Bouncing Universes in Loop Quantum Cosmology

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Rovelli and WE, arXiv:1310.8654 [gr-qc].

Quantum Gravity: The Singularity Problem

Singularities generically appear in general relativity. It seems likely that these singularities indicate that general relativity can no longer be trusted, and so it is necessary to turn to a theory of quantum gravity where the singularities are hoped to be resolved.

Cosmology is a relatively simple setting where singularities appear, and it can be checked whether cosmological singularities can be resolved by quantum gravity effects or not.

A closely related issue is that expanding cosmologies typically begin with a big-bang singularity. It is tempting to set the initial conditions at the initial time of the big bang, but observables typically diverge or vanish at this initial time. Can this be fixed by quantum gravity?

Quantum Gravity: Observable Tests

It is generally expected that quantum gravity effects will only become important when

- the space-time curvature becomes very large,
- or at very small scales / very high energies.

Since we cannot probe sufficiently small distances with accelerators, or even with cosmic rays, the best chance of testing any theory of quantum gravity appears to be by observing regions with high space-time curvature.

The two obvious candidates are black holes and the early universe. However, since the strong gravitational field near the center of astrophysical black holes is hidden by a horizon, it seems that observations of the early universe are the best option.

The Cosmic Microwave Background

Recently, the Planck collaboration has released a wealth of data on the cosmic microwave background (CMB). Further information regarding the polarization of the CMB will be released soon.
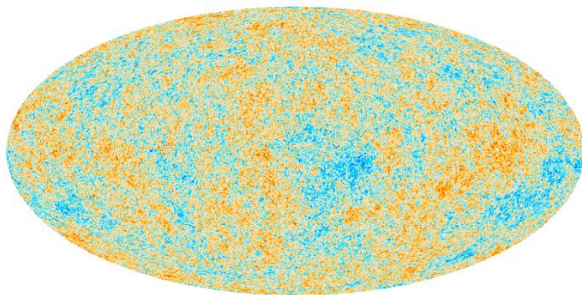
In addition, the detection of B modes in the CMB by the South Pole telescope raises the prospect that primordial B modes —seeded from primordial gravitational waves— may be detected in the near future.

Cosmological Observables

The variables $\mathcal{R}_k$ and h_k are the comoving curvature perturbations and the tensor perturbations, in Fourier space. The scalar power spectrum is

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 \sim k^{n_s-1},$$

and $n_s = 1$ for scale-invariant perturbations; equivalently $\mathcal{R}_k \sim k^{-3/2}$.

The relative amplitudes of tensor and scalar modes are given by the tensor-to-scalar ratio r,

$$r = \frac{\Delta_h^2(k)}{\Delta_{\mathcal{R}}^2(k)}.$$

Observations of temperature anisotropies in the CMB by the Planck collaboration, and of its polarization by WMAP, indicate that

$$n_s = 0.9603 \pm 0.0073, \qquad (68\%),$$

$$r < 0.120, \qquad (95\%).$$

Outline

Loop Quantum Gravity

Loop quantum gravity (LQG) is a background independent approach to the problem of quantum gravity.

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- Operators in LQG act on space itself rather than fields living on a space.

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The fundamental operators are holonomies of the Ashtekar-Barbero SU(2) connection A_a^i, and area operators given by fluxes of the densitized triads E_i^a.

- Operators corresponding to geometrical properties like lengths, areas, volumes and angles have a discrete spectra,
- There is a minimum non-zero area eigenvalue, $\Delta \sim \ell_{\mathrm{Pl}}^2$.

Loop Quantum Cosmology

In loop quantum cosmology (LQC), the main inputs are to use the same variables as in LQG —holonomies of A_a^i and areas— and also the existence of a minimum area, Δ.
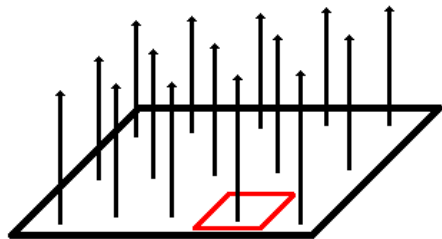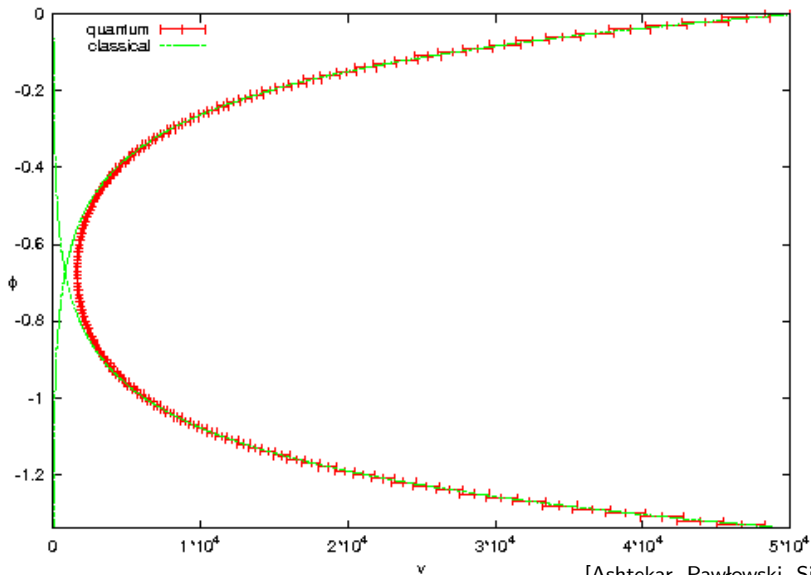

So, in the Hamiltonian constraint $\mathcal{C}_H$, the field strength $F(A)_{ab}{}^k$ is expressed in terms of holonomies of A_a^i around a loop in the $a - b$ plane with an area of Δ.

This gives a difference equation (rather than a differential equation) in terms of $\nu \sim a^3$ for the LQC Hamiltonian of the flat FLRW space-time, [Bojowald; Ashtekar, Lewandowski, Pawłowski, Singh, . . .]

$$\partial_\varphi^2 \Psi(\nu, \varphi) \sim \nu^2 \left[\Psi(\nu + 4, \varphi) - 2\Psi(\nu, \varphi) + \Psi(\nu - 4, \varphi) \right].$$

A Bouncing Universe

[Ashtekar, Pawłowski, Singh]

Effective Equations

The wave function remains sharply peaked throughout its evolution, even at the bounce point. Therefore, for sharply peaked states, their trajectories can be described by the "effective equations"

$$H^2 = \frac{8\pi G}{3}\rho \left(1 - \frac{\rho}{\rho_c}\right),$$

$$\dot{H} = -4\pi G \left(\rho + P\right)\left(1 - \frac{2\rho}{\rho_c}\right),$$

$$\dot{\rho} + 3H\left(\rho + P\right) = 0.$$

$H = \dot{a}/a$ is the Hubble rate and the bounce occurs when the matter energy density equals the critical energy $\rho_c \sim \rho_{\mathrm{Pl}}$. [Taveras]

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For example, for a dust-dominated universe ($P = 0$),

$$a(t) = \left(6\pi G\rho_o t^2 + \frac{\rho_o}{\rho_c}\right)^{1/3}.$$

Perturbations in Loop Quantum Cosmology

One approach that makes it possible to perform a loop quantization of both the background and the perturbations is to generalize the 'separate universes' approach to LQC. [Wands, Malik, Lyth, Liddle]

a(1) φ(1)	a(2) φ(2)	a(3) φ(3)
a(4) φ(4)	a(5) φ(5)	a(6) φ(6)
a(7) φ(7)	a(8) φ(8)	a(9) φ(9)

The idea is to work on a lattice, where each cell is homogeneous, and the gravitational and matter fields vary from one cell to another. Then, after some gauge-fixing, the standard LQC techniques can be used in each homogeneous cell. It turns out that interaction terms are easy to handle.

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Then, for states where the perturbations are small, the scalar and diffeomorphism constraints for each cell, $\widehat{\mathcal{H}}(\vec{z})$ and $\widehat{\mathcal{H}_a}(\vec{z})$, weakly commute with the Hamiltonian $\widehat{\mathcal{C}_H}$.

Effective Equations of Lattice LQC

In the FLRW models, the effective equations provide an excellent approximation to the dynamics of sharply peaked states, throughout the entire evolution including the bounce.

Some simple arguments indicate that the effective equations will be a good approximation for the perturbations too, so long as their wavelengths remain much larger than the Planck length.

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Defining

$$z = a\sqrt{\rho + P}/H,$$

the effective equation for $v = z\mathcal{R}$ coming from lattice LQC is

$$v'' - \left(1 - \frac{2\rho}{\rho_c}\right)\nabla^2 v - \frac{z''}{z}v = 0.$$

The same effective equation is obtained by following the anomaly freedom approach. [Cailleteau, Mielczarek, Barrau, Grain]

Some Cosmologies

There exist several cosmologies that predict scale-invariant scalar perturbations, among them

- inflation,
- the matter bounce,
- the ekpyrotic universe.

The matter bounce and the ekpyrotic universe are two alternatives to inflation where scale-invariant perturbations are generated in a contracting universe.

The presence of a bounce is necessary for these two models, so they seem to fit in quite nicely with LQC.

The Goal

The goal now is to use cosmological observations to test LQC.

As observations become more precise, one of these three cosmologies (or perhaps another) is likely to become favoured.

- How does LQC change the predictions in these scenarios when quantum gravity effects are included?
- Are there LQC corrections (that may be small) to the predictions that can be tested observationally?

In this talk, I will focus on holonomy corrections, which are the cause of the bounce in LQC.

We will now look at inflation, the matter bounce and the ekpyrotic universe in the context of LQC.

Inflation in LQC

Assuming the presence of an inflaton with the appropriate mass, the a prior likelihood of inflation occurring in LQC is greater than $1 - 10^{-5}$. [Ashtekar, Sloan; Corichi, Karami]

A quantum gravity extension of the inflationary scenario has been performed via a hybrid quantization, where the perturbations are Fock-quantized on an LQC background. [Fernández-Méndez, Mena Marugán, Olmedo; Agulló, Ashtekar, Nelson]

Potential observable consequences of LQC in the context of inflation have been studied in several settings, however observational signatures are typically very small. [Bojowald, Calcagni, Tsujikawa; Agulló, Ashtekar, Nelson; Linsefors, Cailleteau, Barrau, Grain, . . .]

The Matter Bounce Scenario

The matter bounce is an alternative to inflation where a contracting dust-dominated universe with

$$P = 0, \qquad \Rightarrow \qquad \rho \sim a^{-3},$$

turns quantum vacuum fluctuations into scale-invariant fluctuations, just as an exponentially expanding universe does. [Wands]

If there is a bounce, continuity arguments give some hope that the perturbations in the expanding post-bounce era will also be scale-invariant. [Finelli, Brandenberger]

However, the specifics of how the perturbations travel through the bounce depend on the detailed dynamics of the bounce, and also whether the equations of motion for the perturbations are modified as they go through the bounce.

Perturbations in the Matter Bounce

The solution to the Mukhanov-Sasaki equation in the pre-bounce epoch, before quantum gravity effects become important, is

$$v_k(\eta) = \frac{\sqrt{\pi \hbar (-\eta)}}{2} H^{(1)}_{\frac{3}{2}}(-k\eta),$$

where the numerical pre-factor is chosen so that v_k are quantum vacuum fluctuations at early times ($\eta \to -\infty$),

$$v_k \sim \sqrt{\frac{\hbar}{2k}} e^{-ik\eta}.$$

Then, when the modes exit the Hubble radius as the bounce is approached, $\mathcal{R}_k = v_k/z$ contains a growing scale-invariant term,

$$\mathcal{R}_k \sim k^{3/2} + \frac{k^{-3/2}}{\eta^3}.$$

Evolution Through the Bounce

In the long wavelength limit, the LQC Mukhanov-Sasaki equation is

$$v_k'' - \frac{z''}{z} v_k = 0, \qquad v_k = A_1 z + A_2 z \int^\eta \frac{\mathrm{d}\bar\eta}{z^2},$$

where $z \sim (t^2/t_{\mathrm{Pl}}^2 + 1)^{5/6}/t$. The solution for $\mathcal{R}_k = v_k/z$ is

$$\mathcal{R}_k \sim A_1 + A_2 \left[t_{\mathrm{Pl}} \left(\arctan(t/t_{\mathrm{Pl}}) + \frac{\pi}{2} \right) - \frac{t}{(t/t_{\mathrm{Pl}})^2 + 1} \right].$$

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The A_i are determined by demanding that for $t \ll -t_{\mathrm{Pl}}$,

$$\mathcal{R}_k \sim k^{3/2} + \frac{k^{-3/2}}{\eta^3},$$

and so $A_1 \sim k^{3/2}$, $A_2 \sim k^{-3/2}$. Then, after the bounce ($t \gg t_{\mathrm{Pl}}$),

$$\mathcal{R}_k \sim k^{3/2} + {\color{red} k^{-3/2}} + \frac{k^{-3/2}}{\eta^3},$$

the dominant constant term for long wavelengths is scale-invariant.

Results for the Matter Bounce in LQC

The same procedure can be followed for the tensor perturbations (although the LQC corrections to the Mukhanov-Sasaki equation for tensor modes are slightly different), and we find that:

- Initial vacuum fluctuations give scale-invariant perturbations in the post-bounce expanding branch,

- The scalar and tensor perturbations have the same tilt,

- The tensor-to-scalar ratio is small, $r \sim 10^{-3}$,

- In order to match observations, $\rho_c \sim 10^{-9} \rho_{\mathrm{Pl}}$.

In most matter bounce models, $r \gtrsim 0.25$, so LQC predictions are different from the expectations coming from general relativity here.

The Ekpyrotic Universe

The ekpyrotic universe is a slowly contracting space-time

$$a(t) = (-t)^p, \qquad 0 < p \ll 1,$$

with one or two scalar fields φ with some potentials $V(\varphi)$ that mimic a perfect fluid

$$P = \omega\rho, \qquad \omega = \frac{2}{3p} - 1.$$

In the contracting branch, the Bardeen potential becomes scale-invariant, though not the comoving curvature perturbations $\mathcal{R}$.
[Khoury, Ovrut, Steinhardt, Turok; Lyth]

If there is more than one scalar field, then the entropy perturbations δs are also scale-invariant. [Finelli; Lehners, McFadden, Turok, Steinhardt]

The Single Scalar Field Case

The Bardeen potential contains a growing mode in the contracting branch that is scale-invariant,

$$\Phi_k \sim k^{-1/2} + \frac{k^{-3/2}}{\eta}.$$

What happens on the other side of the bounce?

By solving the LQC Mukhanov-Sasaki equation in the ekpyrotic universe, we find that in the expanding branch of the universe

$$\Phi_k \sim k^{-1/2} + k^{1/2} + \frac{k^{-3/2}}{\eta}.$$

Since the last term will decay rapidly in the expanding universe, the dominant term for long wavelengths goes as $k^{-1/2}$ and has a blue spectrum.

The Two Scalar Field Case

If there are two scalar fields, then entropy perturbations can become important. Assuming initial quantum vacuum perturbations, as the bounce is approached the growing mode of δs_k is scale-invariant.

Entropy perturbations can source curvature perturbations. From this entropic mechanism, the curvature perturbations become [Lehners, McFadden, Turok, Steinhardt; Buchbinder, Khoury, Ovrut, ...]

$$\mathcal{R}_k \sim k^{-1/2} + \frac{k^{1/2}}{\eta} + k^{3/2} + k^{-3/2},$$

already in the contracting branch. Now we can use LQC to determine the form of the comoving curvature perturbations in the expanding branch,

$$\mathcal{R}_k \sim k^{-3/2} + k^{-1/2} + k^{1/2} + k^{3/2} + \frac{k^{1/2}}{\eta},$$

and we find that the dominant term is scale-invariant.

Results for Ekpyrotic LQC

- The ekpyrotic scenario with a single scalar field is not viable in LQC.

- The two scalar field realization of the ekpyrotic universe agrees with observations: a nearly scale-invariant power spectrum, and no tensor modes.

- The predicted values for the tilt and amplitude of the scalar perturbations depend on the specific form of the potentials for the scalar fields.

- The dominant term in the constant mode of $\mathcal{R}_k$ can be determined in the classical regime before the bounce; LQC corrections do not affect it.

Conclusions

- LQC naturally gives a bouncing universe: the big-bang singularity is resolved. This is also true for space-times beyond FLRW.

- Linear perturbations can be included by working with a lattice of homogeneous cosmologies.

- In LQC, inflation, the matter bounce and the ekpyrotic universe with two scalar fields are all in agreement with the results of Planck.

- Observations of tensor perturbations with $r \gtrsim 0.01$ would rule the last two cosmologies out, while inflation with a quadratic potential predicts an r near the upper bound given by Planck.

- Next steps: inflationary models going beyond the $V(\varphi) \sim \varphi^2$ potentials, non-Gaussianities, anisotropies, and more.

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<div align="center">Thank you for your attention!</div>