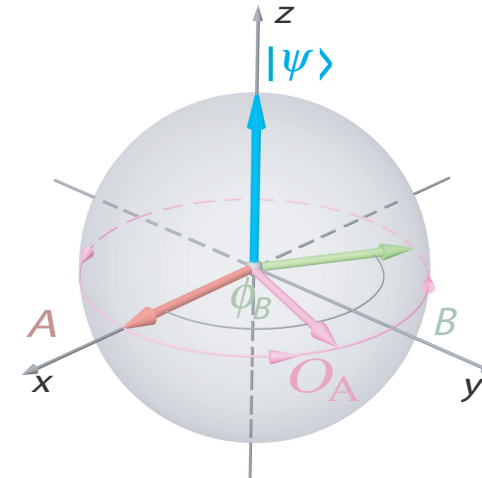
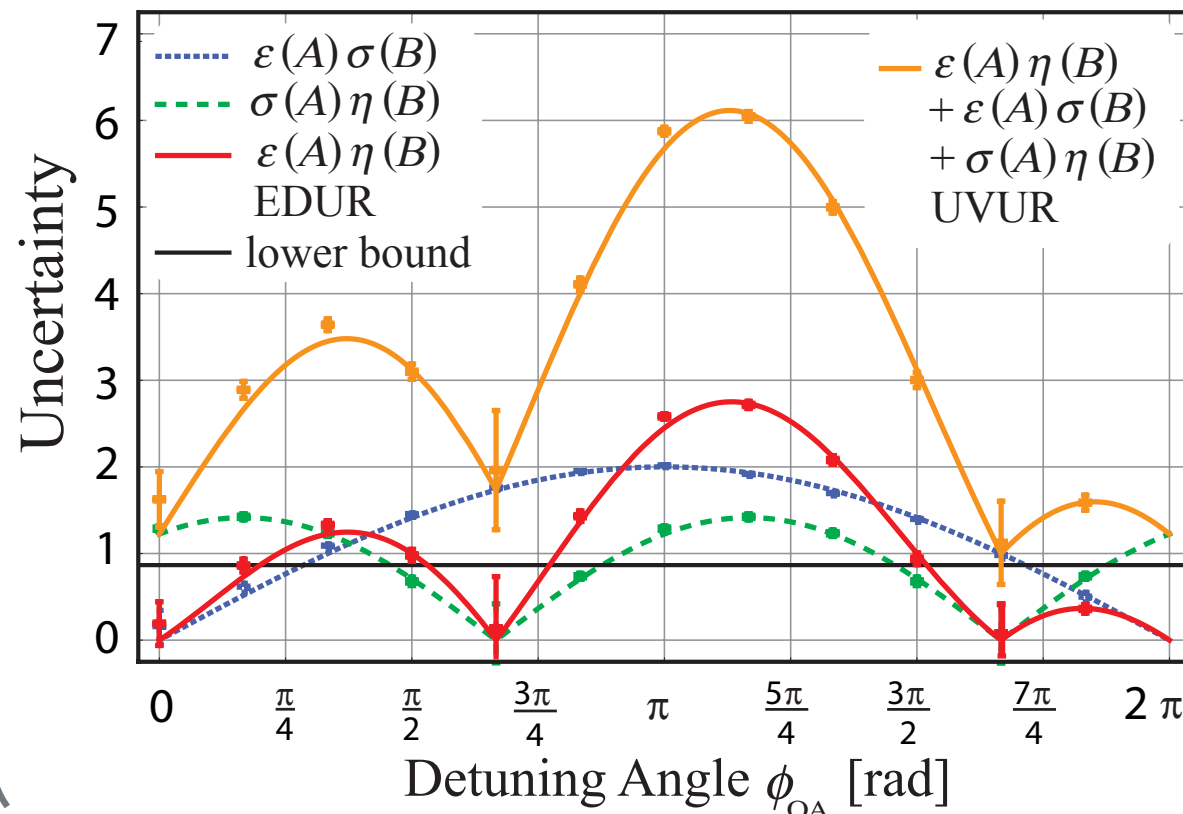
B – Variation : Experimental Results $\phi_B \in [0, 2\pi]$



$$\phi_B = \frac{2\pi}{3}$$

$$\sigma(A) = 0.9995(4)$$

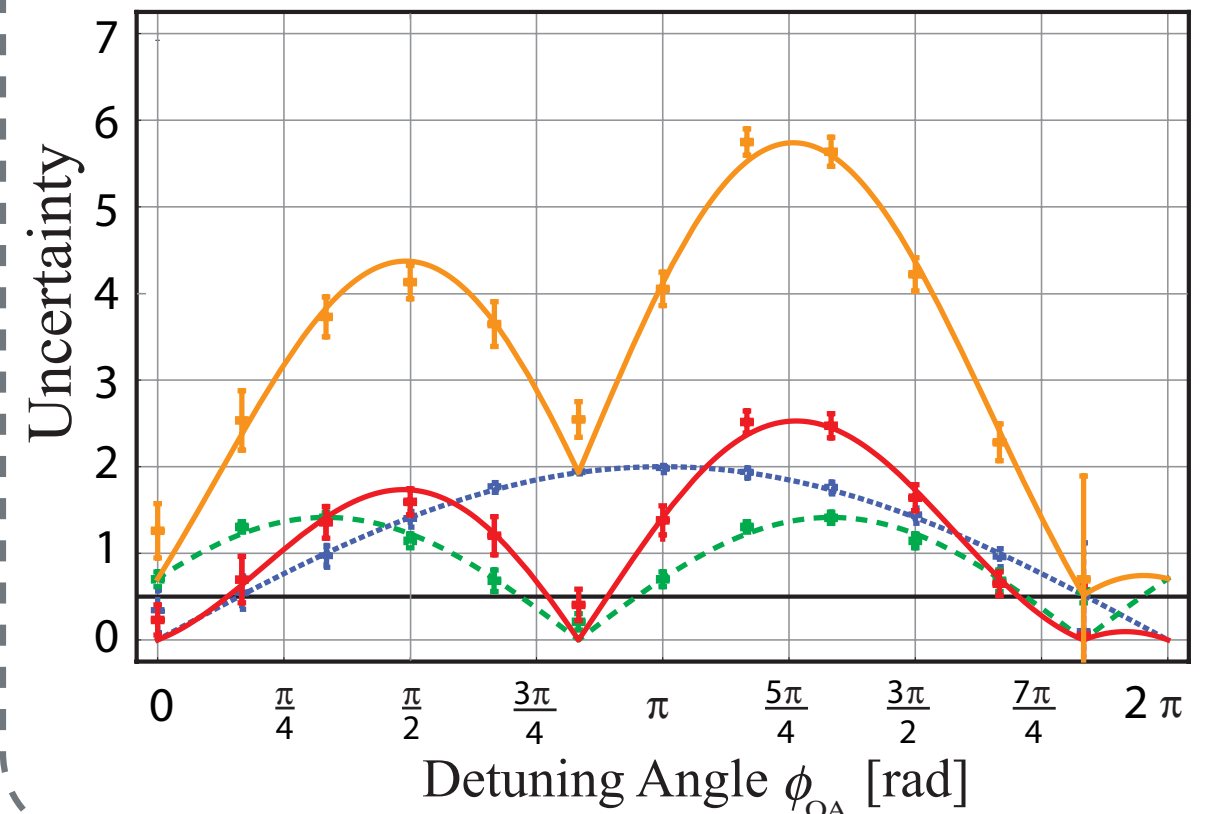
$$\sigma(B) = 0.9999(1)$$



$$\phi_B = \frac{5\pi}{6}$$

$$\sigma(A) = 0.9995(4)$$

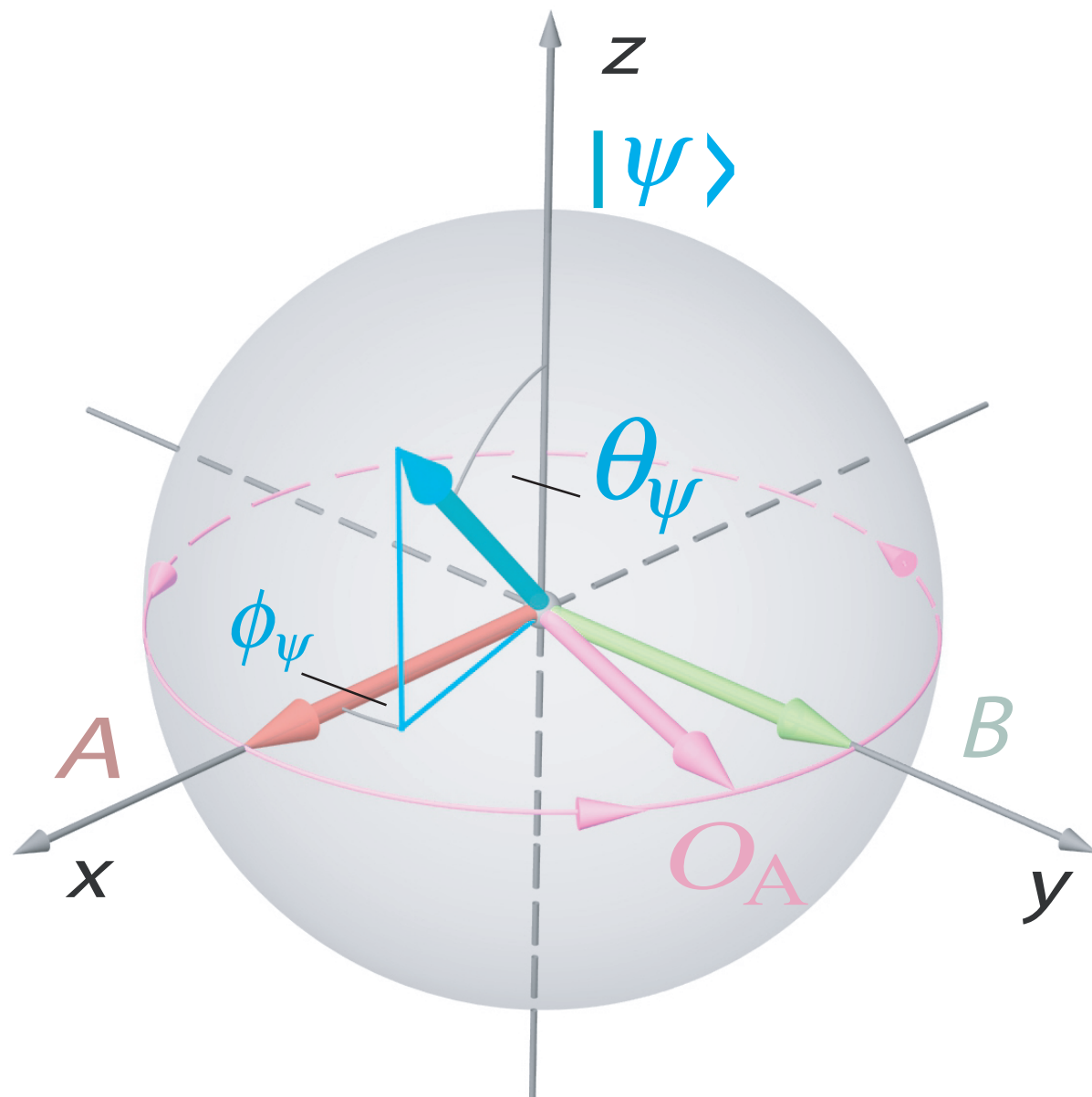
$$\sigma(B) = 0.9999(1)$$



disturbance shifted, asymmetry occurs & sum approaches limit

G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparation*

Variation of Initial State $|\psi\rangle$: Results $\phi_\psi \in [0, 2\pi]$, and $\theta_\psi \in [0, \pi]$



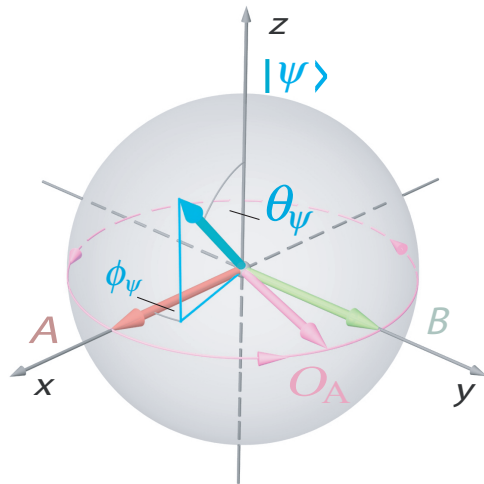
$$\left. \begin{aligned} \epsilon(A) &= 2 \sin \frac{\phi_{OA}}{2} \\ \eta(B) \sqrt{2} |\cos \phi_{OA}| \end{aligned} \right\} \text{state-independent}$$

$$\sigma(A) = \sqrt{1 - \cos^2 \phi_\psi \sin^2 \theta_\psi}$$

$$\sigma(B) = \sqrt{1 - \sin^2 \phi_\psi \sin^2 \theta_\psi}$$

$$\frac{1}{2} \langle \psi | [A, B] | \psi \rangle = |\cos \theta_\psi|$$

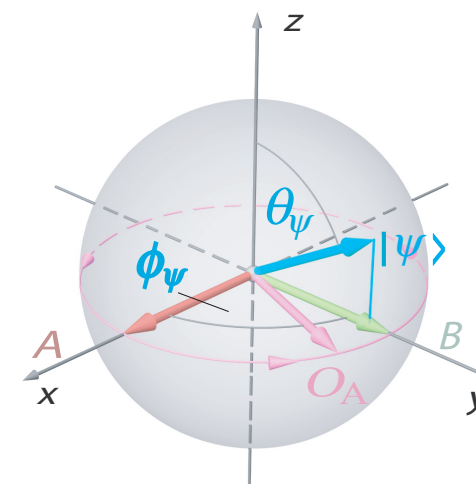
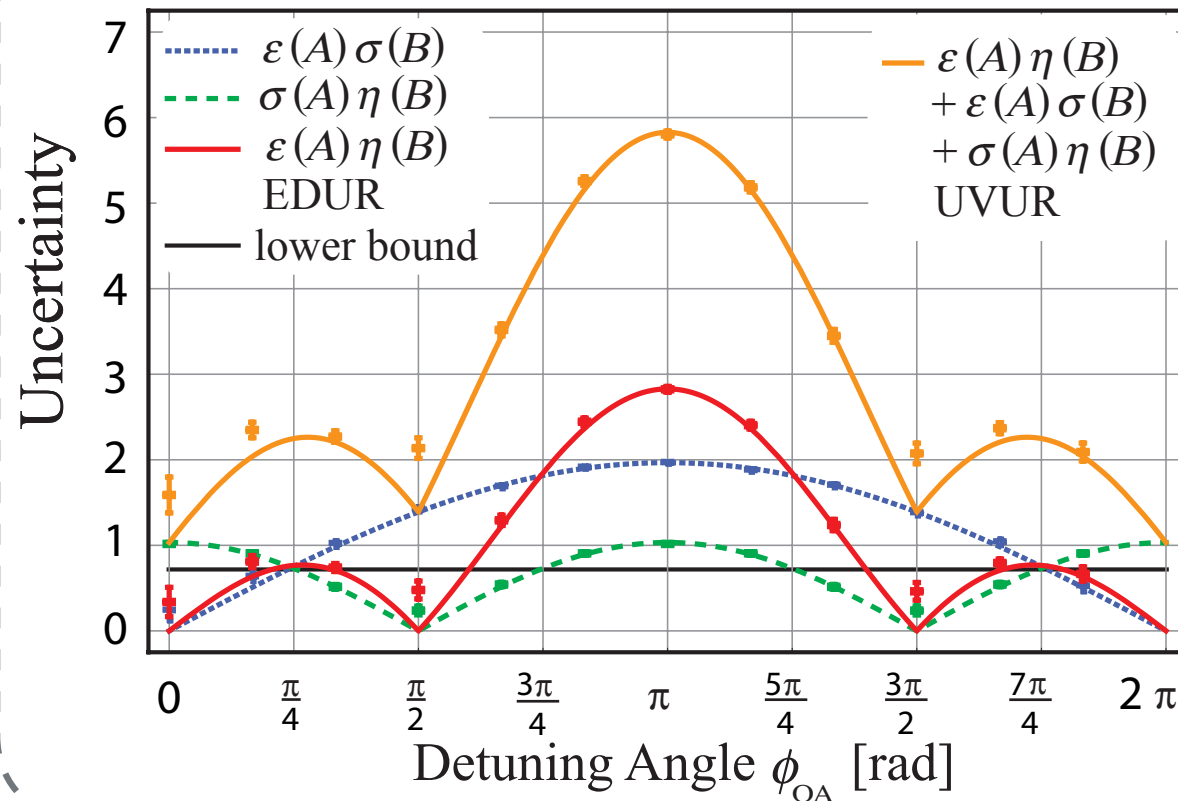
Variation of Initial State $|\psi\rangle$: Results $\phi_\psi \in [0, 2\pi]$, and $\theta_\psi \in [0, \pi]$



$$\theta_\psi = \frac{\pi}{4}, \phi_\psi = \frac{\pi}{12}$$

$$\sigma(A) = 0.714(9)$$

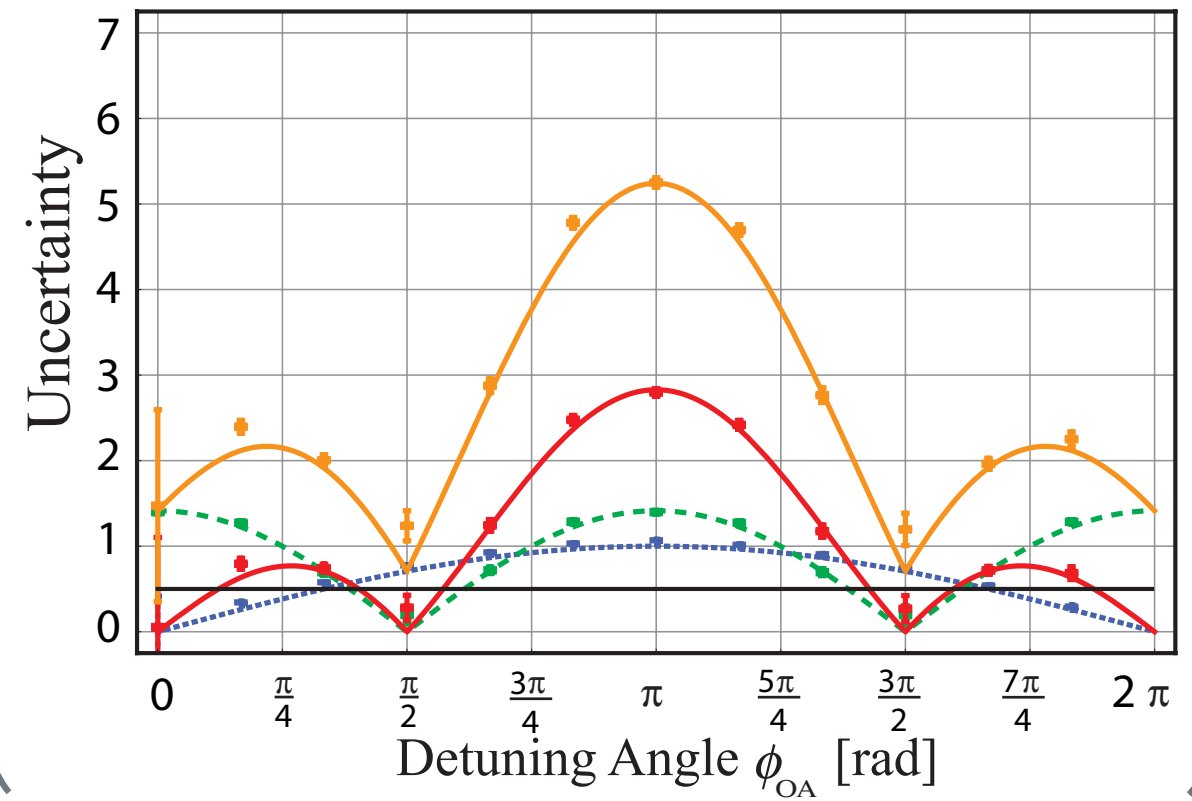
$$\sigma(B) = 0.988(2)$$



$$\theta_\psi = \frac{\pi}{3}, \phi_\psi = \frac{\pi}{2}$$

$$\sigma(A) = 1.000(1)$$

$$\sigma(B) = 0.528(11)$$

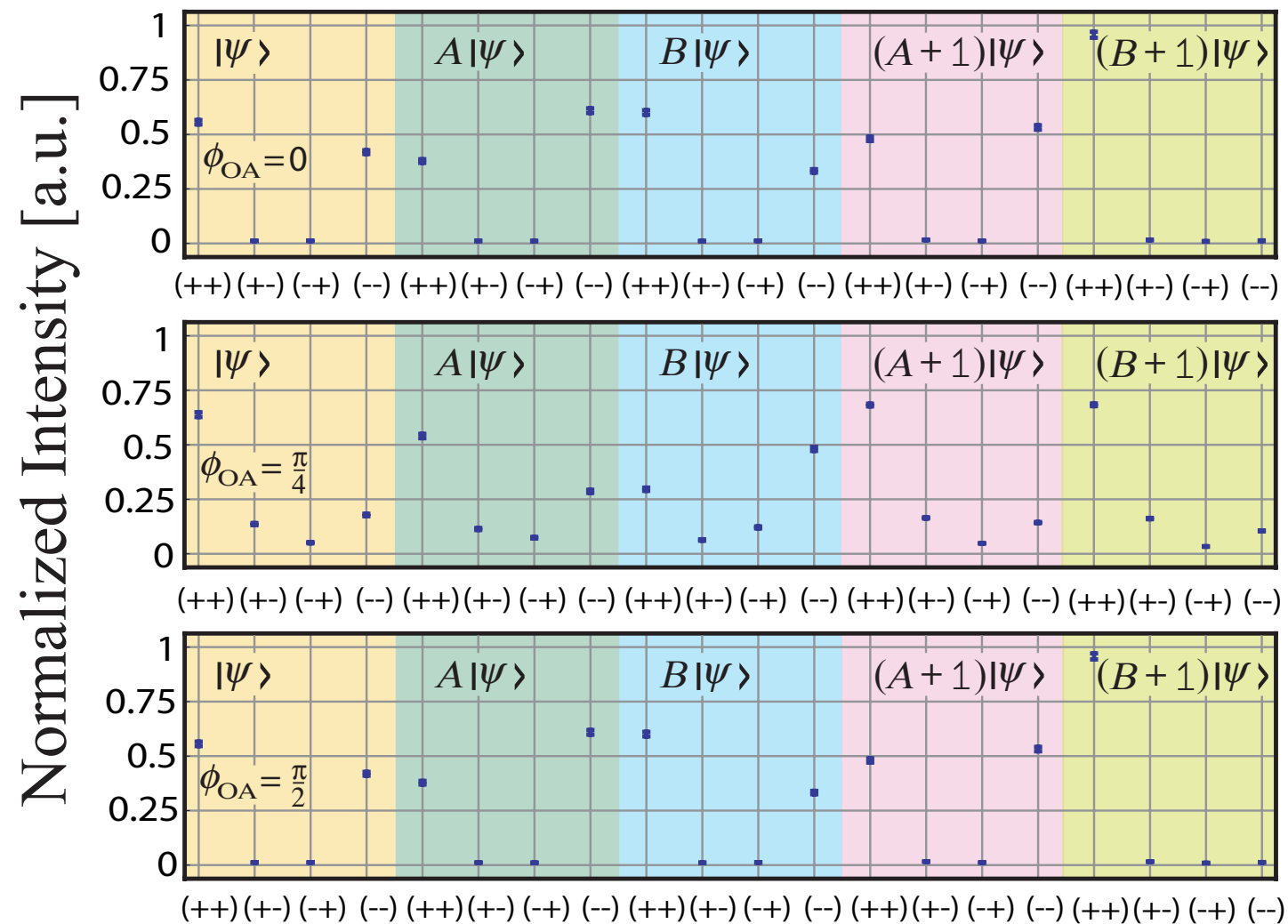


limit, standard deviations and UVUR change

error & disturbance state independent, EDUR unchanged

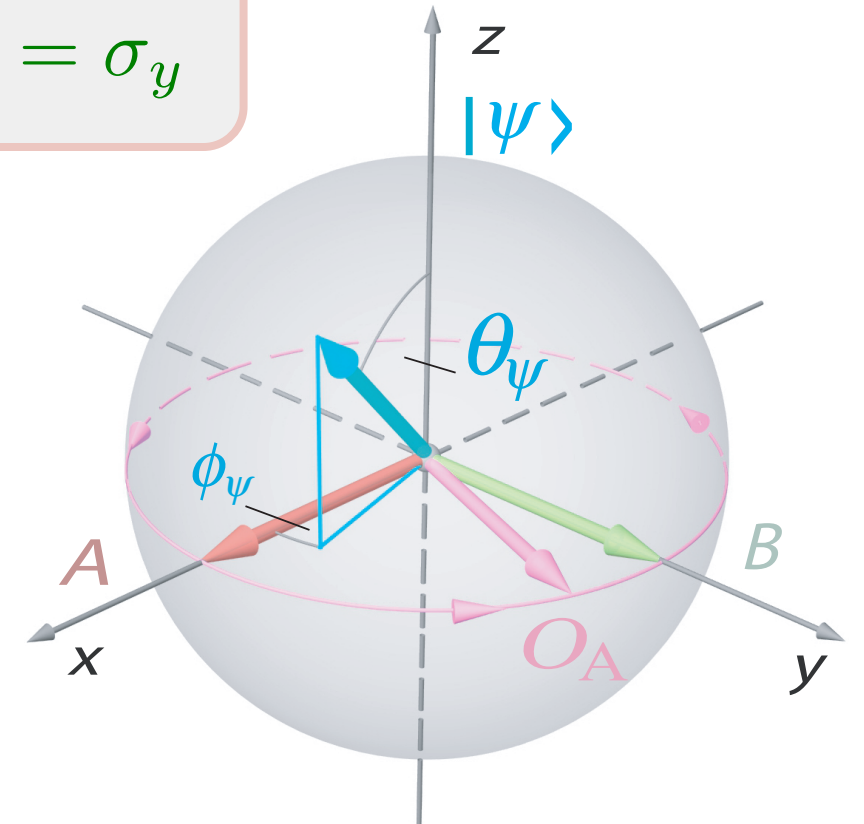
G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparation*

Experimental Data 5 input states



$$A = \sigma_x$$

$$B = \sigma_y$$



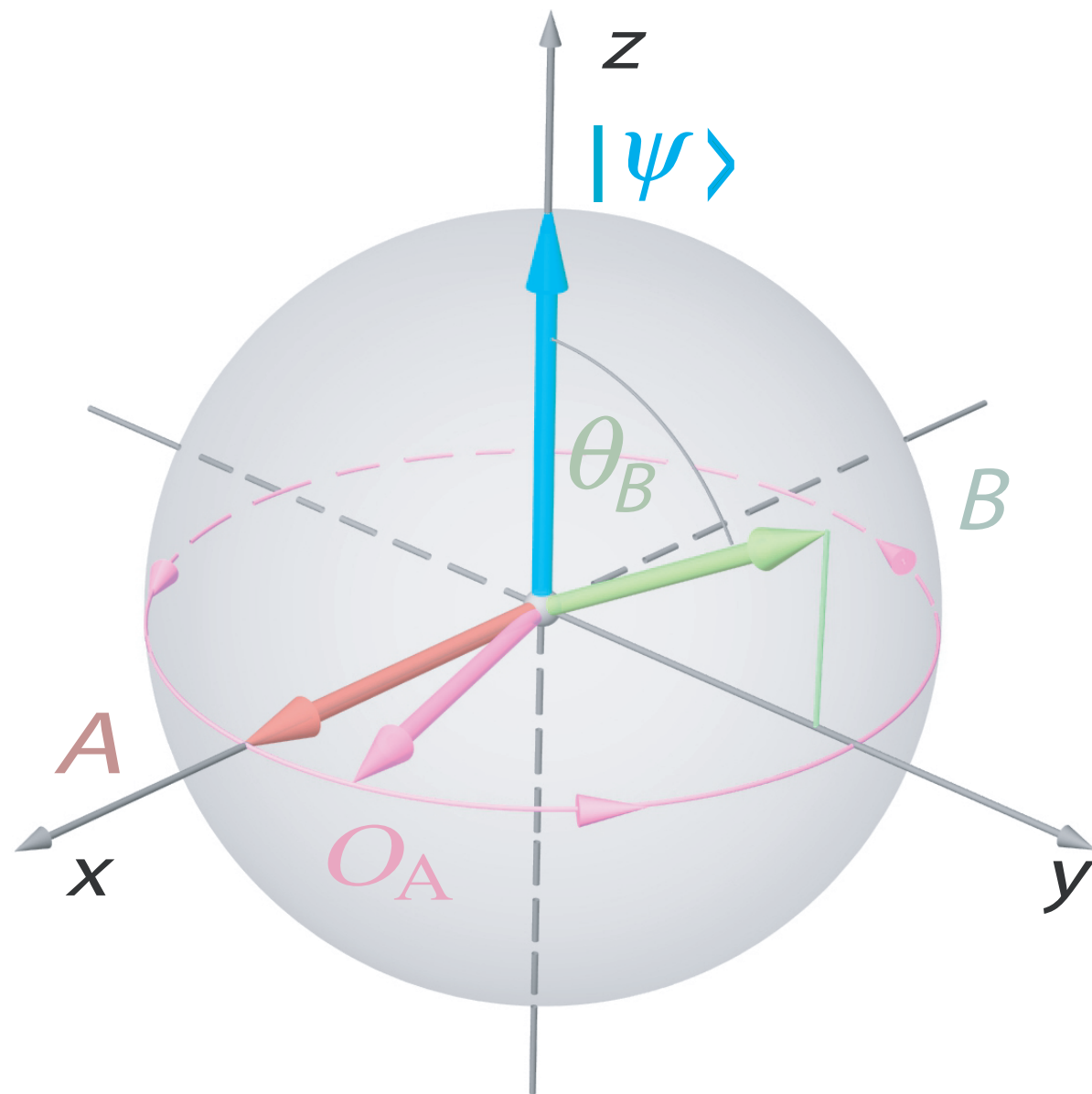
$$A| \psi \rangle \neq B| \psi \rangle \neq | -z \rangle$$

$$\frac{(I_{++} + I_{+-}) - (I_{-+} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi_i | O_A | \psi_i \rangle$$

$$\frac{(I_{++} + I_{-+}) - (I_{+-} + I_{--})}{I_{++} + I_{-+} + I_{+-} + I_{--}} = \langle \psi_i | X_B | \psi_i \rangle$$

G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparation*

B – Variation : Experimental Results $\theta_B \in [0, \pi]$



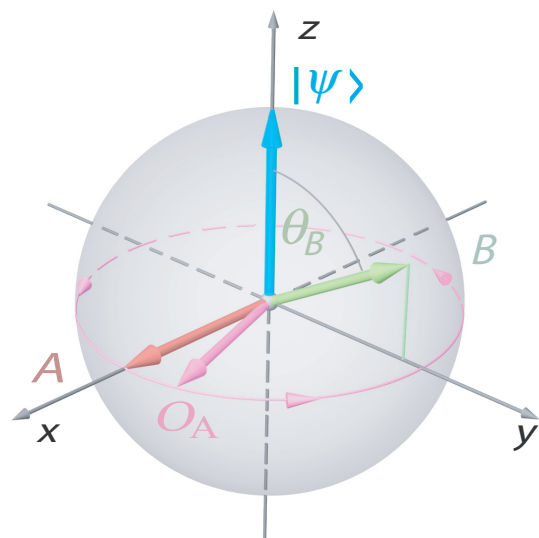
$$\epsilon(A) = 2 \sin \frac{\phi_{OA}}{2}$$

$$\eta(B) = \sqrt{2} |\sin(\phi_{OA} - \phi_B)|$$

$$\sigma(A) = 1, \quad \sigma(B) = \sin \theta_B$$

$$\frac{1}{2} \langle \psi | [A, B] | \psi \rangle = \sin \theta_B$$

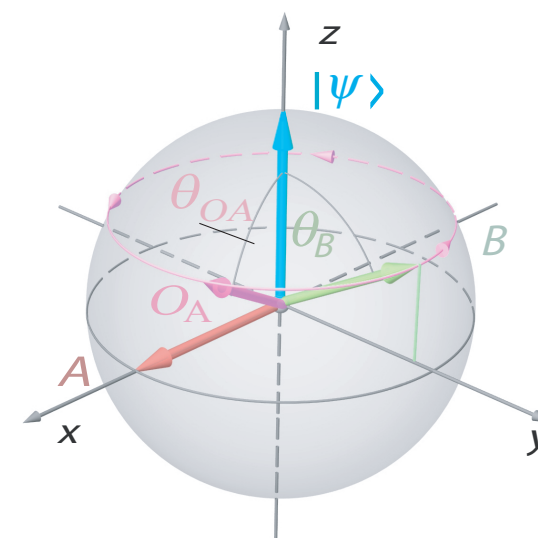
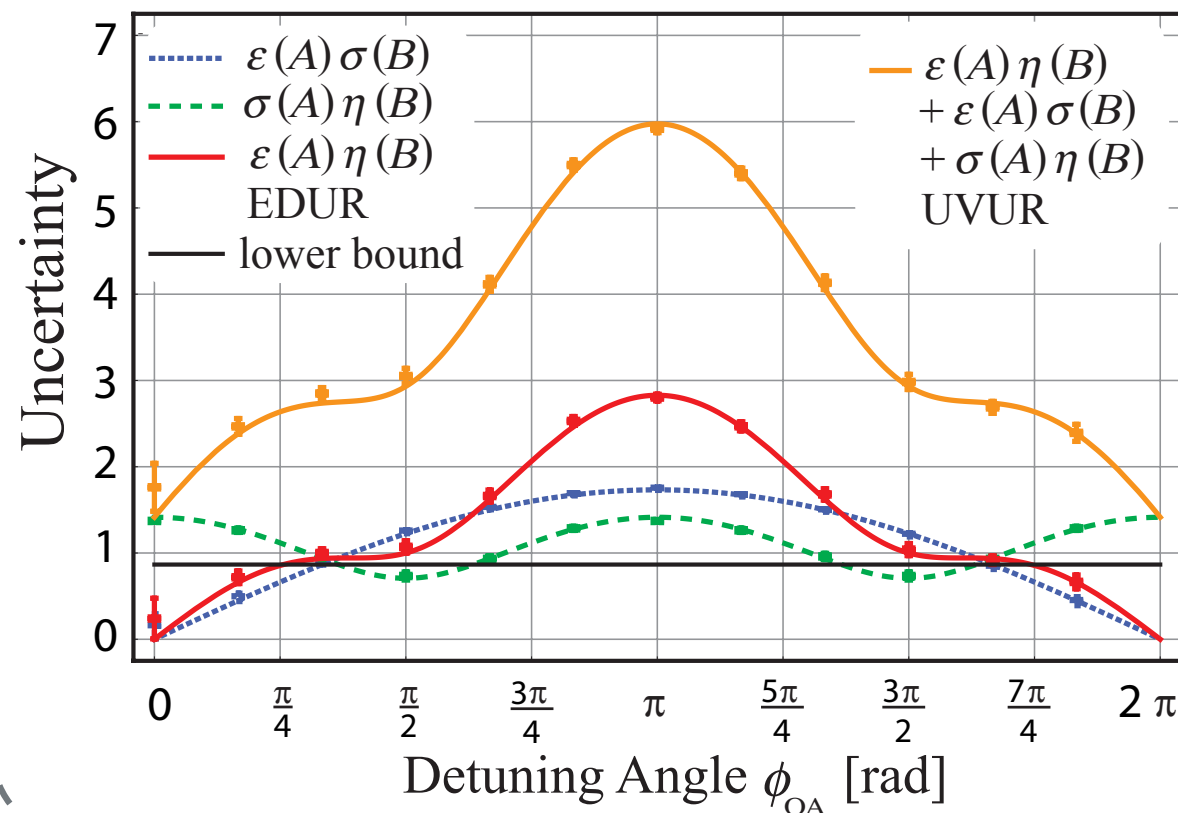
B – Variation : Experimental Results $\theta_B \in [0, \pi]$



$$\theta_B = \frac{\pi}{3}$$

$$\sigma(A) = 0.982(2)$$

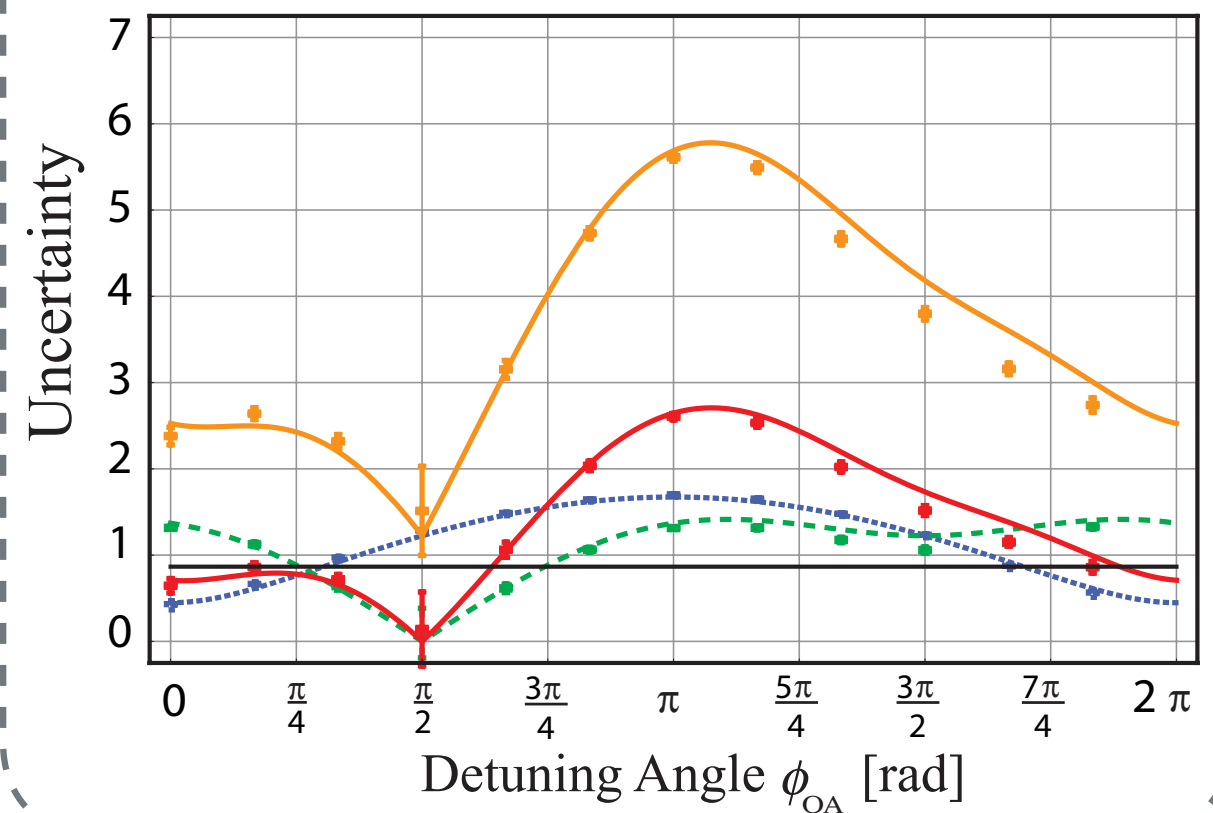
$$\sigma(B) = 0.868(6)$$



$$\theta_{OA} = \theta_B = \frac{\pi}{3}$$

$$\sigma(A) = 0.982(2)$$

$$\sigma(B) = 0.868(6)$$

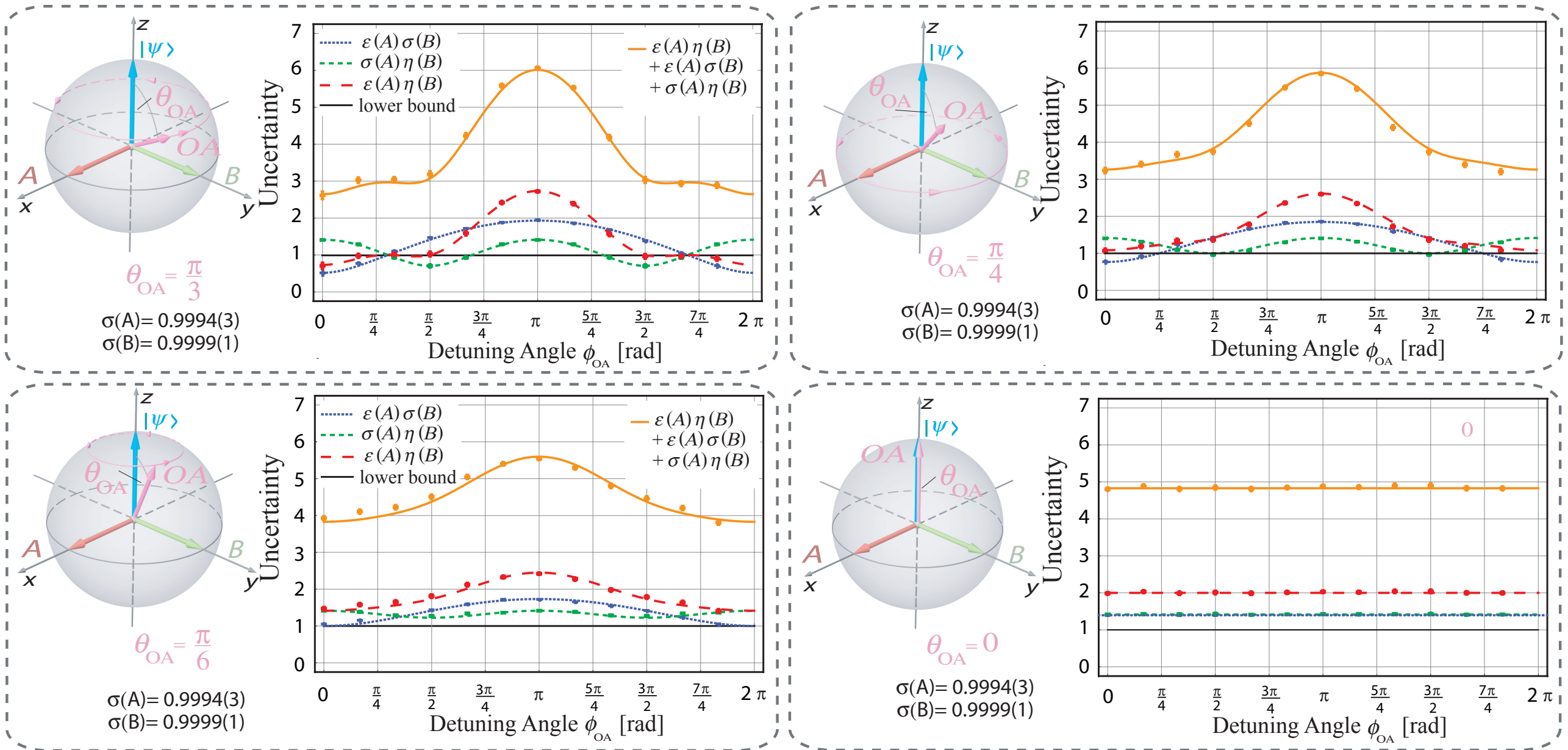


$$\eta(B) \neq 0 \quad \forall \phi_{OA}$$

$$\epsilon(A) \neq 0 \quad \forall \phi_{OA}$$

G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparations*

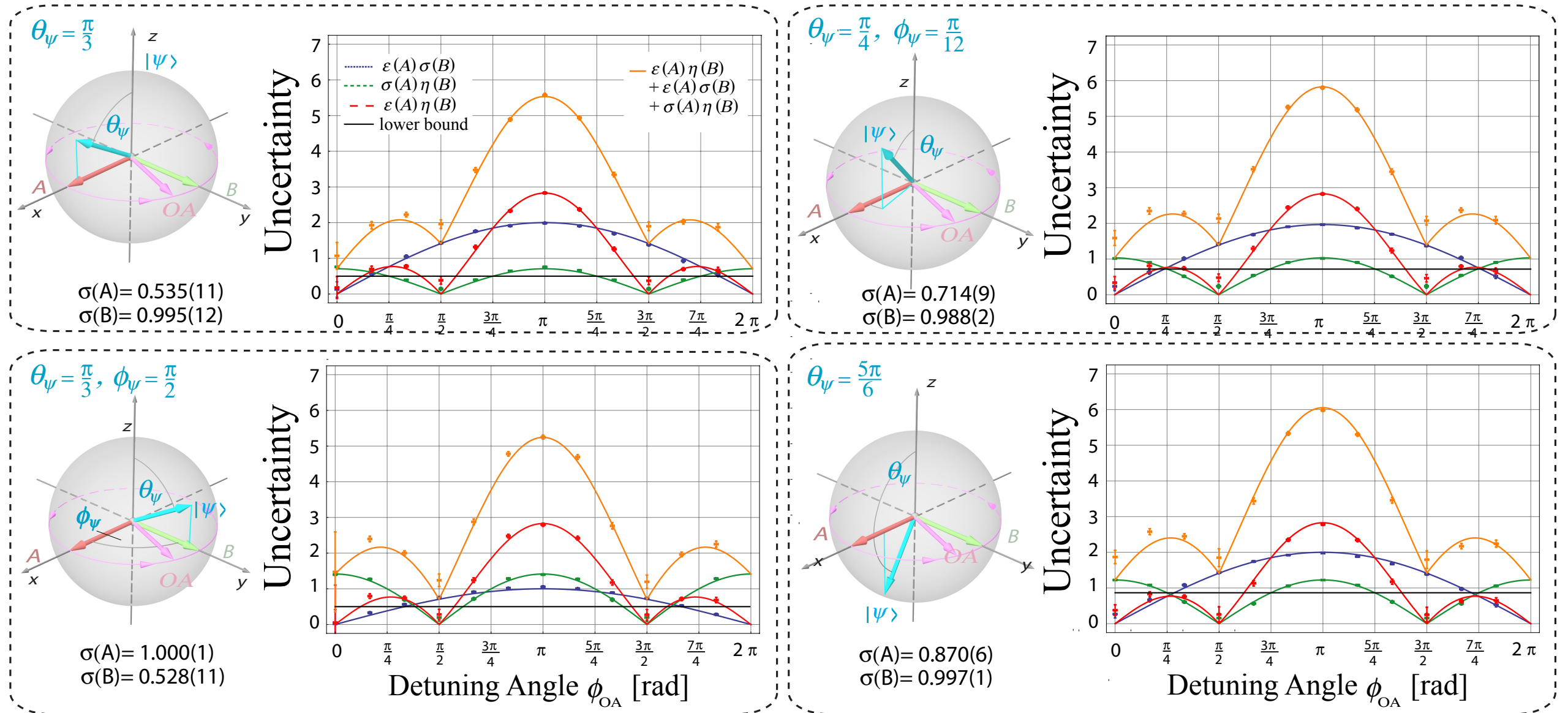
Experimental Results $\phi_{OA} \in [0, 2\pi]$, and $\theta_{OA} \in [0, \pi]$



$\epsilon(A) \ \& \ \eta(B) \neq 0 \ \forall \ \phi_{OA}$ unless $\theta_{OA} = \pi/2$, limit remains unchanged

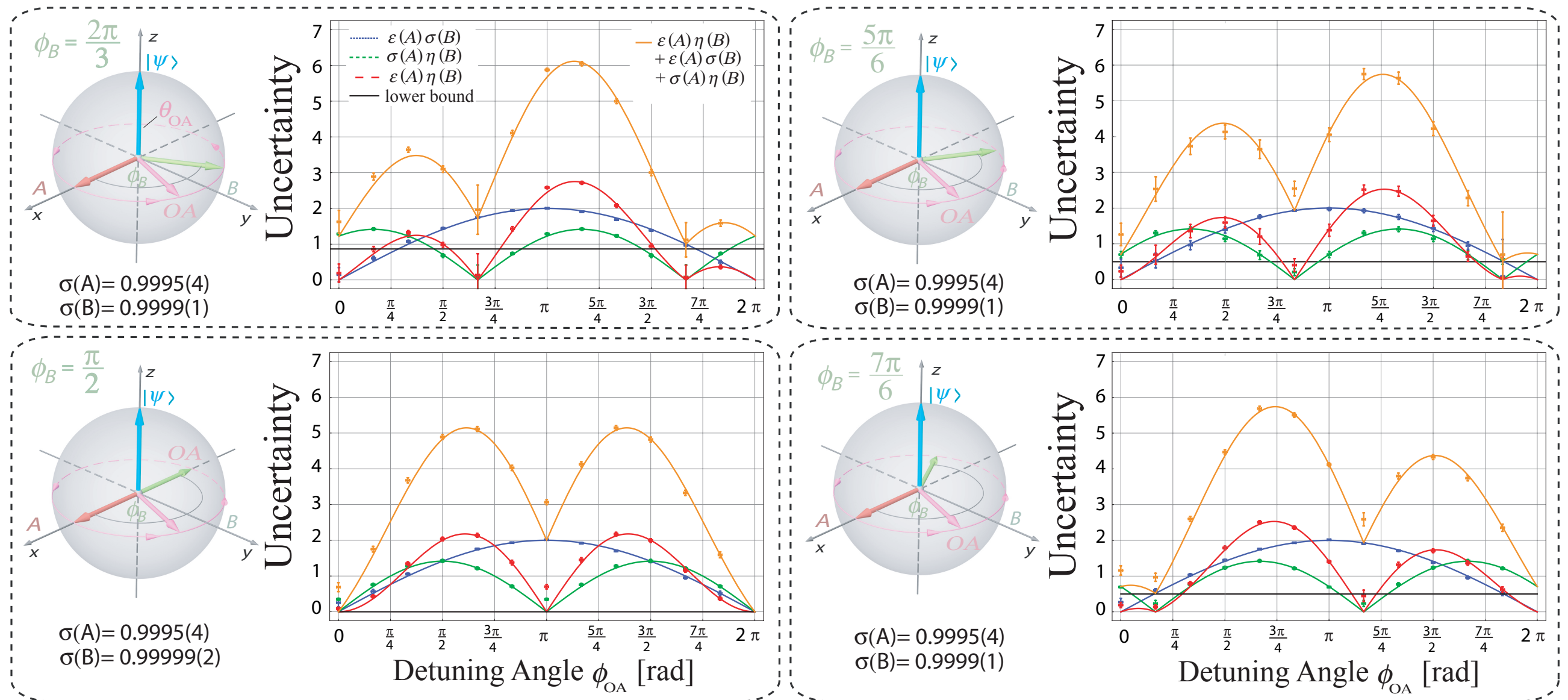
G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparations*

Variation of Initial State $|\psi\rangle$: Results $\phi_\psi \in [0, 2\pi]$, and $\theta_\psi \in [0, \pi]$



- limit, standard deviations and UVUR change
- error & disturbance state independent, EDUR unchanged

B – Variation : Experimental Results $\phi_B \in [0, 2\pi]$



disturbance shifted, asymmetry occurs & sum touches limit

G. Sulyok, S. Sponar, J. Erhart, G. Badurek, M. Ozawa, Y. Hasegawa, *in preparations*

Steinberg Experiment - Weak Values of Single-Photon Polarization

PRL 109, 100404 (2012)

PHYSICAL REVIEW LETTERS

week ending
7 SEPTEMBER 2012



Violation of Heisenberg's Measurement-Disturbance Relationship by Weak Measurements

Lee A. Rozema, Ardavan Darabi, Dylan H. Mahler, Alex Hayat, Yasaman Soudagar, and Aephraim M. Steinberg
Centre for Quantum Information & Quantum Control and Institute for Optical Sciences, Department of Physics, 60 St. George Street,
University of Toronto, Toronto, Ontario, Canada M5S 1A7
(Received 4 July 2012; published 6 September 2012)

While there is a rigorously proven relationship about uncertainties intrinsic to any quantum system, often referred to as “Heisenberg’s uncertainty principle,” Heisenberg originally formulated his ideas in terms of a relationship between the precision of measurement and the disturbance it must create. Although this latter relationship is not rigorously proven, it is commonly believed (and taught) as an aspect of the broader uncertainty principle. Here, we experimentally observe a violation of Heisenberg’s “measurement-disturbance relationship”, using weak measurements to characterize a quantum system before and after it interacts with a measurement apparatus. Our experiment implements a 2010 proposal of Lund and Wiseman to confirm a revised measurement-disturbance relationship derived by Ozawa in 2003. Its results have broad implications for the foundations of quantum mechanics and for practical issues in quantum measurement.

DOI: [10.1103/PhysRevLett.109.100404](https://doi.org/10.1103/PhysRevLett.109.100404)

PACS numbers: 03.65.Ta, 03.67.Ac, 42.50.Xa



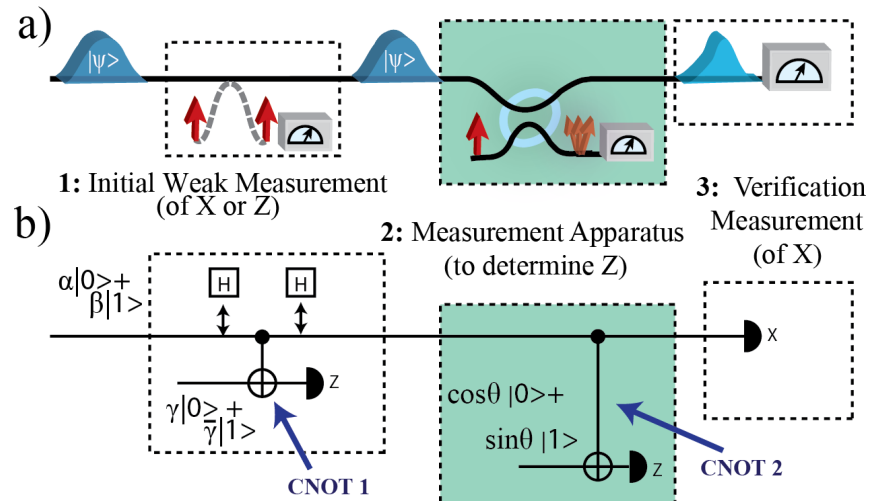
A. M. Steinberg *et al.*, *Phys.Rev.Lett.* **109**, 100404 (2012). University of Toronto

07.01.2014

FUF

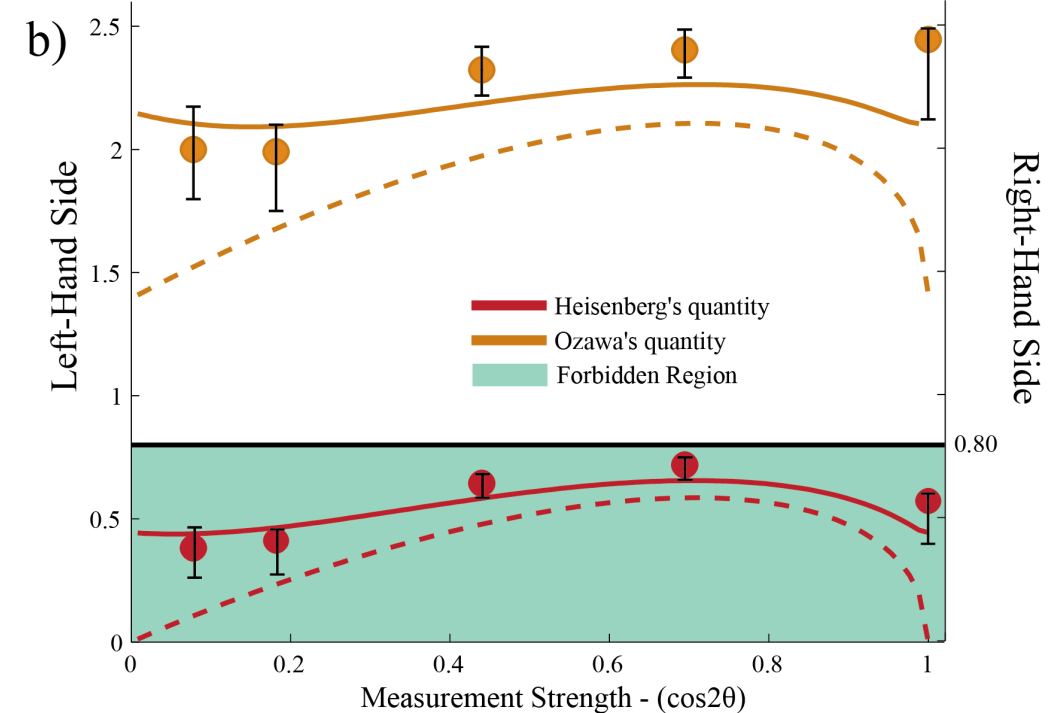
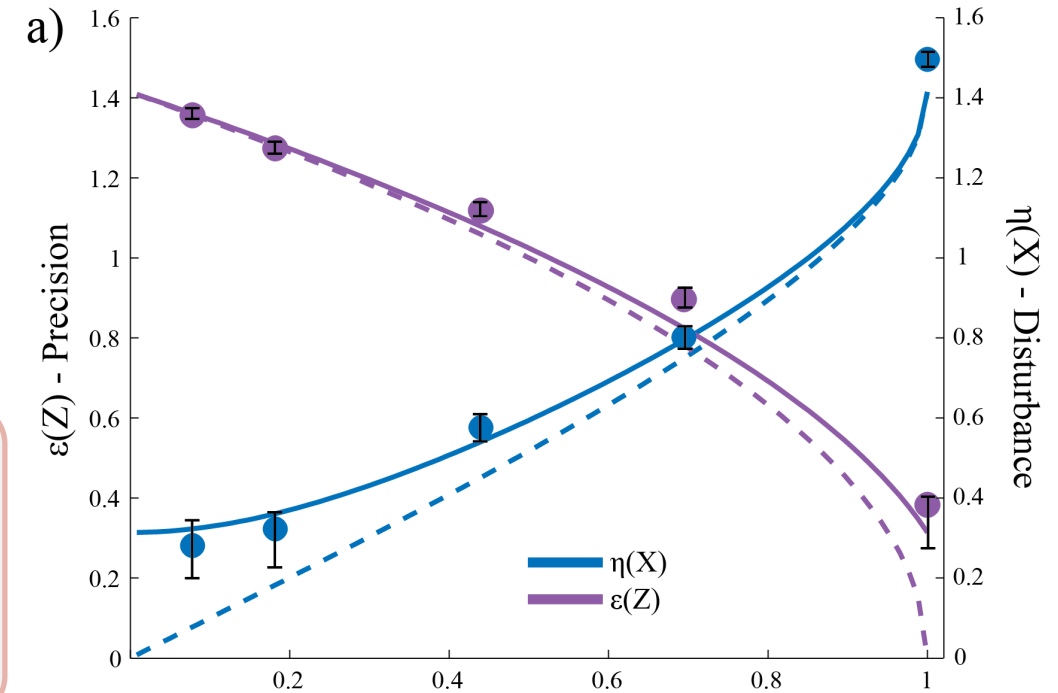
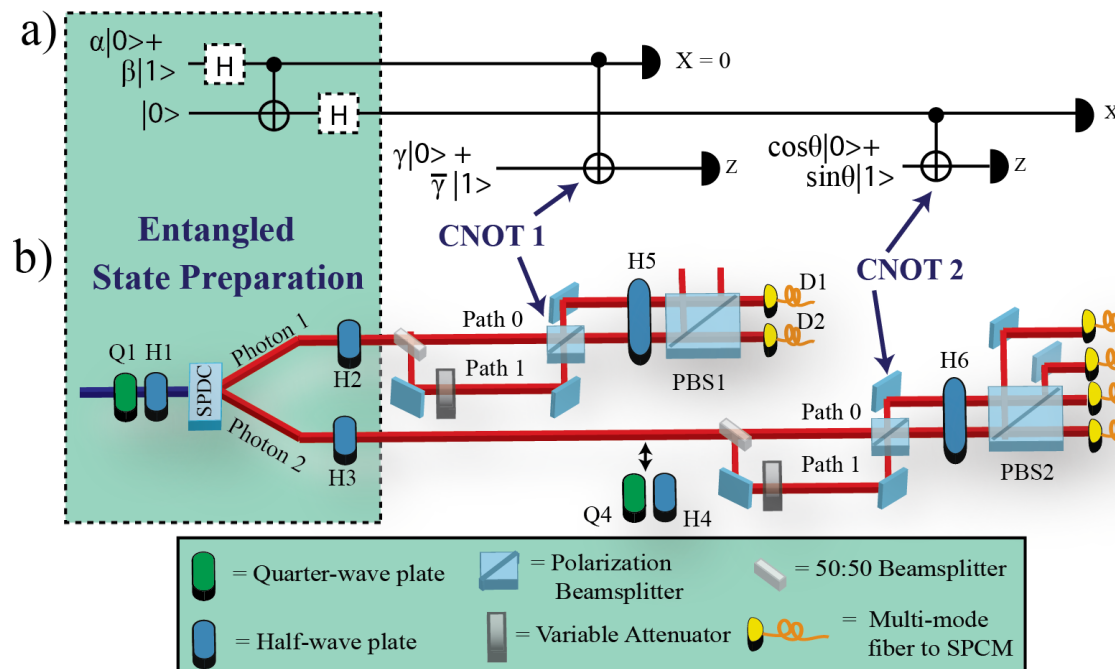
Seminar Neue Arbeiten am Atominstitut

Steinberg Experiment - Weak Values of Single-Photon Polarization



$$\epsilon(A)^2 = \langle (U^\dagger (I \otimes M) U - A \otimes I)^2 \rangle = \sum_{\delta A} (\delta A)^2 P_{WV}(\delta A)$$

$$\eta(B)^2 = \langle (U^\dagger (B \otimes I) U - B \otimes I)^2 \rangle = \sum_{\delta B} (\delta B)^2 P_{WV}(\delta B)$$



A. M. Steinberg *et al.*, *Phys. Rev. Lett.* **109**, 100404 (2012). University of Toronto

Thank you for your attention

ongoing Discussion on Atom Interferometry (AI) Experiments

re-interpretation of atom interferometry experiments:

as measurement of the **gravitational redshift** on the quantum clock operating at the Compton frequency: $\omega_C = mc^2/\hbar \sim 2\pi \times 3.0 \times 10^{25}$ (m is (Cs) atom rest mass)

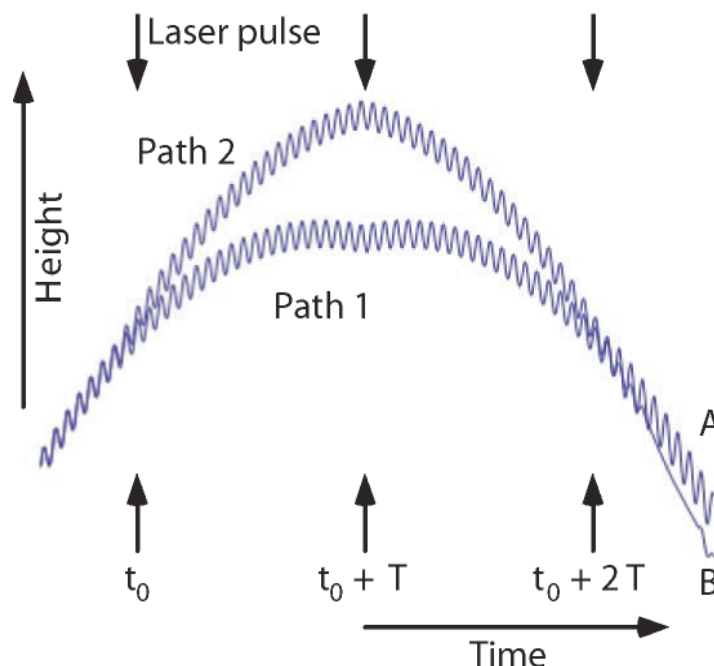
accuracy using clocks: 7×10^{-5} $\longleftrightarrow$ accuracy AI: 7×10^{-9}

$$\Delta\varphi = \overbrace{\int \omega_c d\tau}^{\text{Compton frequency}} + \Delta\varphi_{\text{redshift}} + \Delta\varphi_{\text{laser}}$$

$$\Delta\varphi = (1 + \beta)k g T^2 - k g T^2 + k g T^2$$

$\beta = 0$ for metric theories of gravity (e.g. GR)

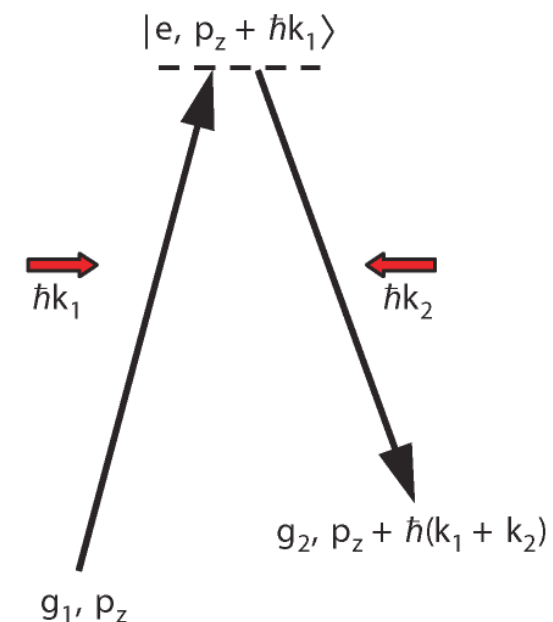
$d\tau$ proper time



$$\Delta\varphi_{\text{time}} = -\Delta\varphi_{\text{laser}} \longrightarrow \Delta\varphi = \Delta\varphi_{\text{redshift}}$$

OR

$$\oint \omega_c d\tau = 0 \longrightarrow \Delta\varphi = \Delta\varphi_{\text{laser}}$$

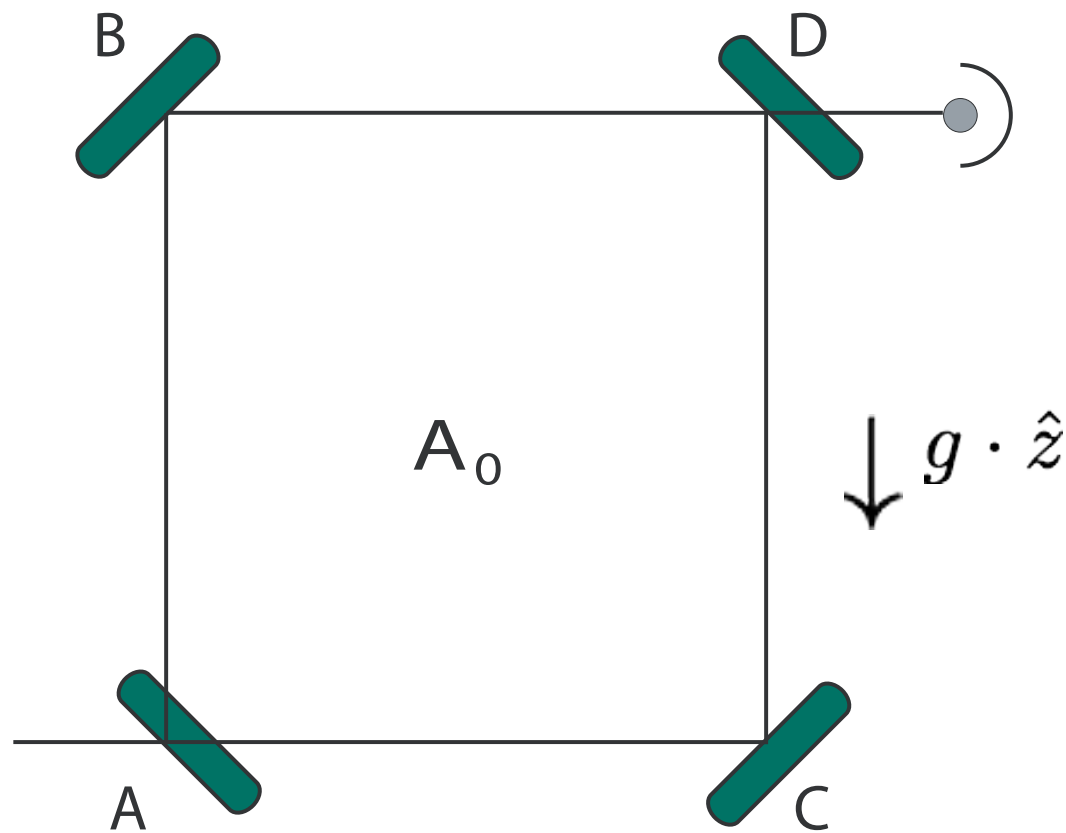


H. Müller, A. Peters, and S. Chu, *Nature* **463**, 926–929 (2010),
C. Cohen-Tannoudji et.al., *Class. Quantum Grav.* **28**, 145017 (2011)

Gravity Quantum Phase & Equivalence Principle (EP)

🔴 effect of gravitational field on quantum particles ?

🟢 Hamiltonian: $E_0 = \frac{\hbar^2 k^2}{2m_I} + m_g g z$



induced phase:

$$\Delta\Phi_{grav} = \Phi_{ACD} - \Phi_{ABD}$$

$$\Delta\Phi_{grav} = -2\pi\lambda \frac{m_I m_g}{\hbar^2} g A_0$$

g gravitational acceleration

m_g gravitational mass

m_I inertial mass

Neutrons: Colella Overhauser Werner (COW) Experiment 1975

$$\Delta\Phi_{COW} = -2\pi\lambda \frac{m_I m_g}{\hbar^2} g A_0 \sin \phi$$

$$\Delta\Phi_{COW} = -q_{grav} \sin \phi$$

$$A_0 = d(d + a \cos \theta) \tan \theta$$

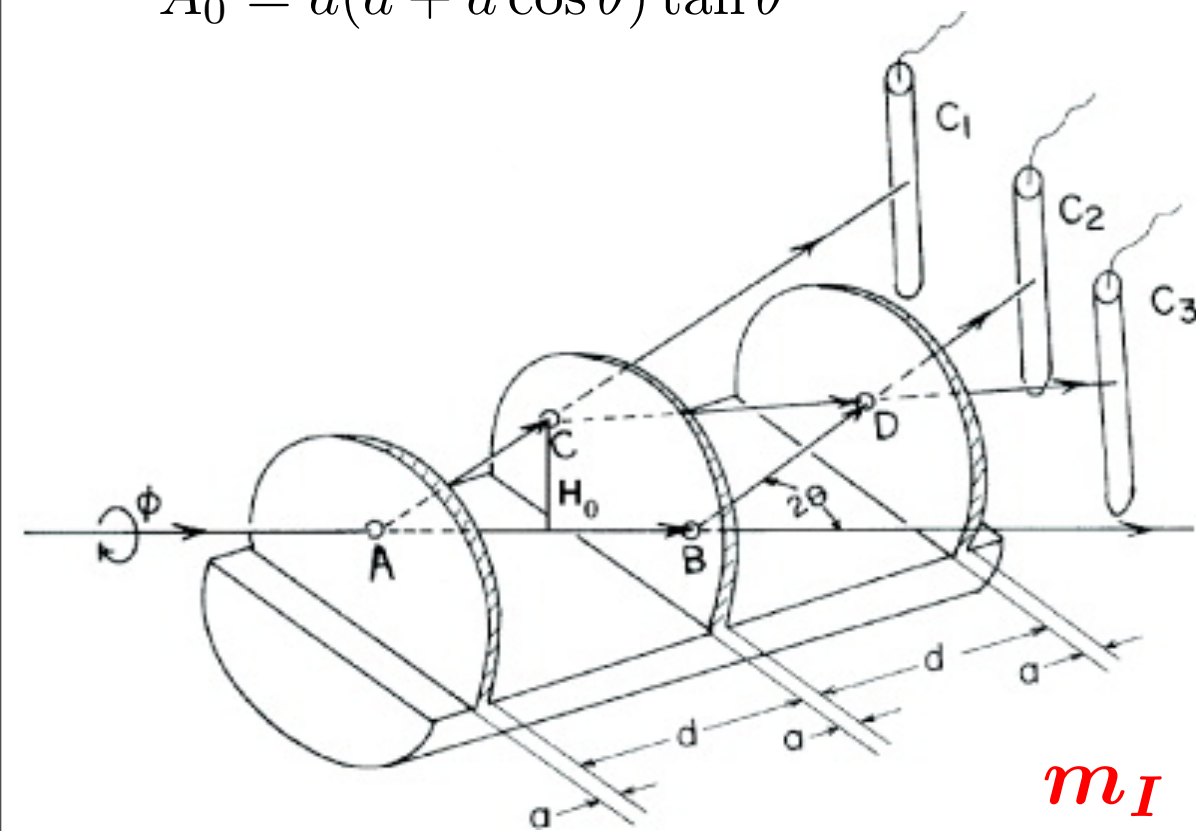
additional terms:

$$\Delta\Phi_{COW} = (-q_{grav} - q_{bend}) \sin \phi + q_{Sagnac} \cos \phi$$

$q_{bend} \rightarrow$ X-ray measurements

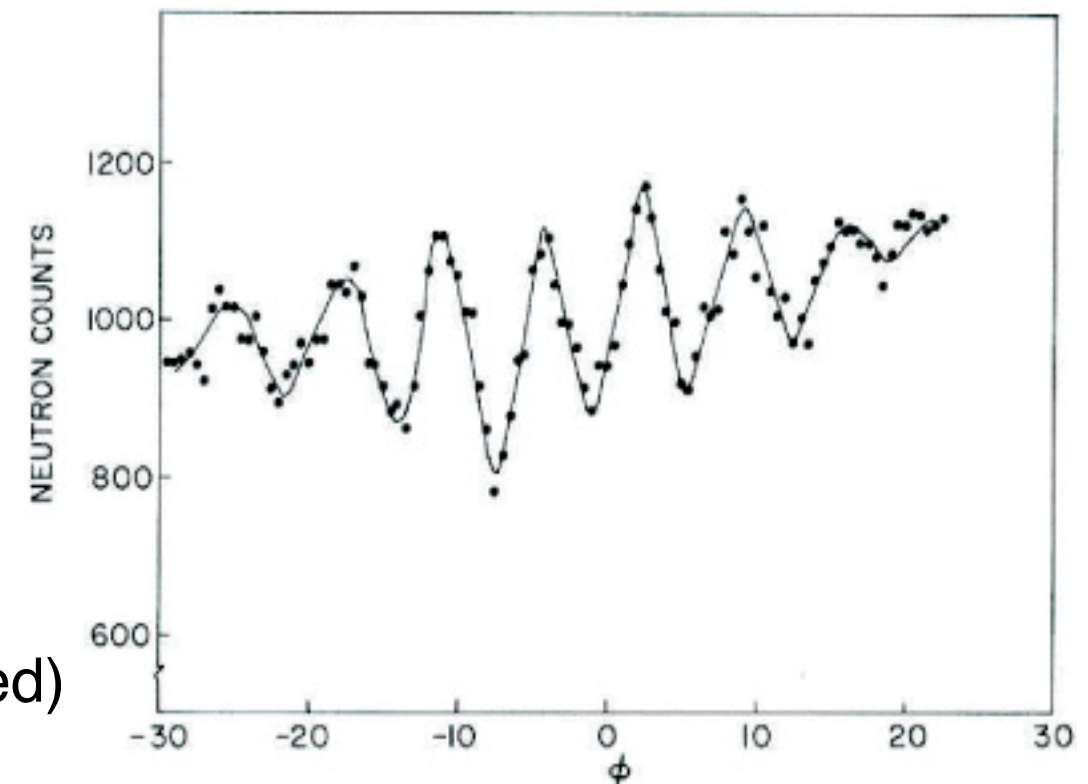
$$q_{Sagnac} = 2\pi m_I \omega_{Earth} A_0 / \hbar \cos \theta_L$$

θ_L colatitude angle



$$m_I = m_g$$

deviation $\sim 4\%$ (1% repeated)



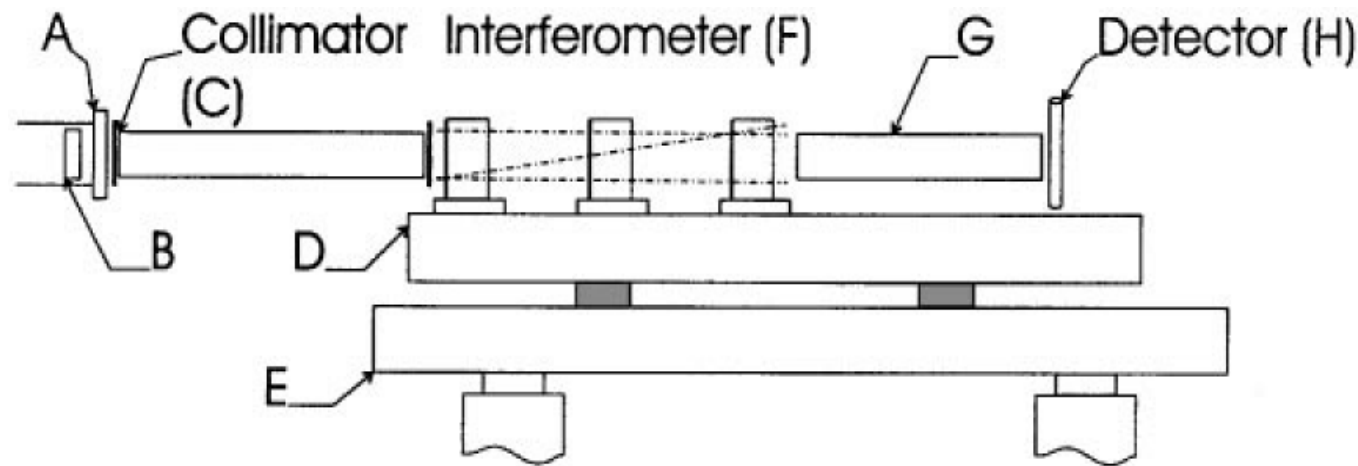
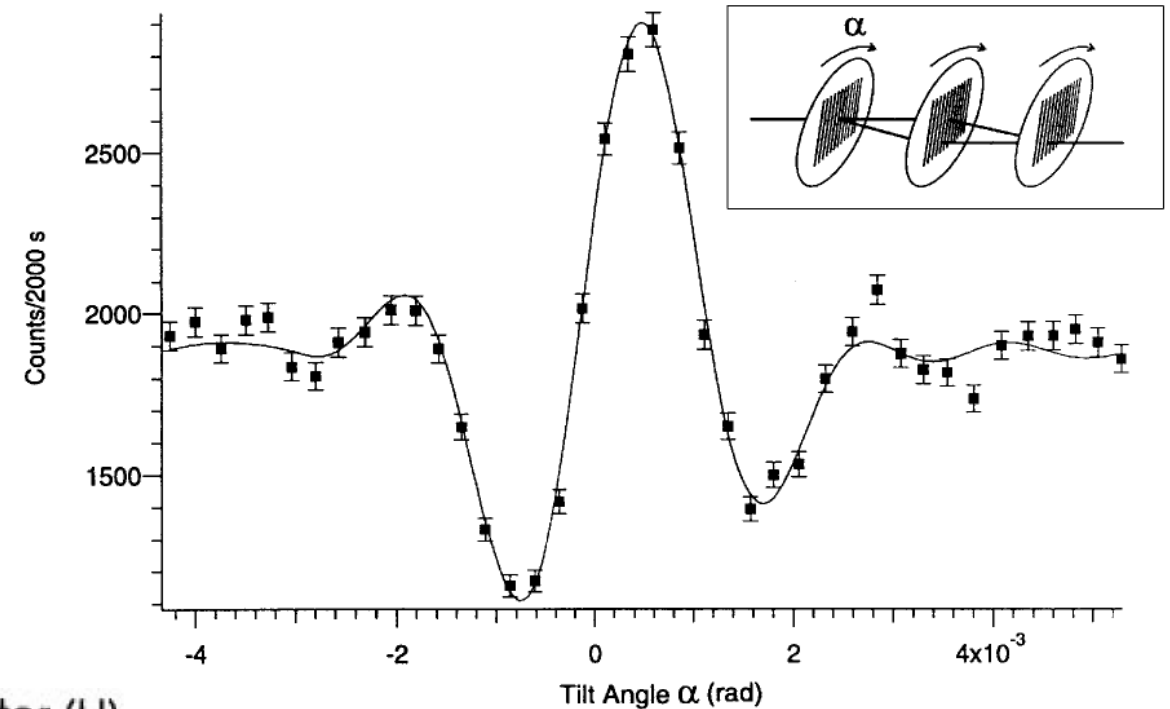
Collela R, Overhauser A W and Werner S A, *Phys. Rev. Lett.* **34**, 1472 (1975)

COW Experiment using Very-Cold Neutrons

interferometer based solely on gratings

$$\Delta\Phi_{grav} = -2\pi\lambda \frac{m_I m_g}{\hbar^2} g A(\lambda) \sin\phi$$

- mean wavelength : 100 Å
- $A(\lambda)$: separation ≥ 0.5 mm, $L = 1$ m
- low countrate : 0.6 s^{-1}



deviation: 1%

$$m_I = m_g$$



van der Zouw et. al., *Nuclear Instruments and Methods in Physics Research A* **440**, 568 (2000)

Discussion

Discussion



Discussion



Discussion

Open Questions:



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