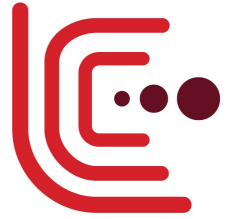


Top quark physics and Electroweak measurements at the LC



Roman Pöschl



Using material from A. Hoang, T. Horiguchi, F. Richard
and many others

See also slides of last weeks top workshop at

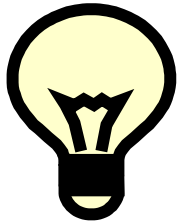
LPNHE

<https://agenda.linearcollider.org/conferenceDisplay.py?ovw=True&confId=6296>

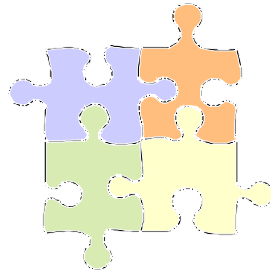
Séminaire LPSC - Grenoble Mars 2014

An enigmatic couple ...

Higgs Boson



Elementary Scalar?



Composite object?

LEPTONS		
Electron Neutrino Mass ~0	Muon Neutrino ~0	Tau Neutrino ~0
Electron .511	Muon 105.7	Tau 1 777
QUARKS		
Up Mass: 5	Charm 1 500	Top ~180 000
Down 8	Strange 160	Bottom 4 250

Top quark

- Higgs and top quark are intimately coupled!

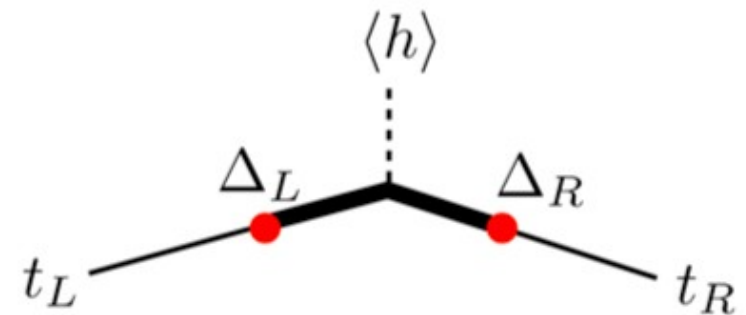
Top Yukawa coupling $O(1)$!

=> Top mass important SM Parameter

- New physics by compositeness?

Higgs and top composite objects?

- **LC perfectly suited to decipher both particles**



Courtesy of S. Rychkov

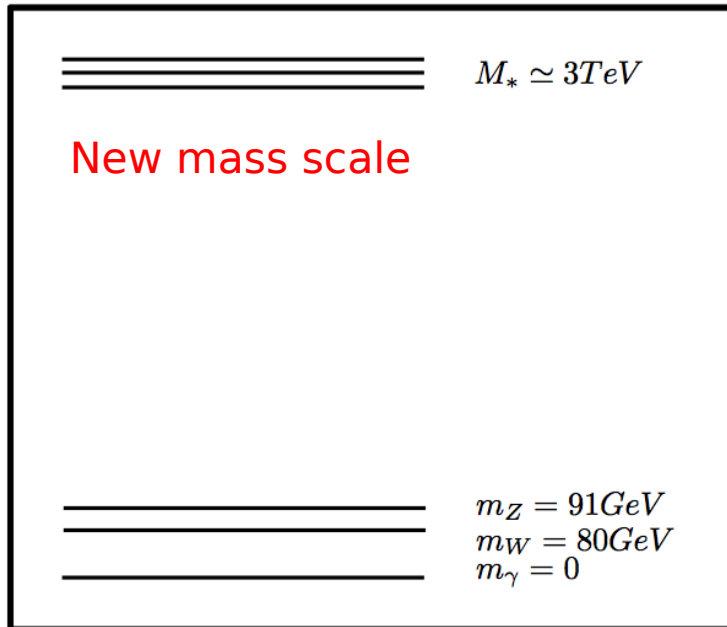
Towards New Physics

à la G.M. Pruna, LC 13, Trento

Compositeness:

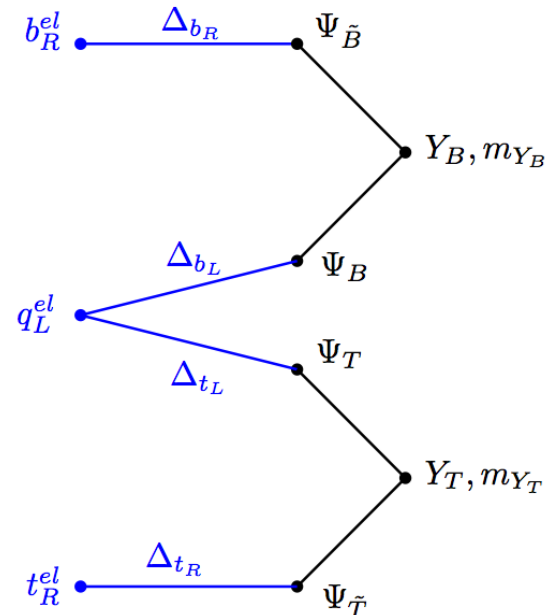
- ... provides elegant solution for naturalness
- ... few tensions with SM predictions
- ... composite Higgs hypothesis has only been marginally studied in comparison with other “fundamental” scenarios
- ... **all** scalar objects observed in nature turned out to be bound states of fermions

Bosonic sector mass spectrum



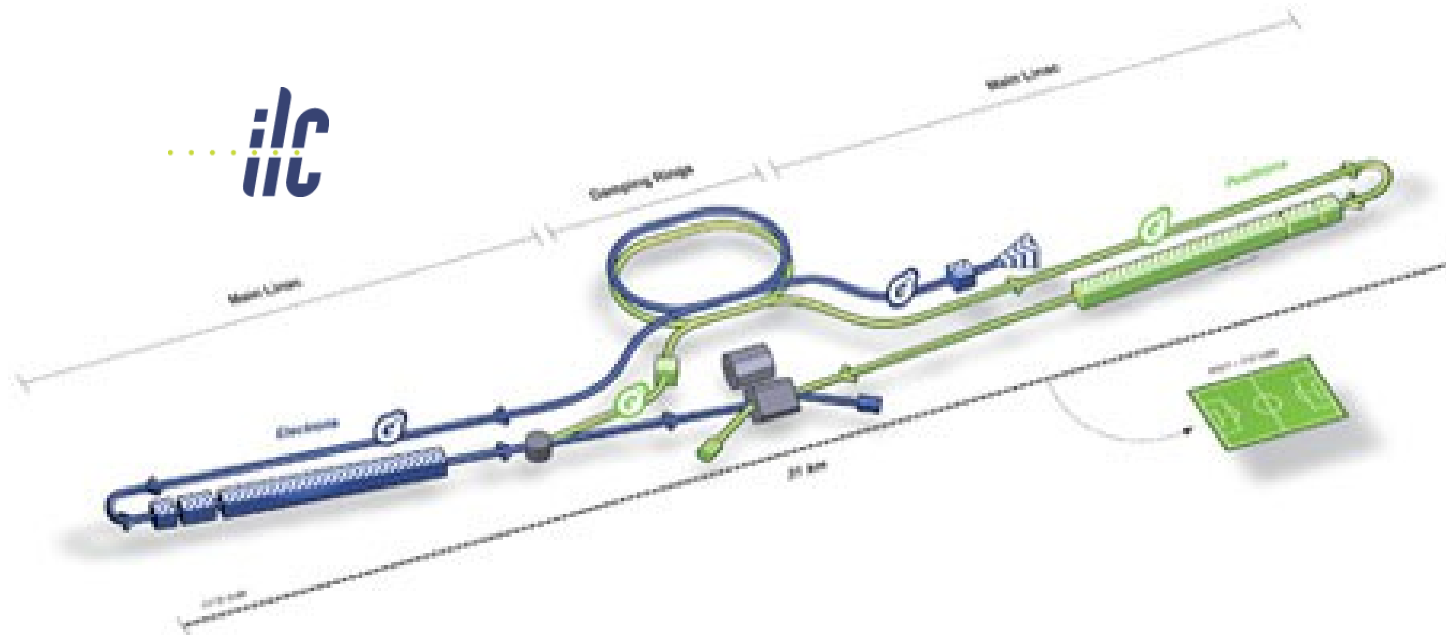
Fermionic resonances

From heavy left handed SM doublet and heavy right handed SM singlet

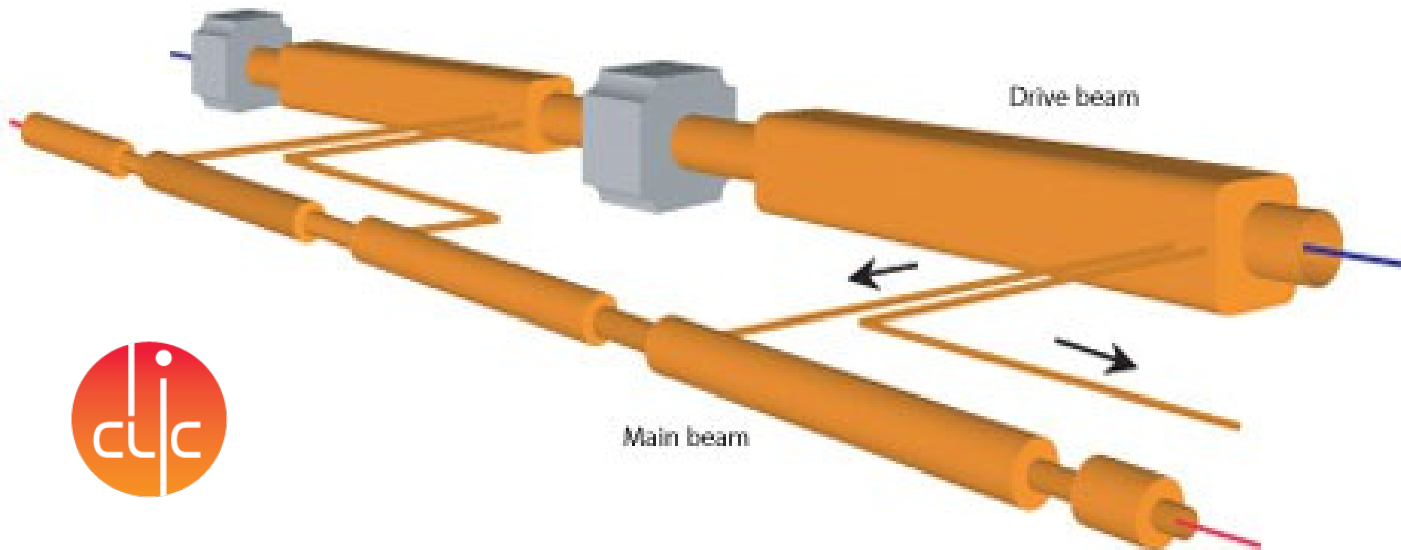


Physics modify Yukawa couplings and Ztt , Zbb
Heavy fermion effect!

(Future) Linear electron-positron colliders



Energy: 0.1 - 1 TeV
Electron (and positron)
polarisation
TDR in 2013
+ DBD for detectors
'Ready for construction'



Energy: 0.5 - 3 TeV
CDR in 2012



Detector requirements

Track momentum: $\sigma_{1/p} < 5 \times 10^{-5} / \text{GeV}$ **(1/10 x LEP)**

(e.g. Measurement of Z boson mass in Higgs Recoil)

Impact parameter: $\sigma_{d0} < [5 \oplus 10/(p[\text{GeV}]\sin^{3/2}\theta)] \mu\text{m}$ **(1/3 x SLD)**

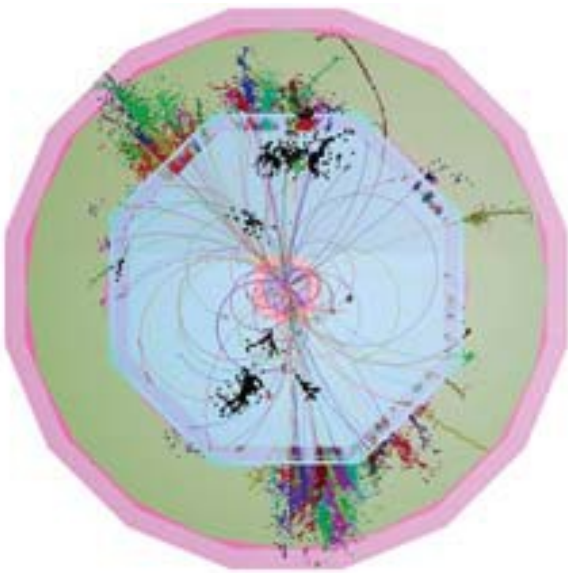
(Quark tagging c/b)

Jet energy resolution : $dE/E = 0.3/(E(\text{GeV}))^{1/2}$ **(1/2 x LEP)**

(W/Z masses with jets)

Hermeticity : $\theta_{\min} = 5 \text{ mrad}$

(for events with missing energy e.g. SUSY)



Final state will comprise events with a large number of charged tracks and jets(6+).

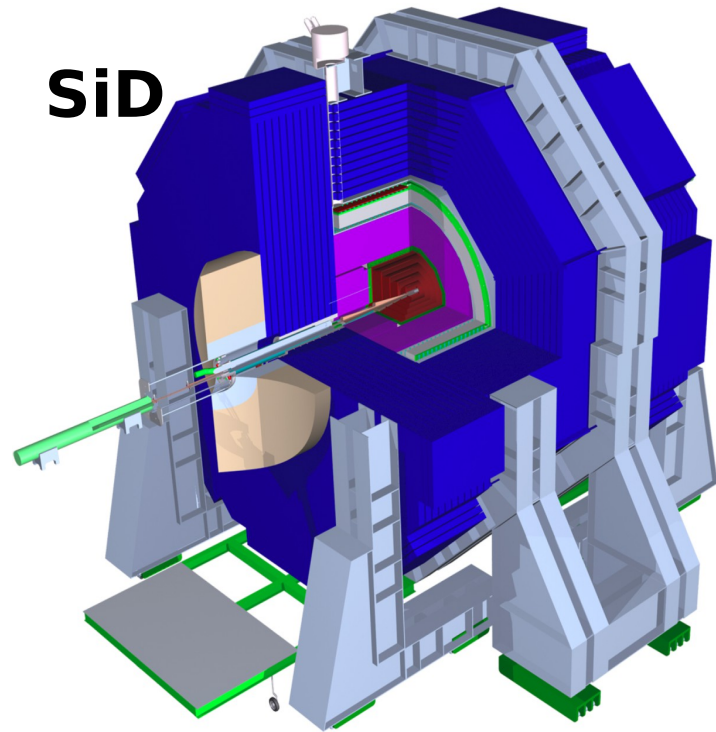
- **High granularity**
- **Excellent momentum measurement**
- **High separation power for particles**

-> Two different approaches

Detector concepts SiD et ILD

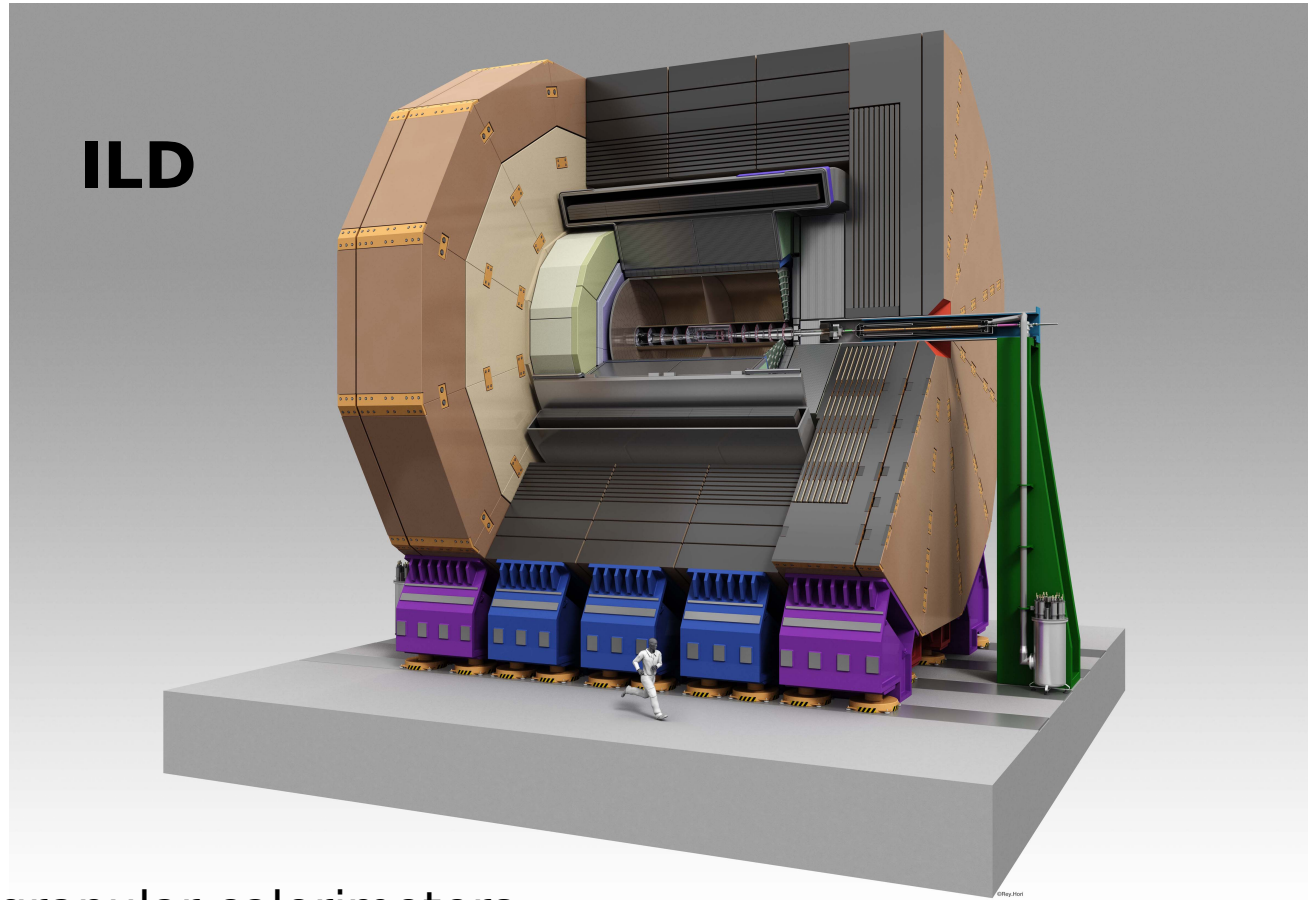
Detector concepts

SiD



Central tracking
with silicon

ILD



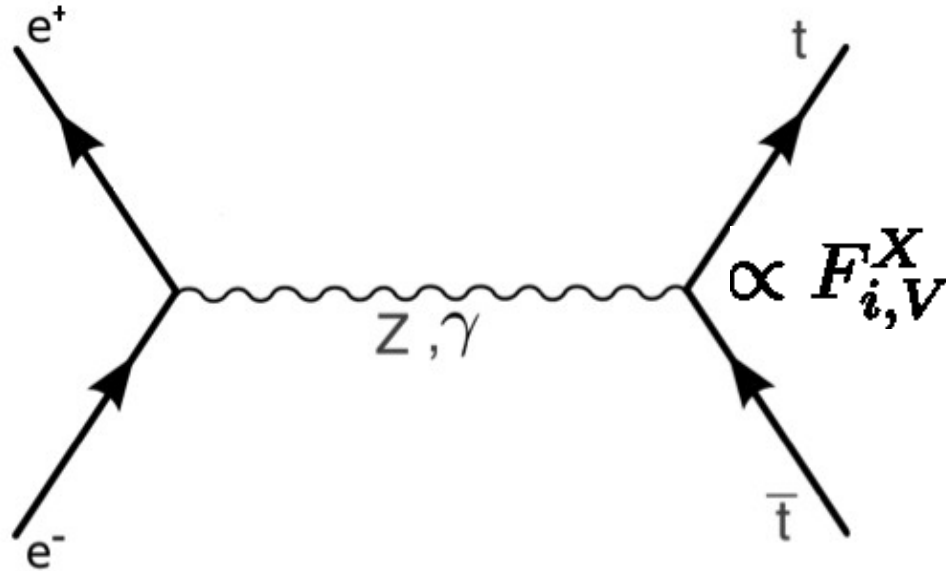
Central tracking
with TPC

Inner tracking with silicon

- LOI's Validated by IDAG in 2009
- Publication of **D**etector **B**aseline **D**esign in 2013, together with TDR
- Concepts based on input from physics studies and detector R&D organised in R&D collaborations

Top quark physics at electron-positron colliders

T. Han



- Top quark production through **electroweak** processes,

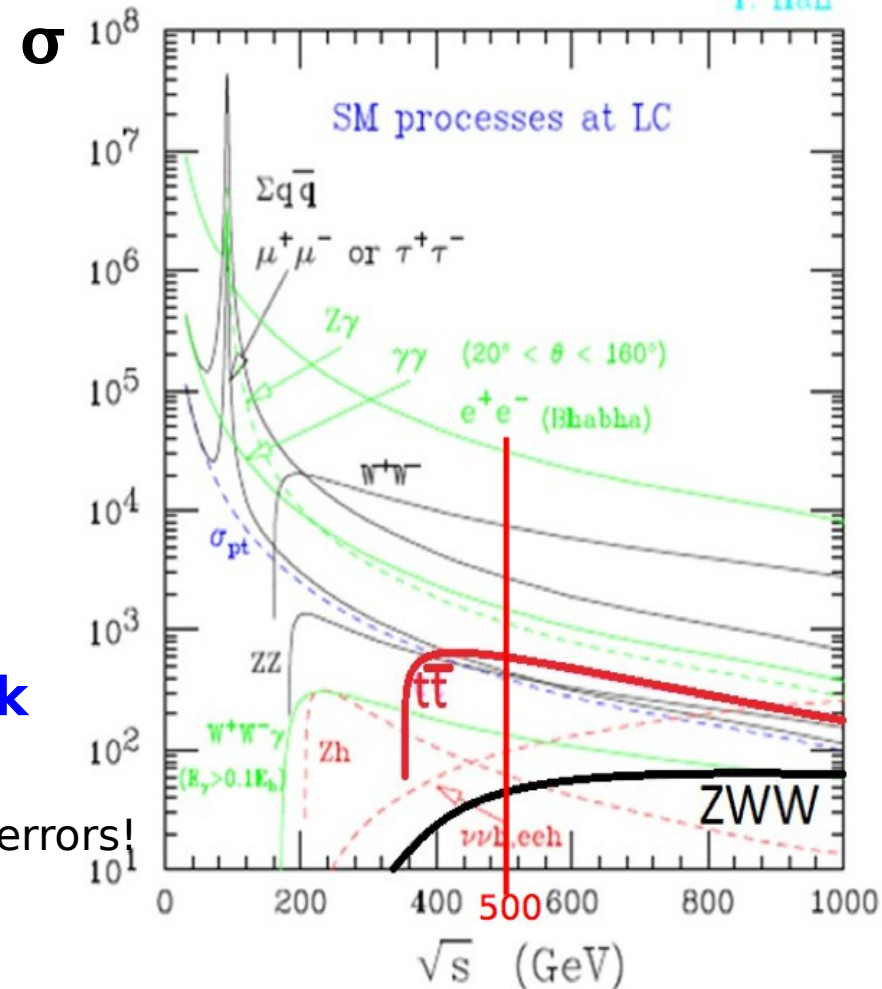
no competing QCD production => Small theoretical errors!

- **High precision measurements**

Top quark mass at ~ 350 GeV through **threshold scan**

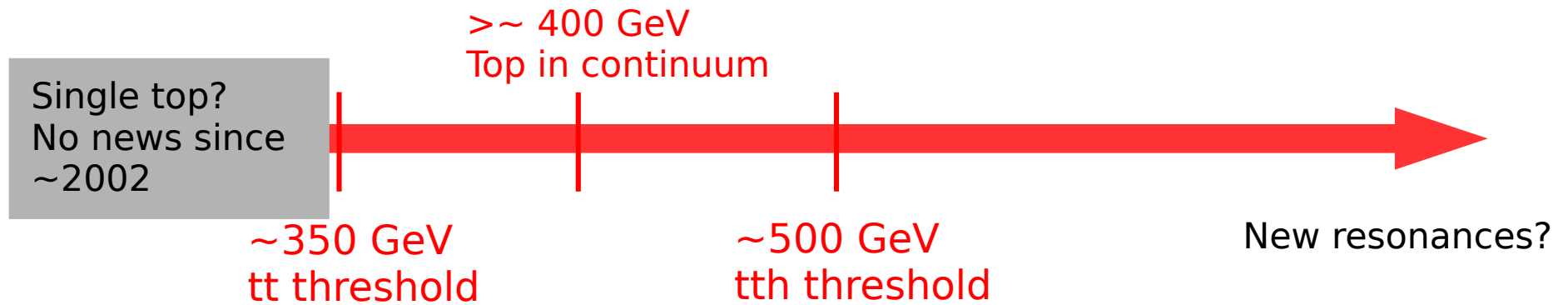
Polarised beams allow to test chiral structure at $t\bar{t}X$ vertex

=> Precision on form factors F



- Studies presented here deal with no or only mildly boosted tops, $\beta \sim 0.7$
- A major **difference between LC and LHC** is that an **LC** will run **triggerless**
- > Unbiased event samples, all event selection happens off-line!

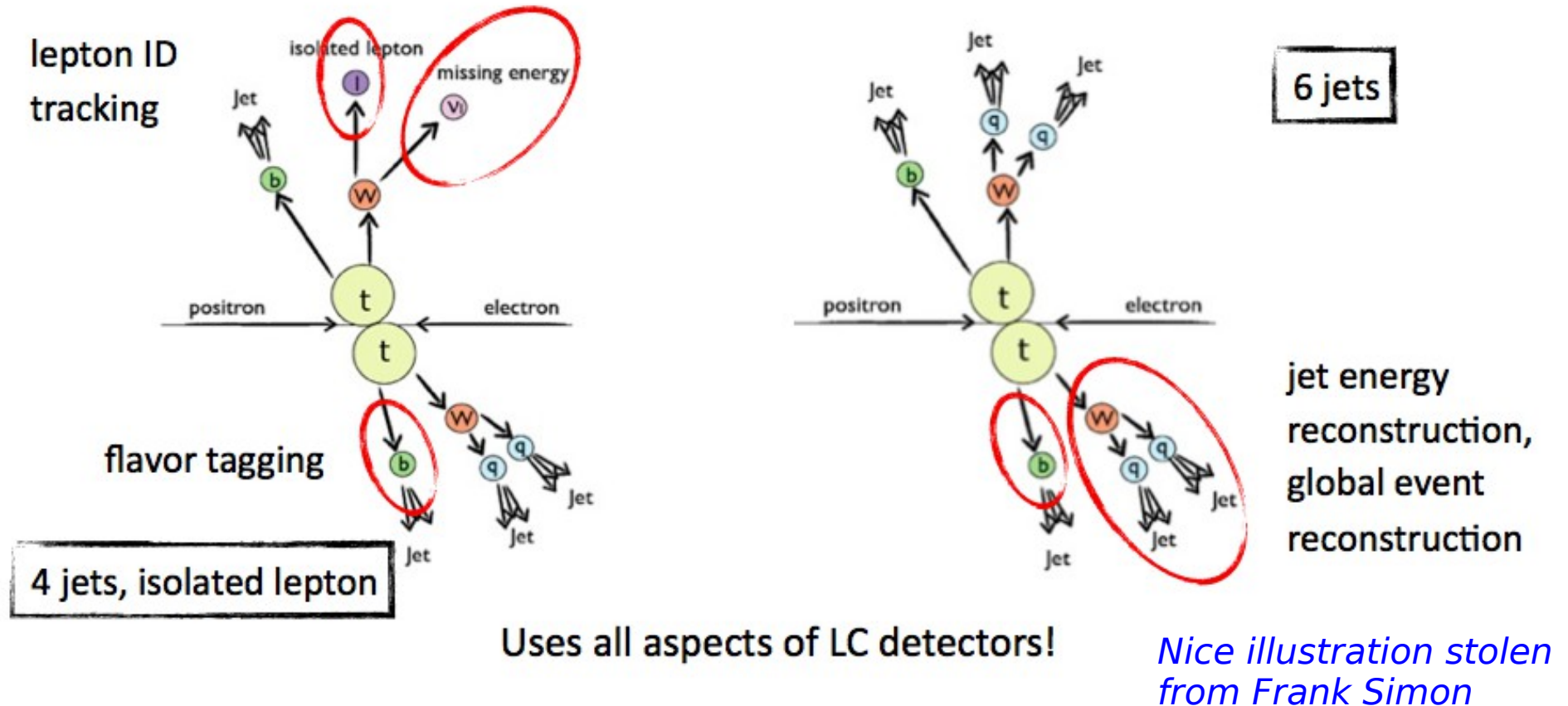
Relevant scales for Top physics and LC Physics programme



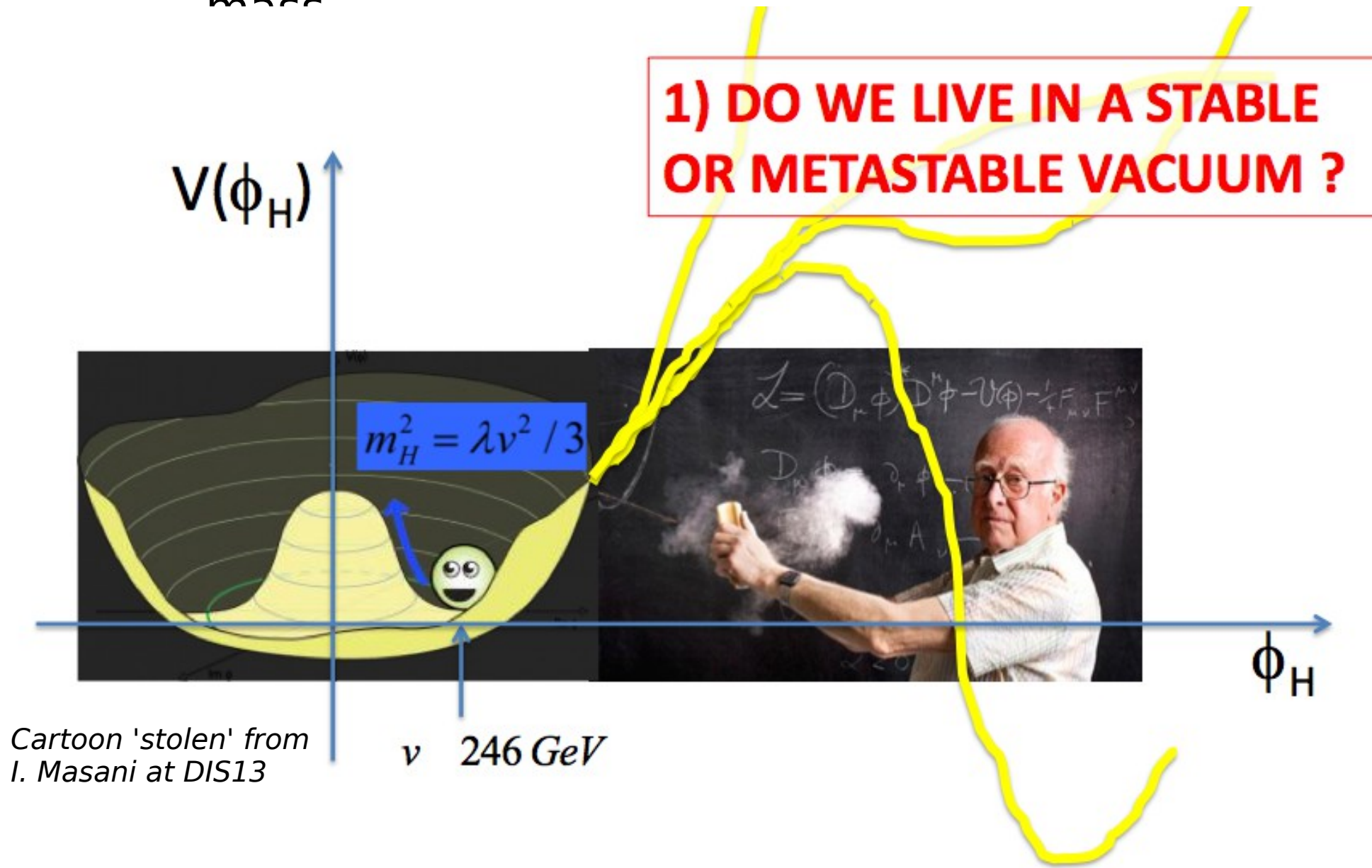
- After TDR and Japanese initiative, programme for ILC under discussion
ILC in staged approach but which is first stage?
- Arguments to start at 350 GeV include Top physics programme

Elements of top quark reconstruction

- By far dominating decays: All-hadronic (46%), semi-leptonic / lepton+jets (45%, 30% w/o τ)
- try to avoid decays into τ , increased uncertainties from additional neutrino



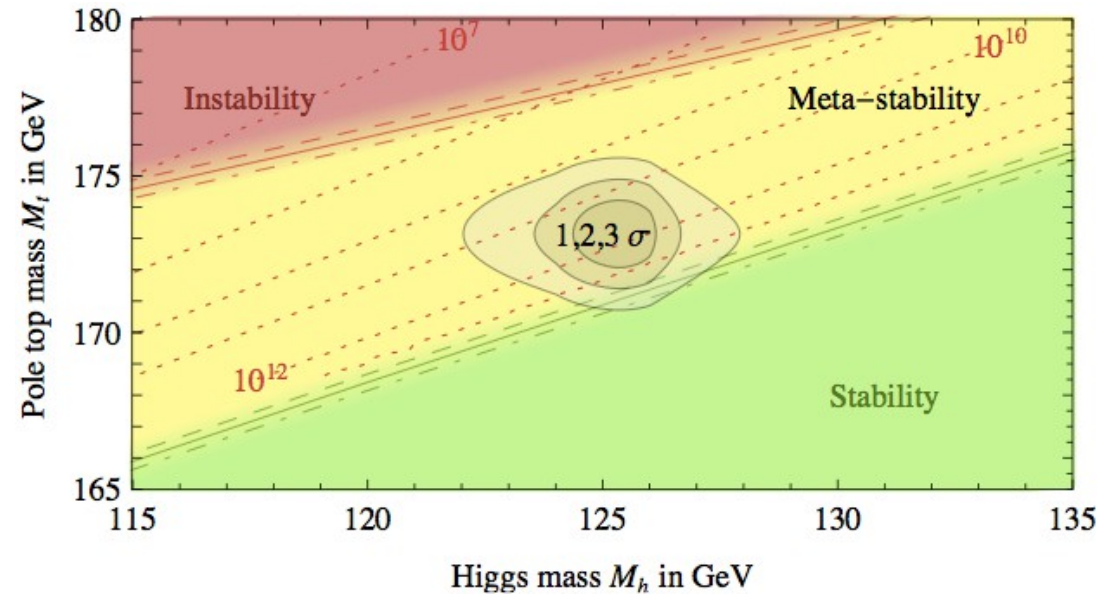
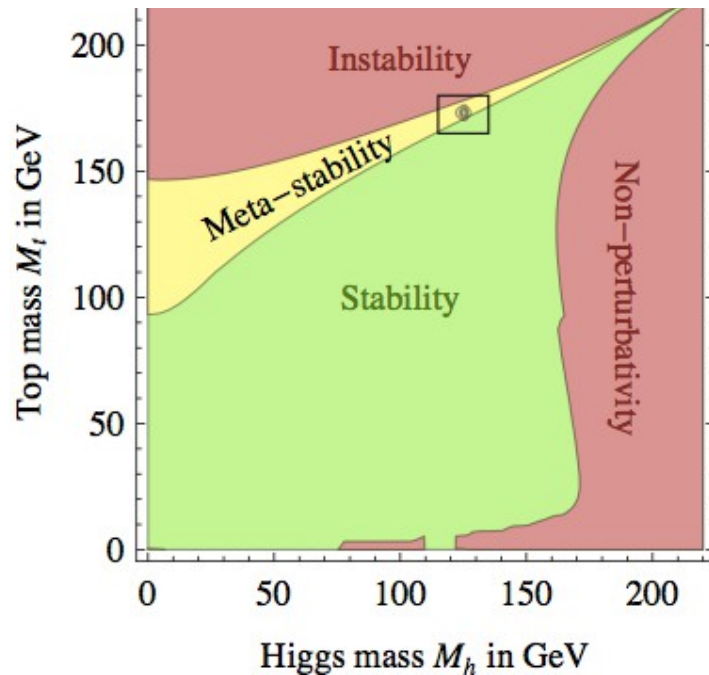
Motivation for precise top quark mass



Vacuum stability and top quark

Degrassi et al.
arXiv:1205.6497

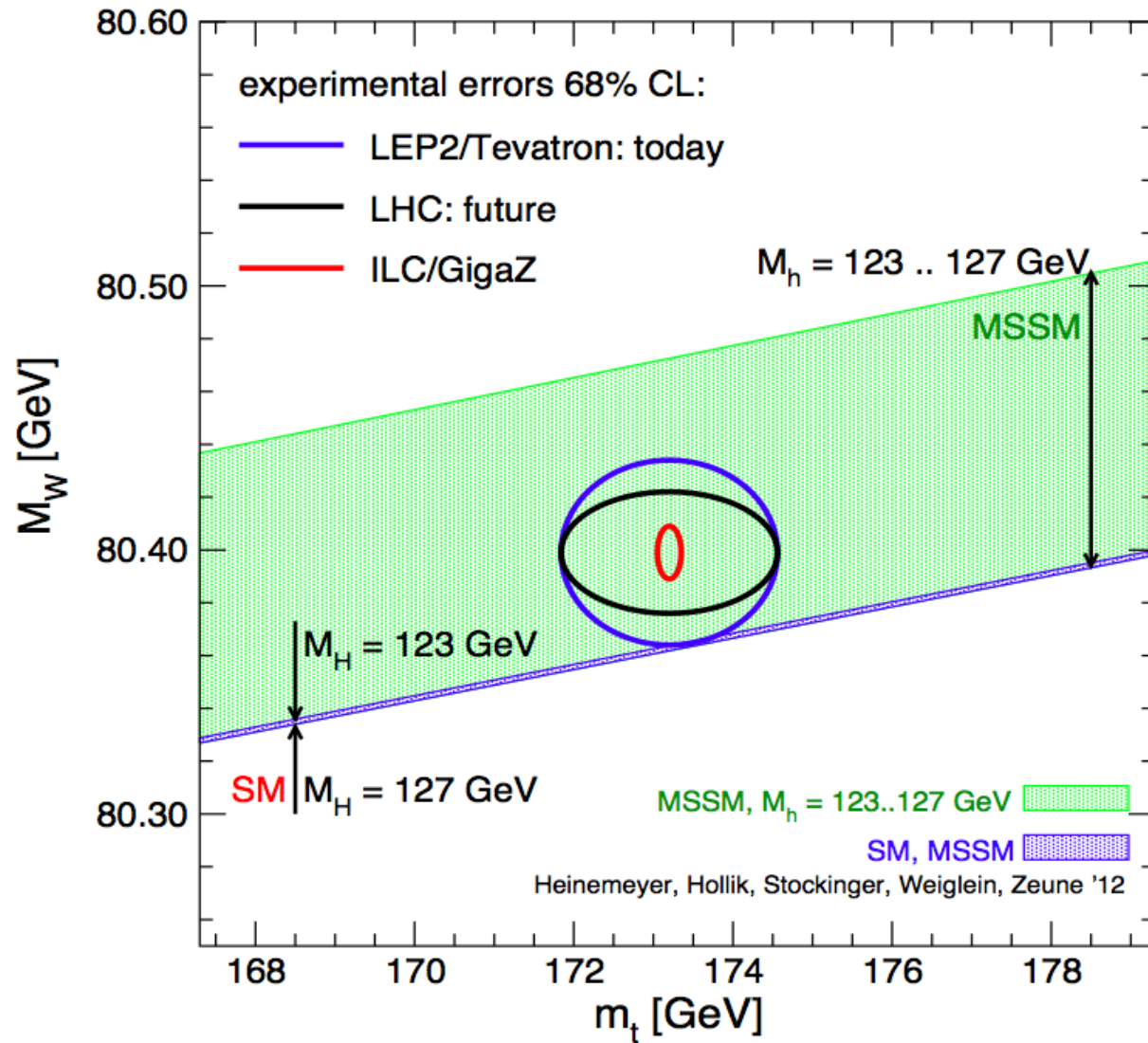
$$M_h [\text{GeV}] > 129.4 + 1.4 \left(\frac{M_t [\text{GeV}] - 173.1}{0.7} \right) - 0.5 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 1.0_{\text{th}} .$$



Type of error	Estimate of the error	Impact on M_h
M_t	experimental uncertainty in M_t	± 1.4 GeV
α_s	experimental uncertainty in α_s	± 0.5 GeV
Experiment	Total combined in quadrature	± 1.5 GeV
λ	scale variation in λ	± 0.7 GeV
y_t	$\mathcal{O}(\Lambda_{\text{QCD}})$ correction to M_t	± 0.6 GeV
y_t	QCD threshold at 4 loops	± 0.3 GeV
RGE	EW at 3 loops + QCD at 4 loops	± 0.2 GeV
Theory	Total combined in quadrature	± 1.0 GeV

Uncertainty on **(pole)**
top quark mass dominates
uncertainty on stability
conditions
(argument is repeated in
literature!)

Top mass Higgs Mass and BSM – SM vs. MSSM



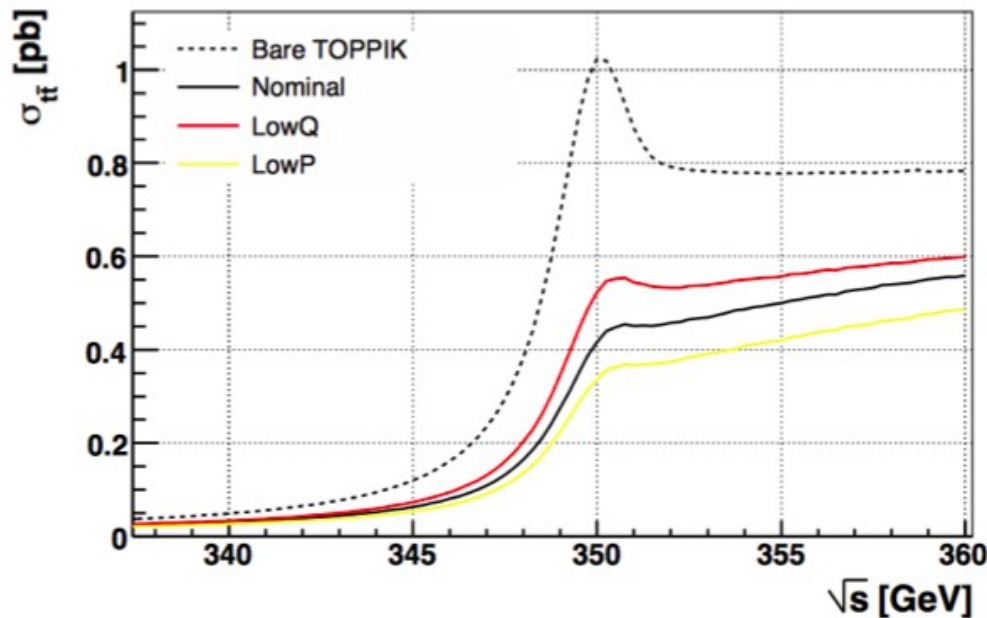
Precise Top (and W) mass
crucial to test compatibility
of measured Higgs mass

MS might not be sufficient
to explain Higgs mass

LHC may not reach sufficient
discriminative power

A lepton collider will

Total $t\bar{t}$ cross section at LC



Principle: m_t from $\sigma_{t\bar{t}}(m_t)$

Advantages:

- ▷ count number of $t\bar{t}$ events
- ▷ color singlet state
- ▷ background is non-resonant
- ▷ physics well understood
(renormalons, summations)
- Top decay protects from non-pert effects

Much of the discriminating power of the approach related to the strong mass-dependence ($t\bar{t}$ resonance).

Peak position very stable in theory predictions (threshold mass scheme).

Typical results:

- $\delta m_t^{\text{exp}} \simeq 50 \text{ MeV}$
- $\delta m_t^{\text{th}} \simeq 100 \text{ MeV}$

What mass?

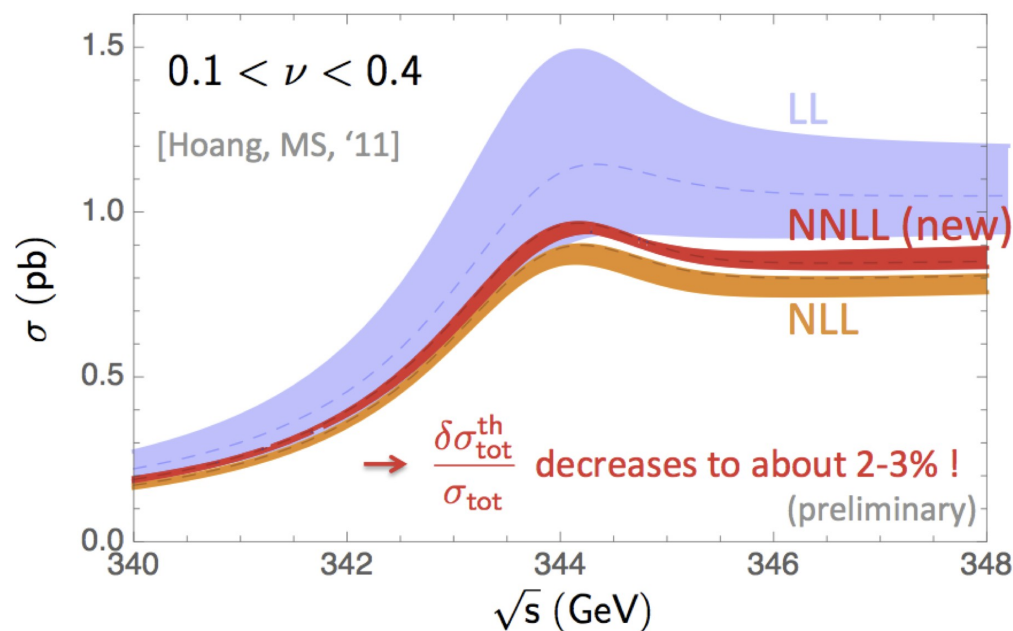
$$\sqrt{s}_{\text{rise}} \sim 2m_t^{\text{thr}} + \text{pert.series}$$

(short distance mass: $1S \leftrightarrow \overline{MS}$)

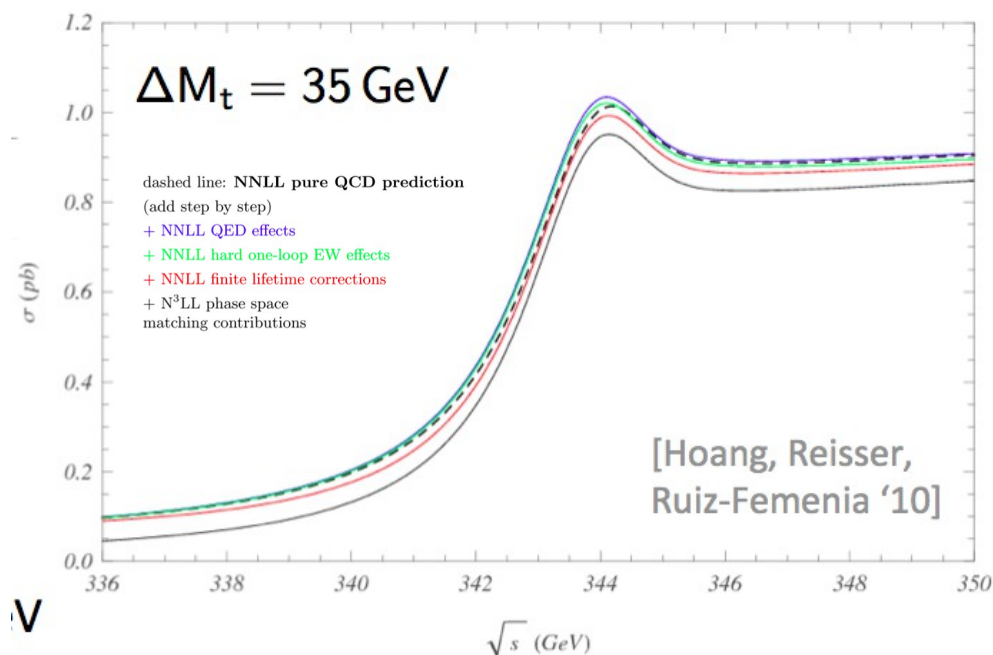
A. Hoang

Top quark mass – Theoretical accuracies

QCD



QCD + electroweak



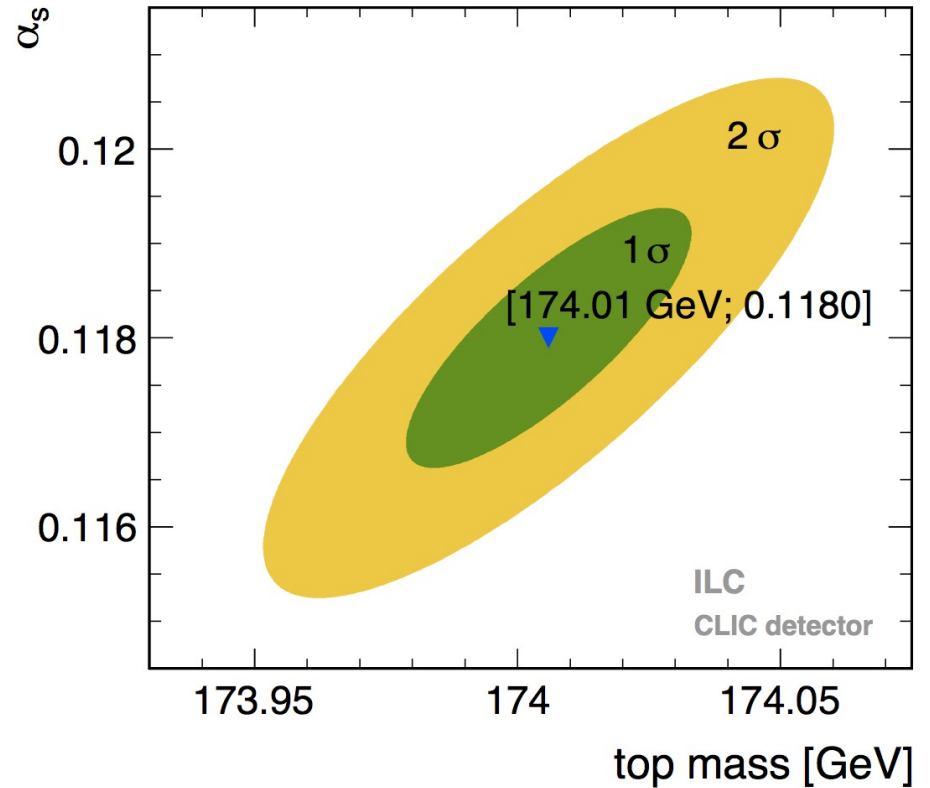
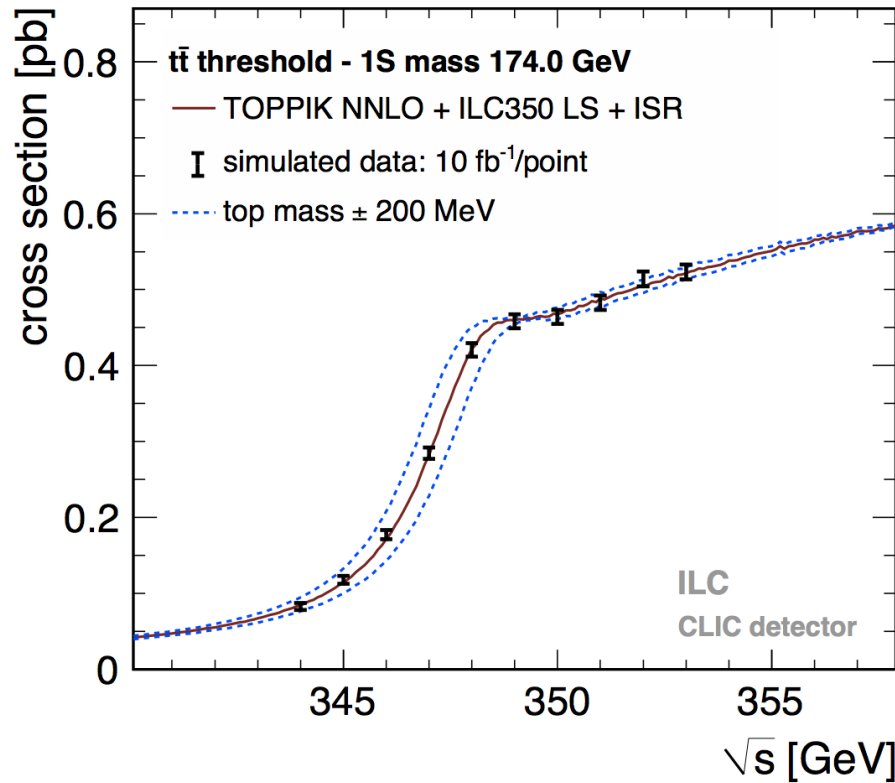
Correct resummation of
Non relativistic logs $\sim v$

Theoretical uncertainties at the 2-3% level
=> Threshold scan theoretically well understood

Top quark mass – Results of full simulation studies I

arXiv:1303.3758

Mass and α_s



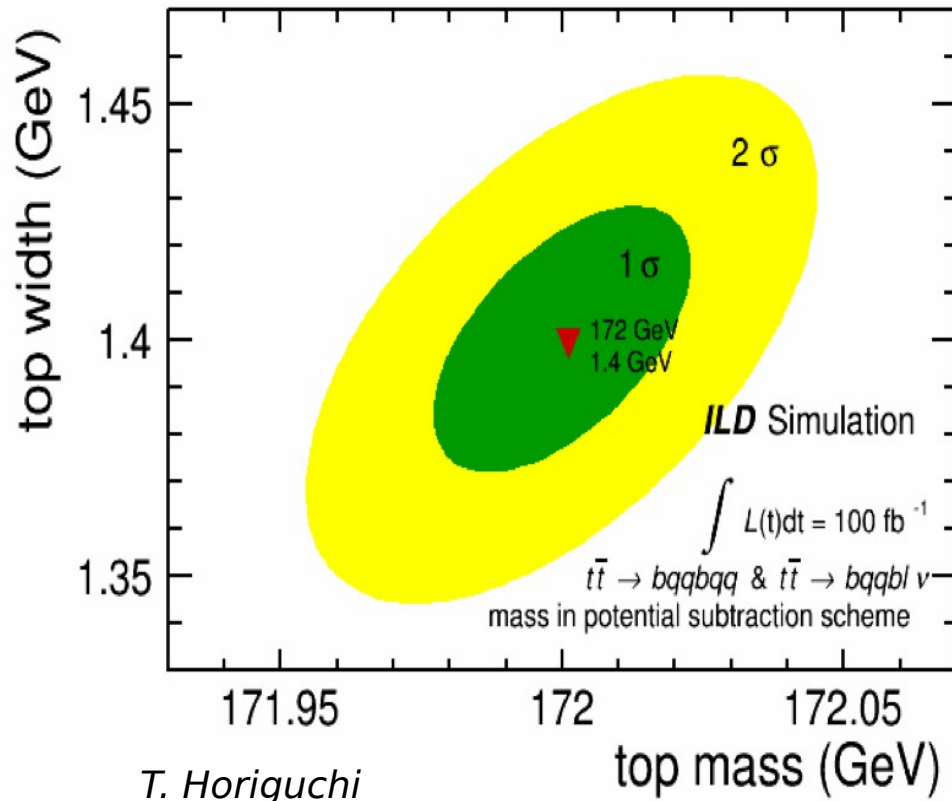
~100 MeV

1S top mass and α_s combined 2D fit

m_t stat. error	27 MeV
m_t theory syst. (1%/3%)	5 MeV / 9 MeV
α_s stat. error	0.0008
α_s theory syst. (1%/3%)	0.0007 / 0.0022

Top quark mass – Results of full simulation studies II

Mass and top width Γ_t (assuming α_s from Lattice QCD)



$m_t^{\text{PS}}(\text{GeV})$

$\Gamma_t(\text{GeV})$

172.001 ± 0.018

1.399 ± 0.026

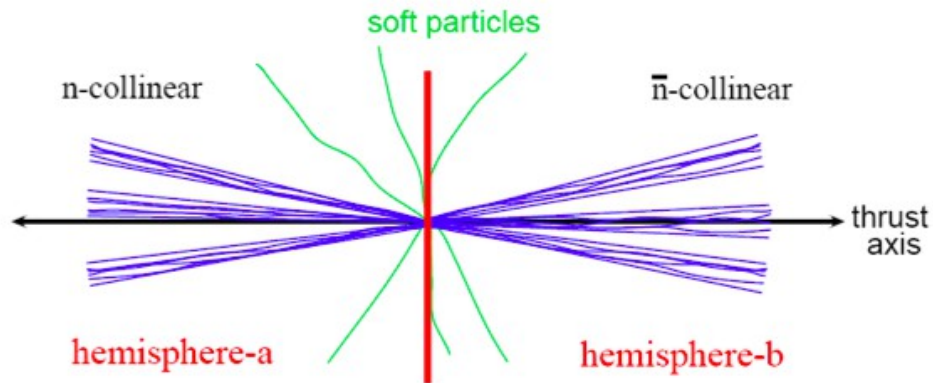
Stat. Error only!!!

Top width is sensitive to anomalous couplings (later)

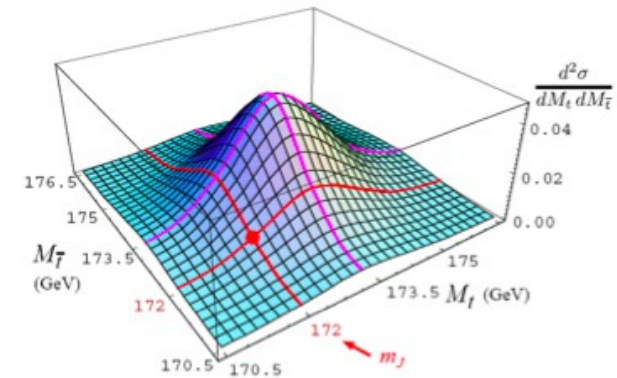
Reconstructed top jets (ILC)

Invariant mass distribution: (boosted tops)

Fleming, Mantry, Stewart, AH (2008)



- Hemisphere top jets
- Related to event-shapes



$$\left(\frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} d\ell^+ d\ell^- B_+\left(\hat{s}_t - \frac{Q\ell^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Q\ell^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(\ell^+, \ell^-, \mu)$$

JET

JET

SOFT

→ Differential strongly top mass-dependent observable.

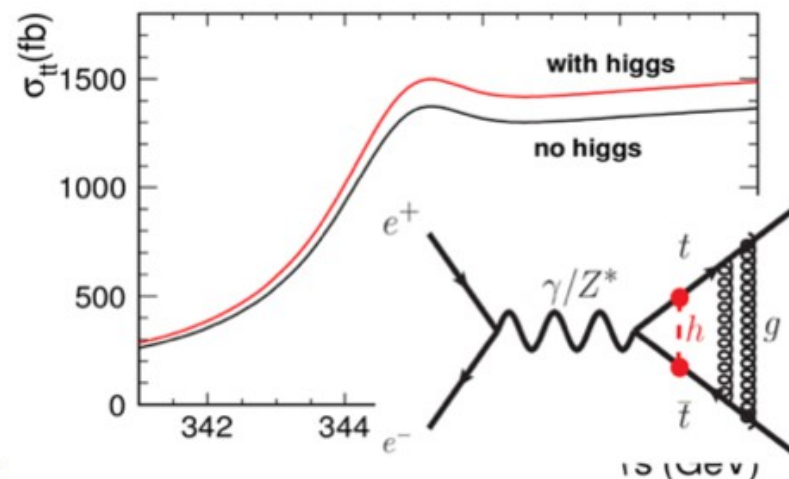
A. Hoang

Top Yukawa coupling at threshold

The cross section is enhanced about **9%** by exchanging the Higgs boson !!

$$\sigma_{tt} \propto |\mathcal{M}_{w/o \text{ higgs}} + y_t^2 \mathcal{M}_{w/ \text{ higgs}}|^2$$

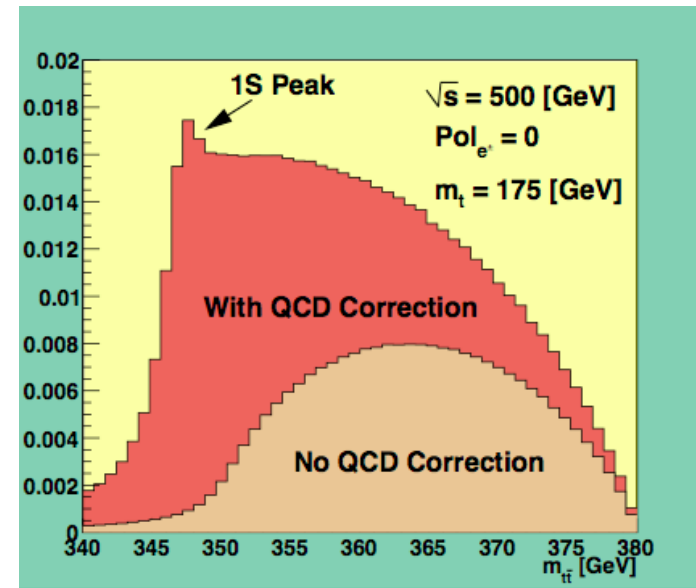
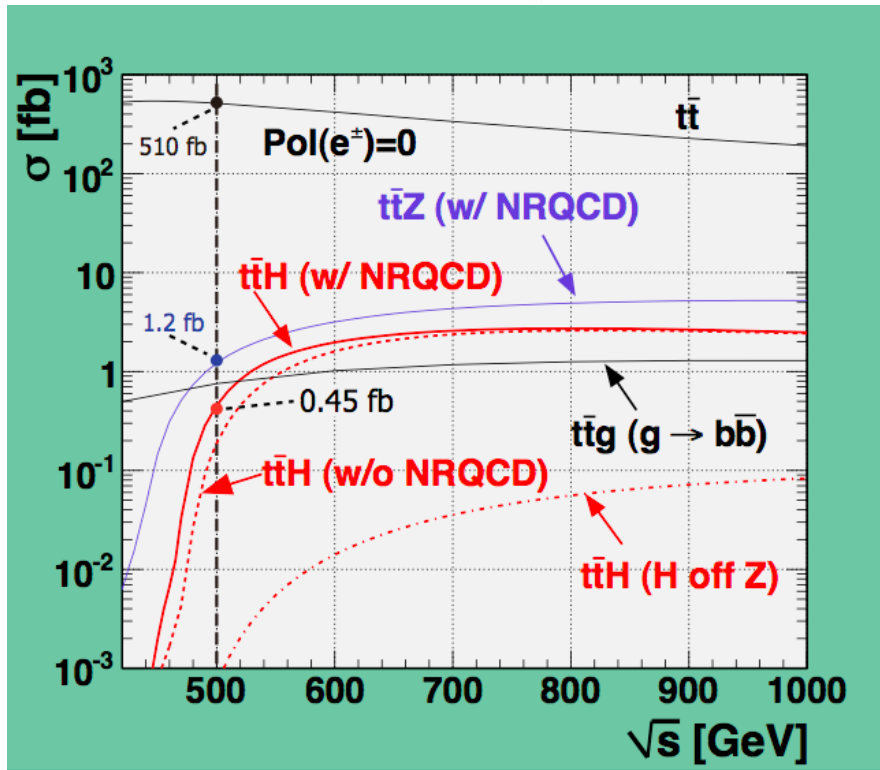
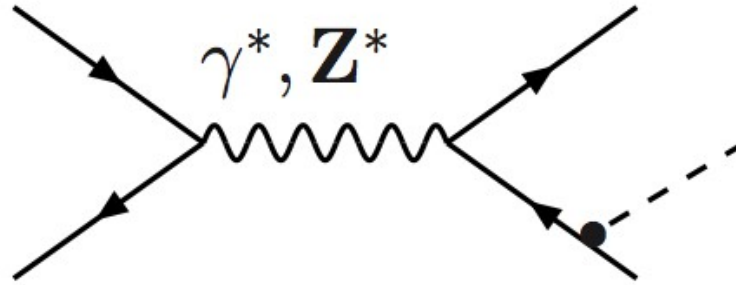
$$\frac{\delta y_t}{y_t} \sim \frac{109 \times \frac{1}{2} \times \frac{\delta \sigma}{\sigma}}{9}$$



Stat. Error (50 fb ⁻¹)	6-Jet (Left)	6-Jet (Right)	4-Jet (Left)	4-Jet (Right)	6-Jet + 4-Jet (Left)	6-Jet + 4-Jet (Right)	Combined (100 fb ⁻¹)
$\frac{\delta \sigma}{\sigma}$	1.2%	1.7%	1.3%	1.9%	0.9%	1.3%	4.3%
$\frac{\delta y_t}{y_t}$	7.2%	10.2%	7.8%	11%	5.3%	7.5%	

T. Horiguchi

Top Yukawa coupling above threshold



~ Factor 2 enhancement
From QCD bound states

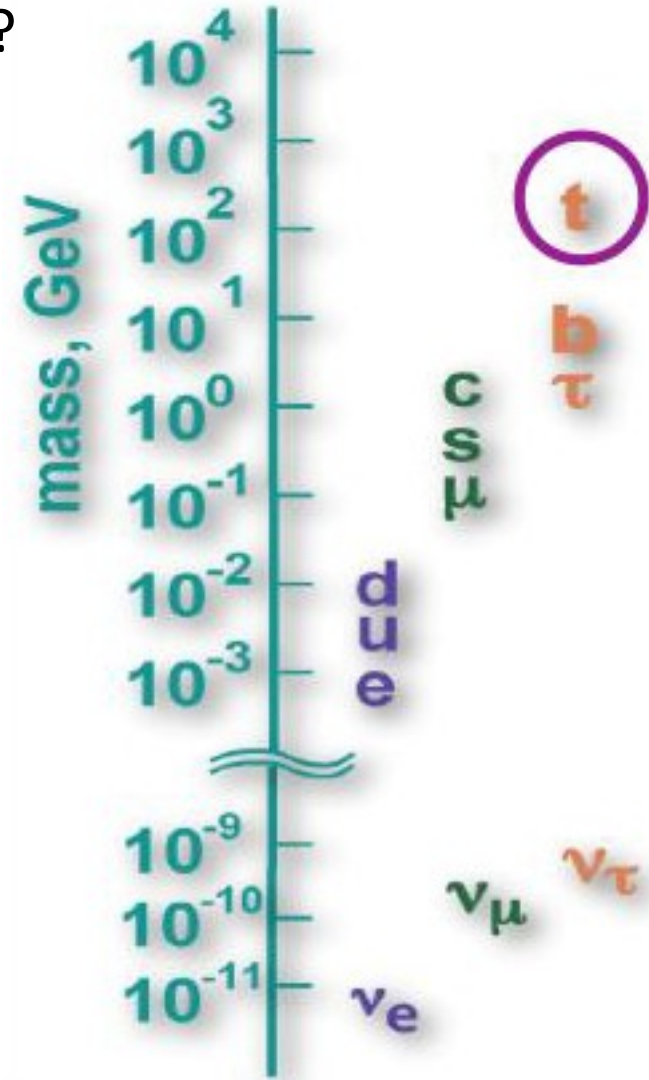
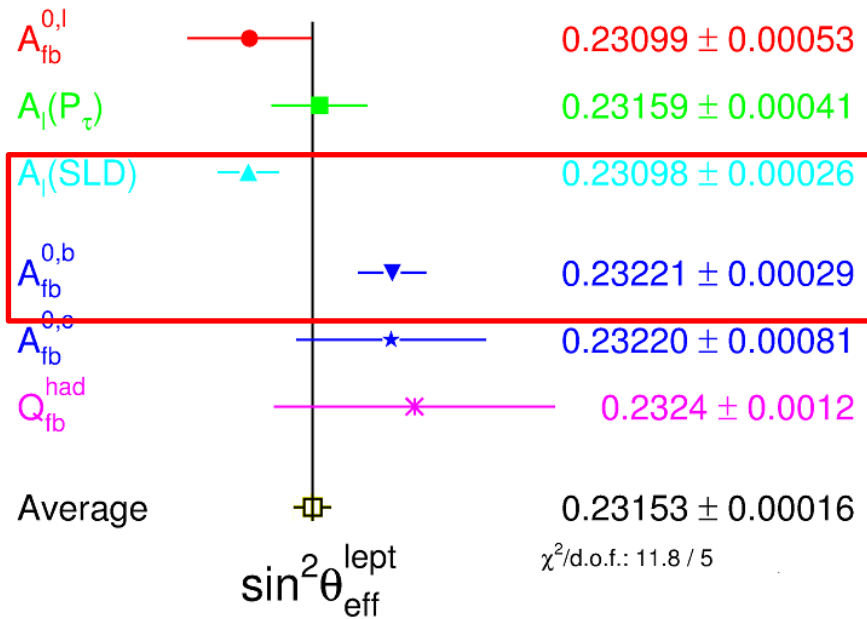
R. Horiguchi et al.
T. Tanabe, T. Price

$\Delta g_{ttH}/g_{ttH}$	500 GeV	500 GeV + 1 TeV
Canonical	14%	3.2%
LumiUP	7.8%	2.0%

← ILC TDR
← Technically possible

The top quark and flavor hierarchy

- Flavor hierarchy ? Role of 3rd generation ?



- A_{FB} anomaly at LEP for b quark

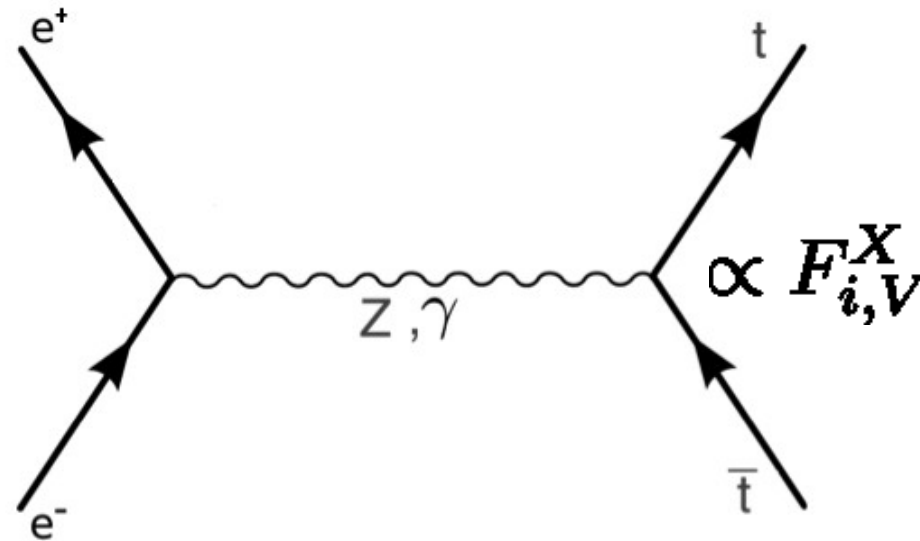
Tensions at Tevatron

- Heavy fermion effect

Strong motivation to study chiral structure of top vertex in high energy e^+e^- collisions

Why is it sooo heavy?

Testing the chiral structure of the Standard Model



$$\Gamma_{\mu}^{ttX}(k^2, q, \bar{q}) = -ie \left\{ \gamma_{\mu} (F_{1V}^X(k^2) + \gamma_5 F_{1A}^X(k^2)) + \frac{\sigma_{\mu\nu}}{2m_t} (q + \bar{q})^{\nu} (iF_{2V}^X(k^2) + \gamma_5 F_{2A}^X(k^2)) \right\}, \quad (2)$$

$$\mathcal{F}_{ij}^L = -F_{ij}^{\gamma} + \left(\frac{-\frac{1}{2} + s_w^2}{s_w c_w} \right) \left(\frac{s}{s - m_Z^2} \right) F_{ij}^Z$$

$$\mathcal{F}_{ij}^R = -F_{ij}^{\gamma} + \left(\frac{s_w^2}{s_w c_w} \right) \left(\frac{s}{s - m_Z^2} \right) F_{ij}^Z,$$

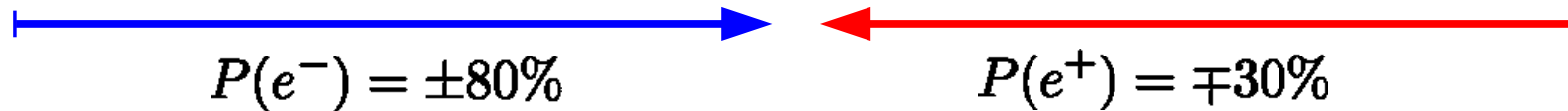
Pure γ or pure Z^0 : $\sigma \sim (F_i)^2 \Rightarrow$ No sensitivity to sign of Form Factors

Z^0/γ interference : $\sigma \sim (F_i) \Rightarrow$ Sensitivity to sign of Form Factors

Disentangling

At ILC **no** separate access to ttZ or $t\bar{t}\gamma$ vertex, but ...

ILC 'provides' two beam polarisations



There exist a number of observables sensitive to chiral structure, e.g.

$$\sigma_I \quad A_{FB,I}^t = \frac{N(\cos\theta > 0) - N(\cos\theta < 0)}{N(\cos\theta > 0) + N(\cos\theta < 0)} \quad (F_R)_I = \frac{(\sigma_{t_R})_I}{\sigma_I}$$

x-section

Forward backward asymmetry

Fraction of right handed top quarks

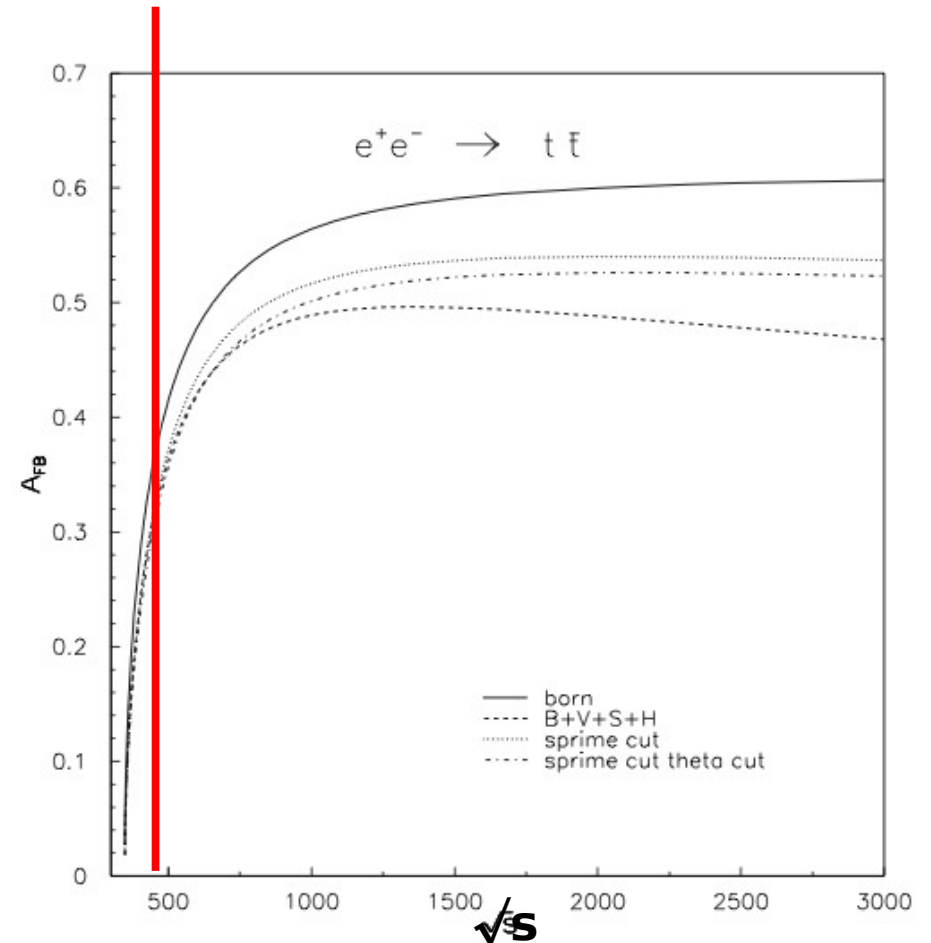
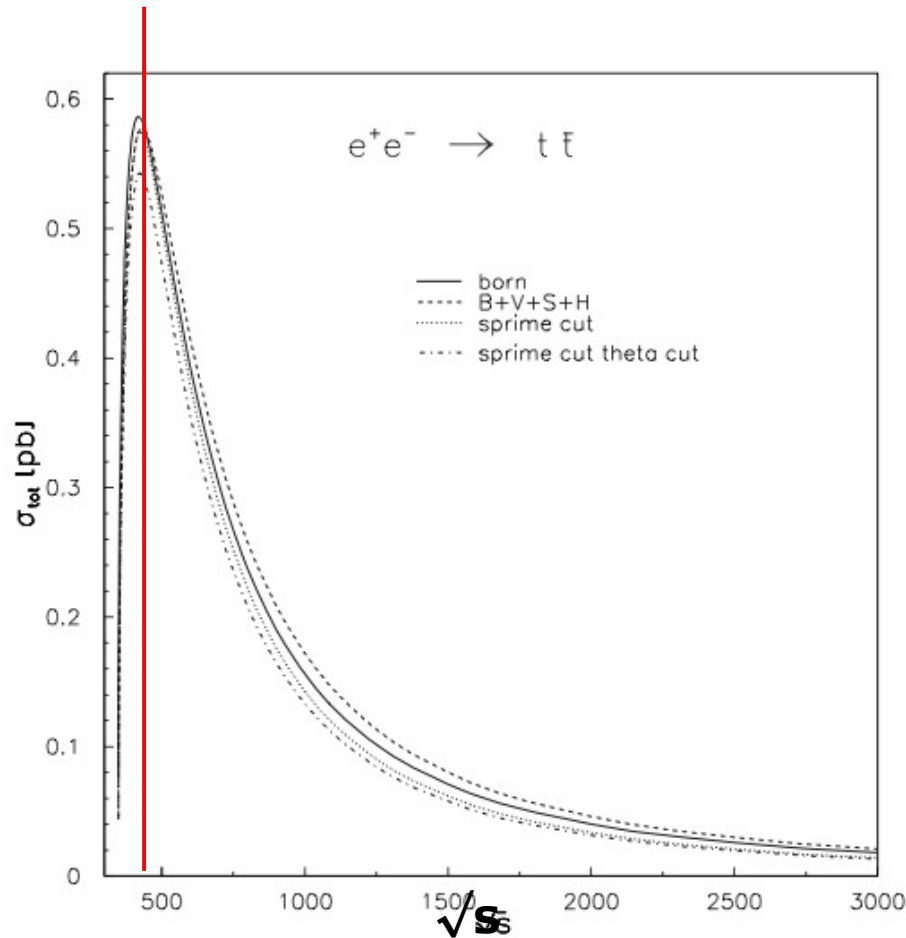


Extraction of six (five) unknowns

$$F_{1V}^\gamma, F_{1V}^Z, F_{1A}^\gamma = 0, F_{1A}^Z$$

$$F_{2V}^\gamma, F_{2V}^Z$$

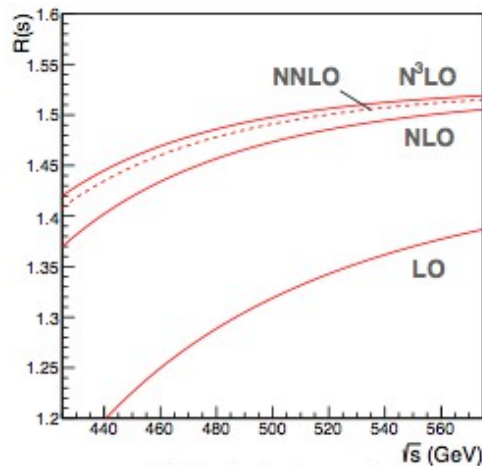
Measuring at 500 GeV



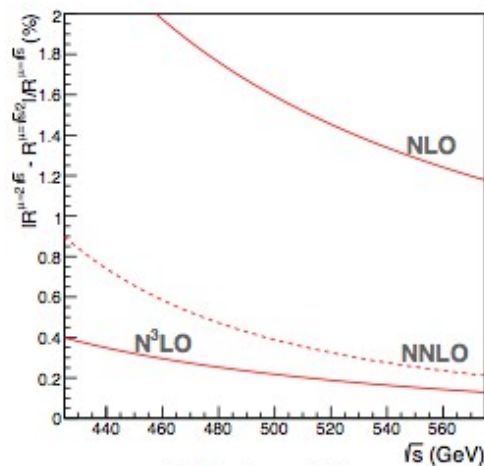
- Cross section close to maximum, A_{FB} well developed
- Other remarks: Need some velocity to get sensitive to chiral observables (see backup slides)

Theoretical uncertainties

*QCD corrections are known up to N³LO



(a) Perturbation series

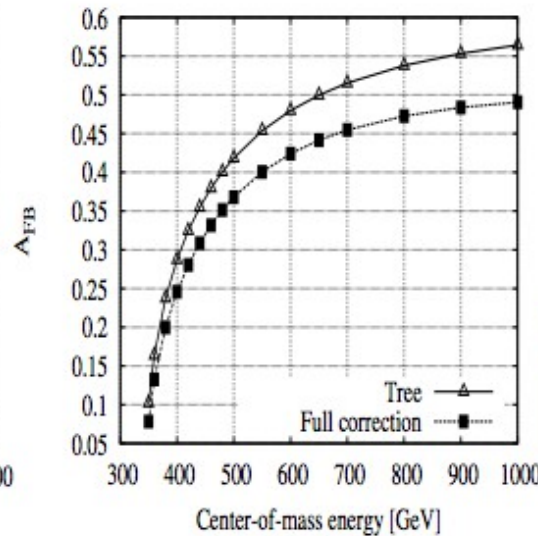
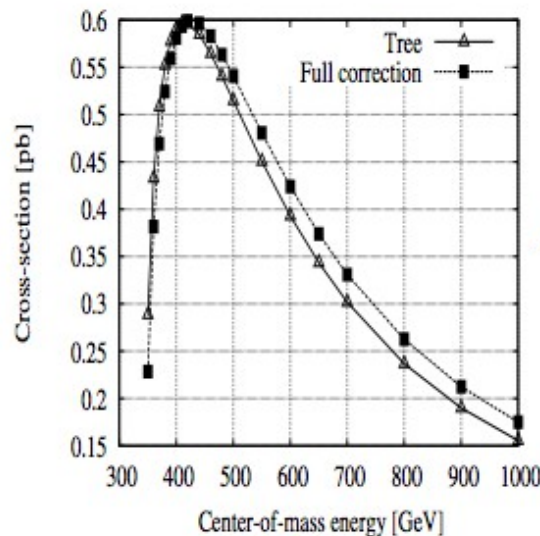


(b) Scale variations

QCD correction (N³LO) is
at the per mil level

Kiyo, Maier, Maierhofer, Marquard, NCP B823 ('09)
Bernreuther, Bonciani, Gehrmann, Heinesch,
Leineweber, NPB750 ('06)
Hoang, Mateu, Zebarjad, NPB813 ('09)

*Electroweak corrections are known at one-loop level



EW correction at one-loop is
~5% for cross section
~10% for A_{FB}

Fleischer, Leike, Riemann, Werthenbach, EJPC31 ('03)
Kheim, Fujimoto, Ishikawa, Kaneko, Kato,
arXiv:1211.1112

Elements of top quark reconstruction

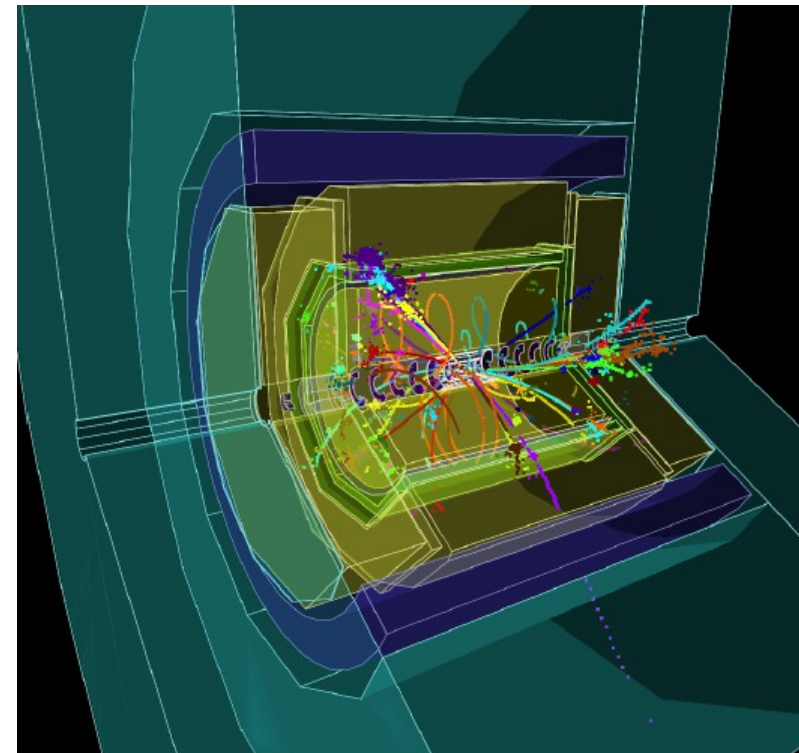
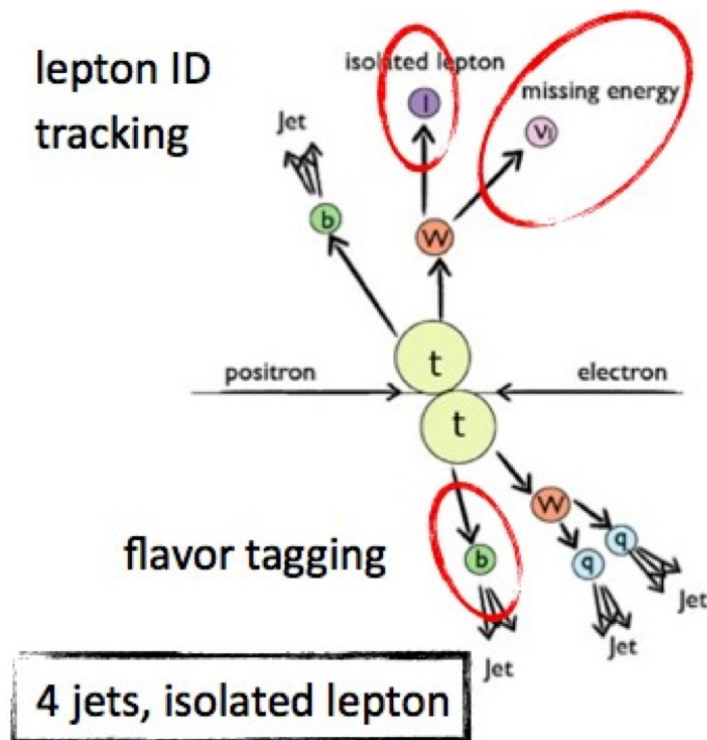
Three different final states:

1) Fully hadronic (46.2%) → 6 jets

2) Semi leptonic (43.5%) → 4 jets + 1 charged lepton and a neutrino

3) Fully leptonic (10.3%) → 2 jets + 4 leptons

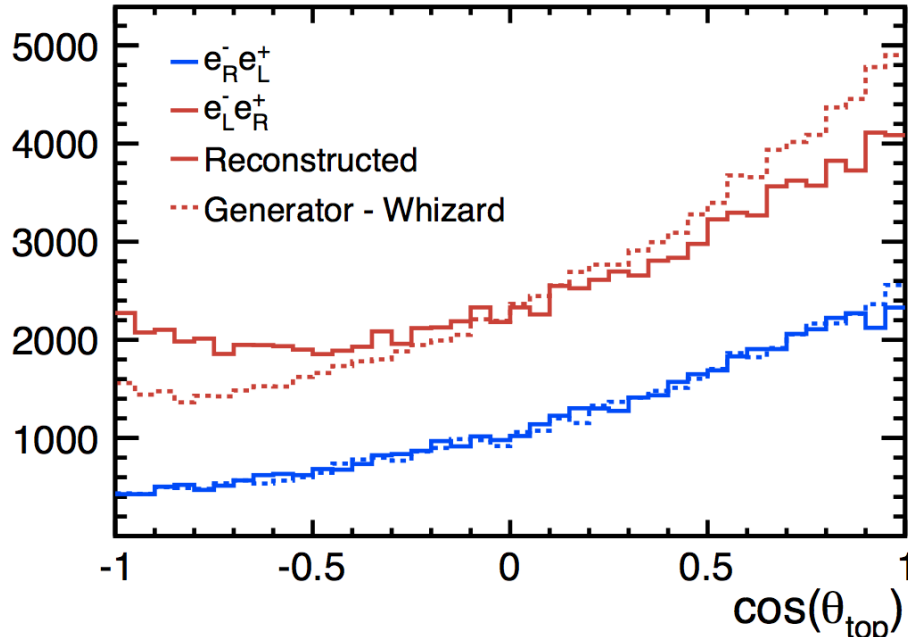
$$t\bar{t} \rightarrow (bW)(bW) \rightarrow (bqq')(bl\nu)$$



Results in the following mainly based on semi-leptonic decay

Do however integrate results from fully hadronic study

Reconstruction of top quark production angle



← Ambiguities in case of **left** handed electron beams
Due to V-A structure at ttX vertex

← Precise reconstruction of θ_{top}
in case of **right** handed electron beams

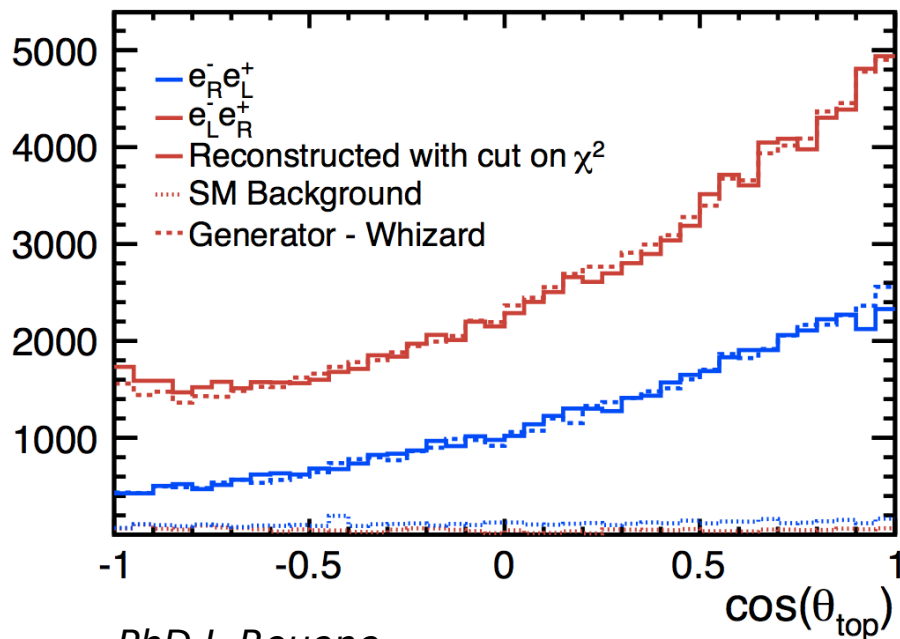
Remedy to address ambiguities:
Select cleanly reconstructed
events by χ^2 analysis
or

Reconstruction of b quark charge

Precise reconstruction for both beam polarisations

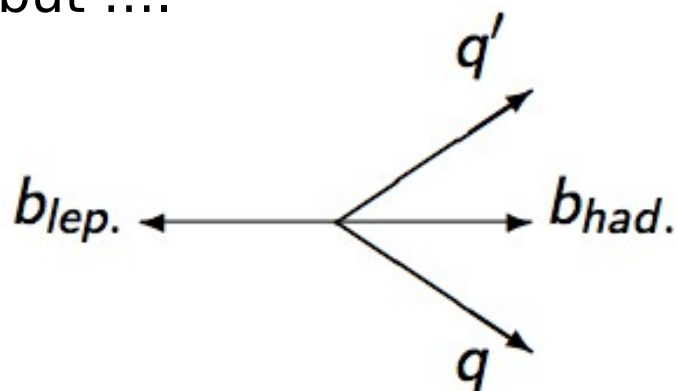
- Efficiency Penalty for e_L
- ϵ_{tot} : $e_R \sim 50\%$, $e_L \sim 30\%$

Precision on $A_{FB} \sim 2\%$



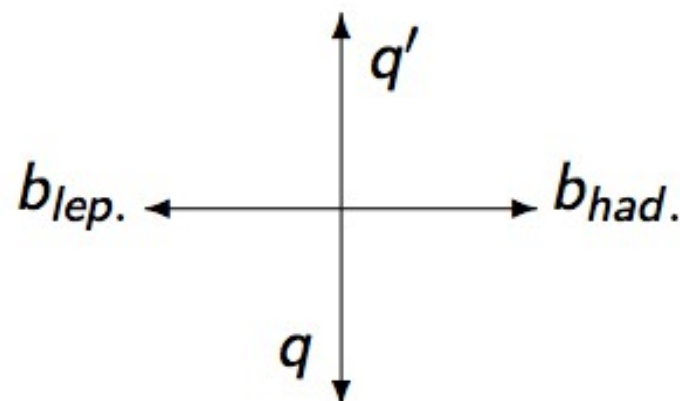
Experimental challenge b-charge reconstruction - Motivation

- To measure AFB in fully hadronic decays there is no choice
- In semi-leptonic decays there is the charged lepton but



Right handed electron beam:

- Hard W in flight direction of Top and soft b's
- Flight direction of t from flight direction of W



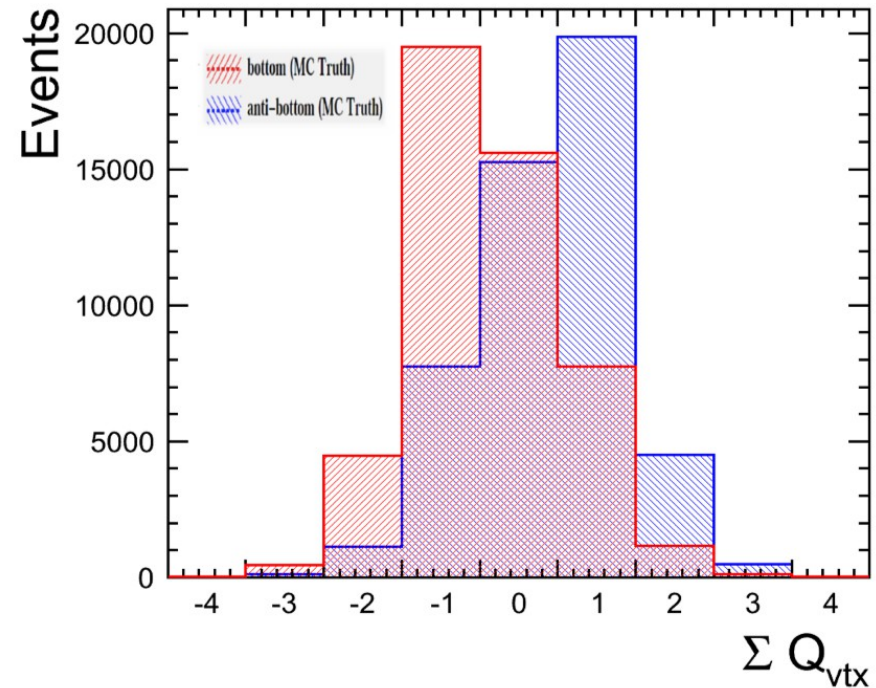
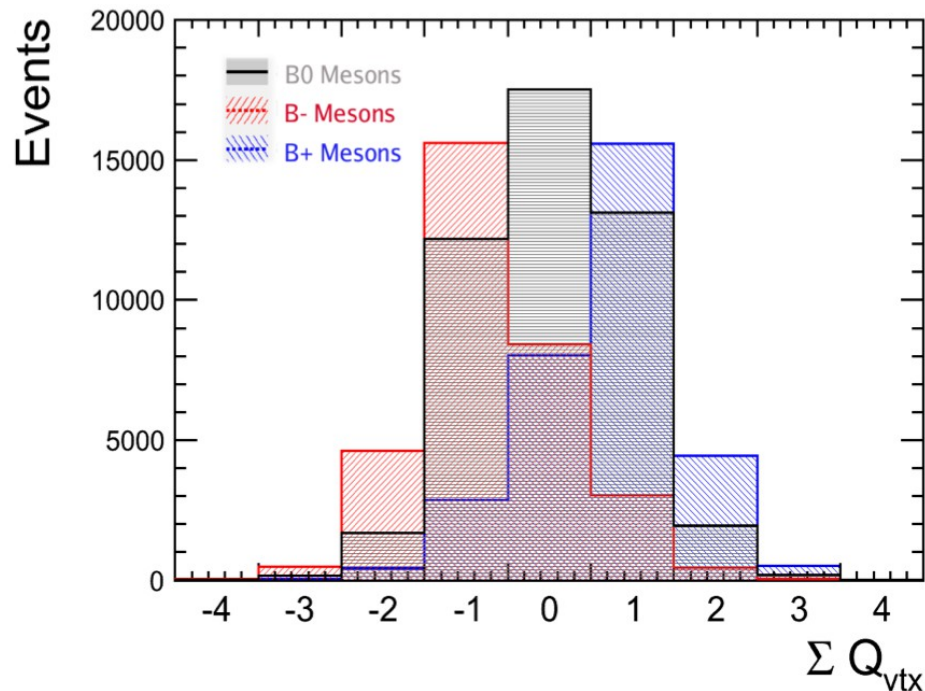
Left handed electron beam:

- Hard b in flight direction of Top and soft W's
 - Flight direction of t from flight direction of b
- => Wrong association ↔ top flip

Measurement of b-charge to resolve ambiguities

Measurement of b quark charge

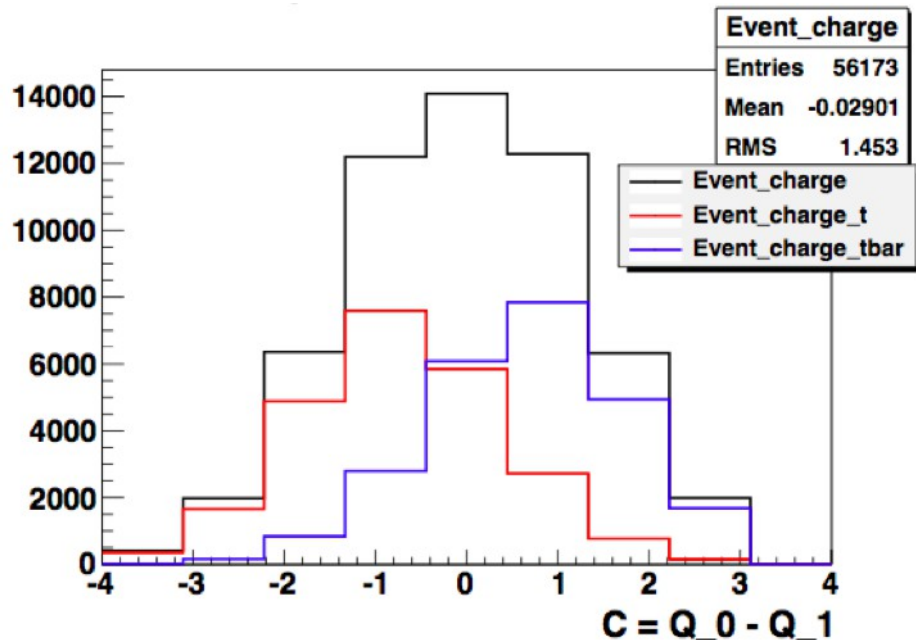
(N.B. At example of fully hadronic analysis, PhD M.S. Amjad)



- Vertex charge measurement mandatory for fully hadronic top decays
- LC vertex and tracking system allows for determination of b-meson (b-quark) charge
- B-quark charge measured correctly in about 60% of the cases
- Can be increased to 'arbitrary' purity on the expense of smaller statistics
- LCFIPlus package not yet optimised for vertex charge measurement

Optimisation of b-quark charge is major topic for future studies

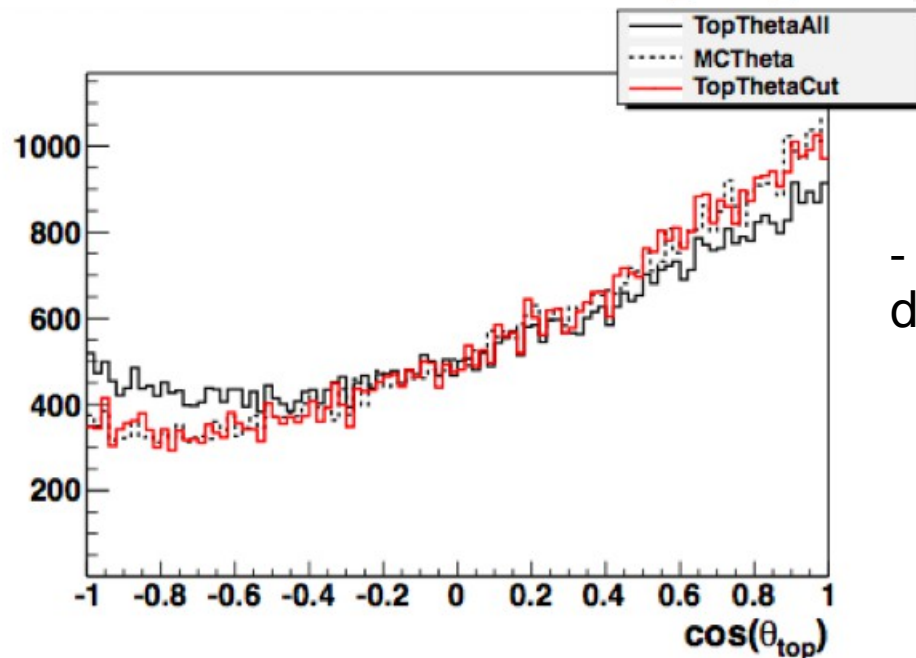
Top polar angle using b charge (SL Analysis)



Event charge $C = b1 - b2$

In SL can compare charge C with lepton charge to select clean sample

Use only events with correct C or $C=0$
(plus another cut on the Lorentz Factor)



- Clean reconstruction of top quark direction

$\epsilon \sim 30\%$

Will improve with improving charge reconstruction

Measurement of top quark polarisation

Measure angle of decay lepton in top quark rest frame

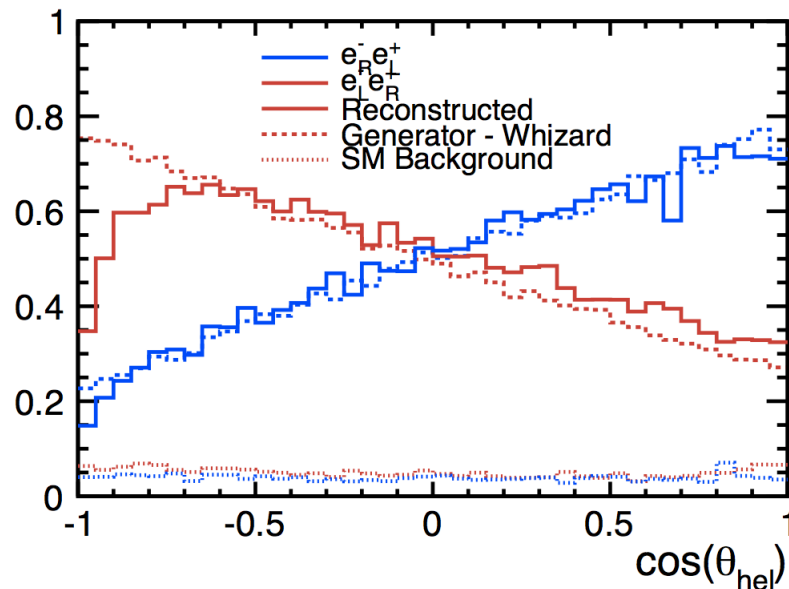
Lorentz transformation benefits from well known initial state

(N.B. : Proposal for hadron colliders applied to lepton colliders)

Differential decay rate

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\theta_\ell} = \frac{1 + \lambda_t \cos\theta_\ell}{2} \quad \text{with } \lambda_t = 1 \text{ for } t_R \text{ and } \lambda_t = -1 \text{ for } t_L$$

Slope measures fraction of $t_{R,L}$ in sample



- Measurement of decay lepton almost 'trivial' at LC
- High reconstruction efficiency for leptons
- Reconstructed slope coincides with generated slope

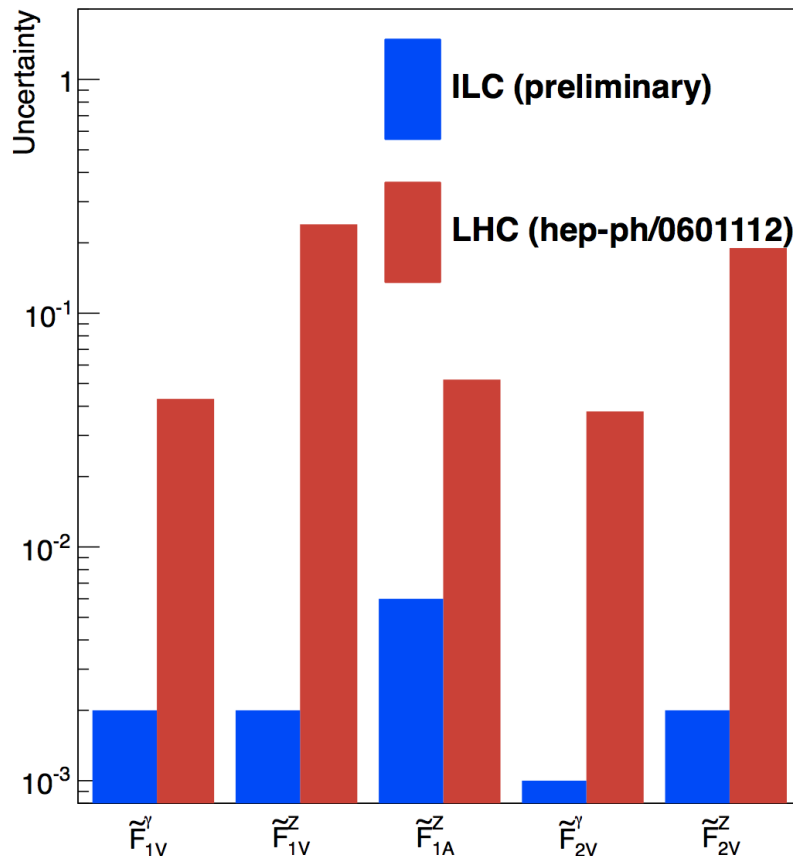
Slope λ_t can be measured to an accuracy of about 3-4%

Results of full simulation study for DBD at $\sqrt{s} = 500$ GeV

ArXiv: 1307.8102

Precision: cross section $\sim 0.5\%$, Precision $A_{FB} \sim 2\%$, Precision $\lambda_t \sim 3-4\%$

Accuracy on CP conserving couplings



- ILC might be up to two orders of magnitude more precise than LHC ($\sqrt{s} = 14$ TeV, 300 fb^{-1})

Disentangling of couplings for ILC
One variable at a time For LHC

- However LHC projections from 8 years old study

- Strong encouragement to update these numbers!

First step is Phys. Rev. Lett. 110 (2013) 172002 by CMS (later)

- Potential for CP violating couplings at ILC under study

ILC will be indeed high precision machine for electroweak top couplings

Discussion of potential systematic uncertainties

Experimental

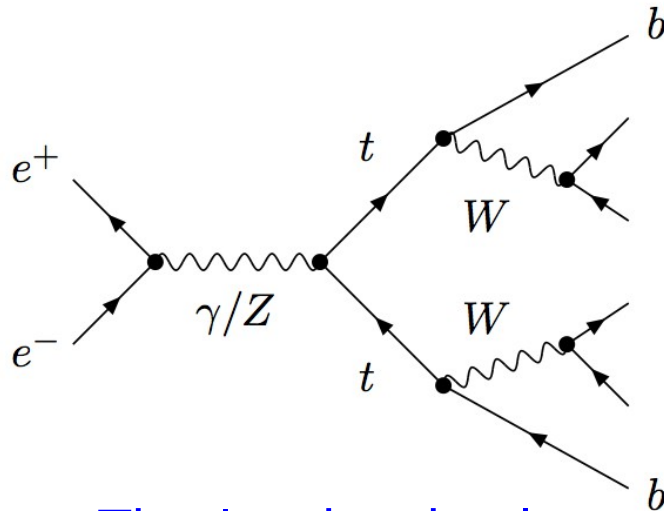
- **Luminosity**: Critical for cross section measurements
Expected precision 0.1% @ 500 GeV
- **Beam polarisation**: Critical for asymmetry measurements
Expected to be known to 0.1% for e- beam
and 0.35% for e+ beam
- **Migrations/Ambiguities**: Critical for AFB:
Need further studies but expect to control them better than the theoretical error
- **Jet energy scale**: Critical for top mass determination
Systematic study CLIC states systematic error \sim statistical error
- **Other effects**: B-tagging, passive material etc.
LEP claims 0.2% error on R_b \rightarrow guiding line for LC

Theory:

- see above and in the following

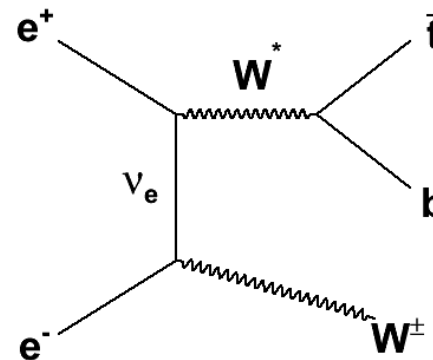
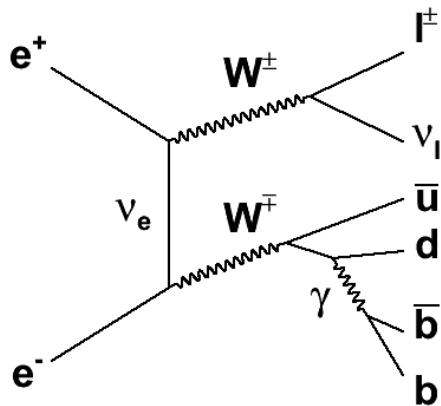
Closer look at $t\bar{t}$ production

That's what we are interested in



Top pair production is effectively $ee \rightarrow 6f$ process

That's what is also contributing to final state!

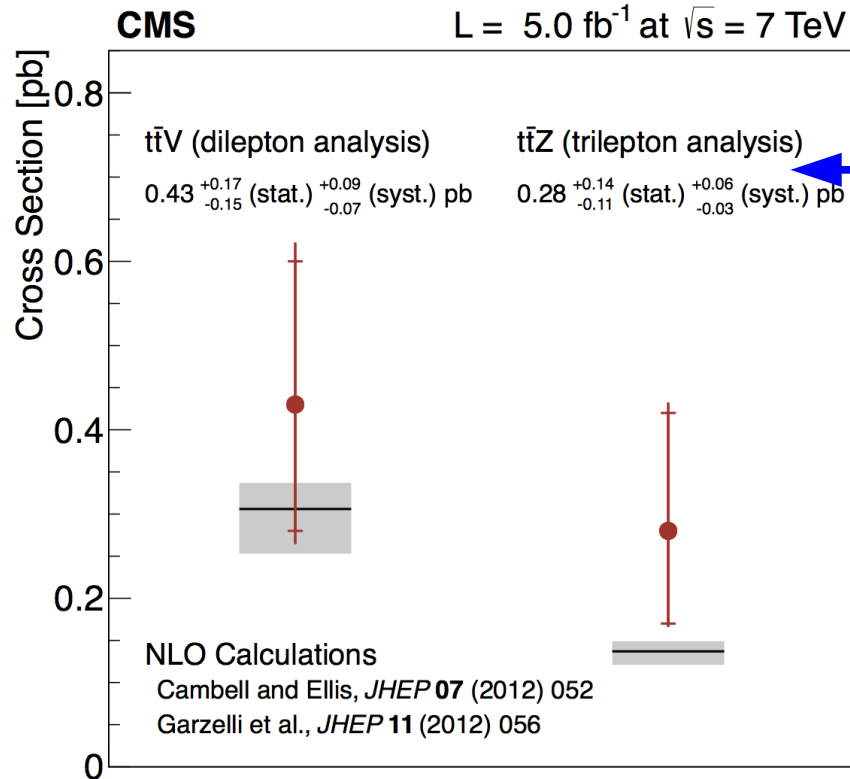
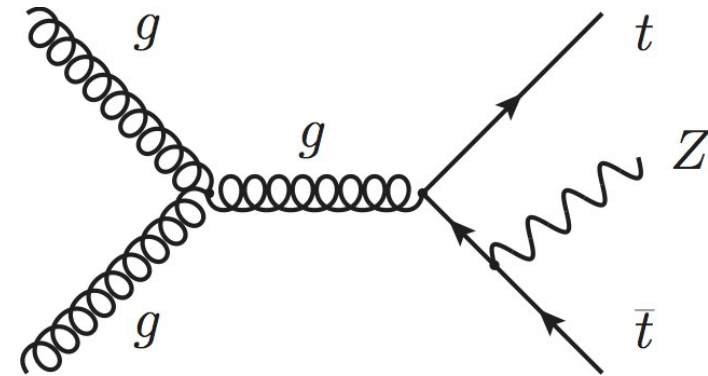
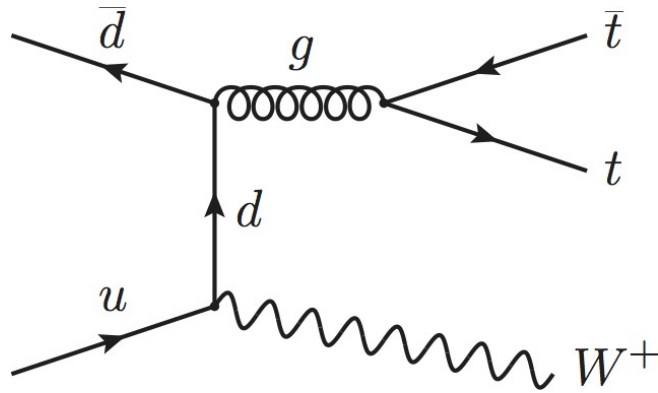


+ s-channel, t-channel only relevant for eL

- Can one really speak about a $t\bar{t}$ cross section?
- If only $6f$ is relevant: What are relations to $t\bar{t}X$ couplings?
- What selection cuts are (theoretically) save?

The race is open !

Recent result on $t\bar{t}V$ by CMS

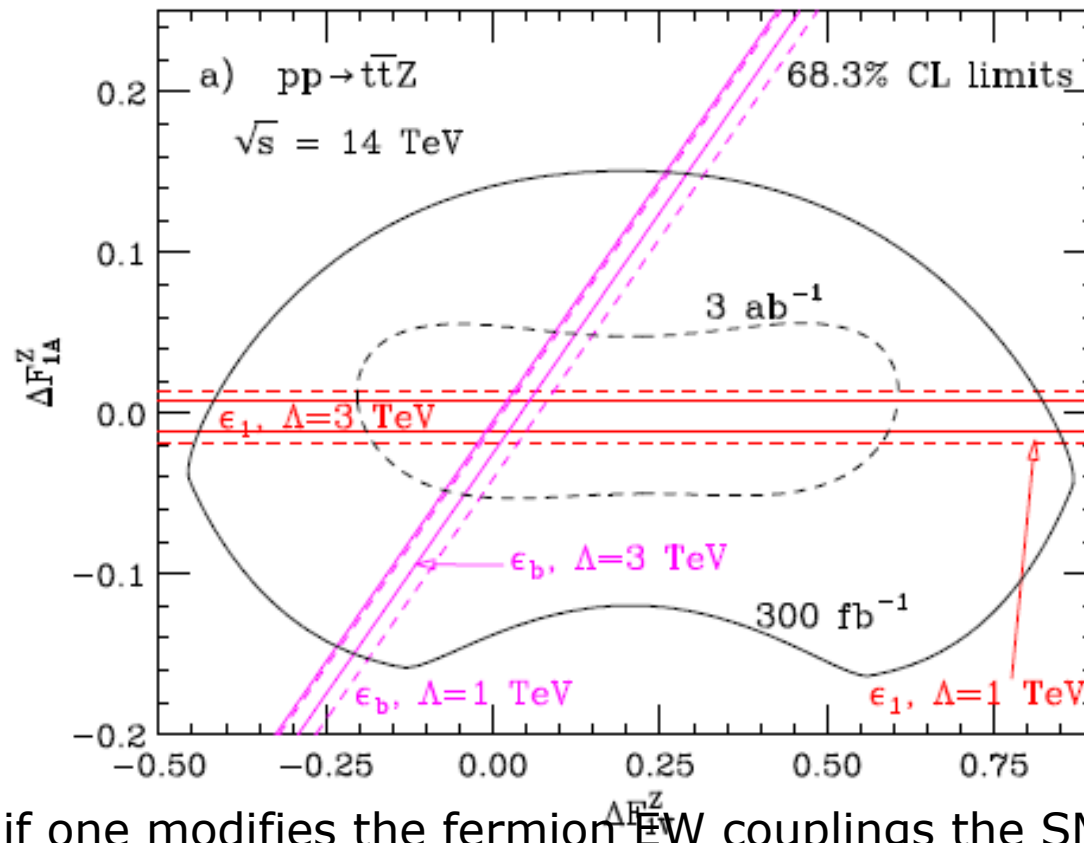


$$\sigma(t\bar{t}Z) = 0.28^{+0.14}_{-0.11} \text{ (stat.) } ^{+0.06}_{-0.03} \text{ (syst.) pb}$$

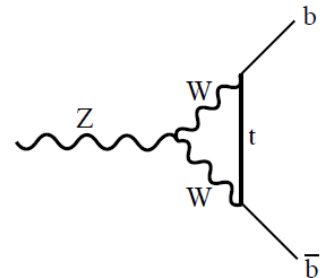
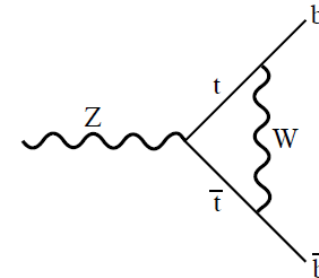
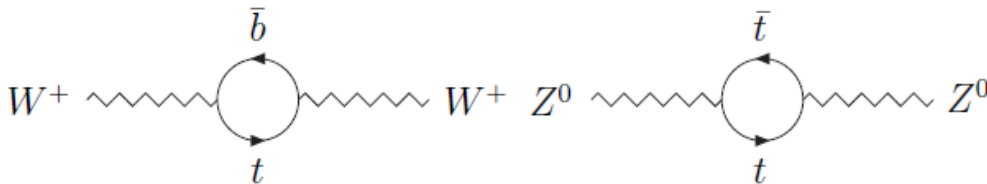
- Clearly, promising result
- How will it evolve with higher Luminosity?
- Revision of 'old' estimations of precisions are needed!

May expect: $\frac{\delta\sigma_{t\bar{t}Z}}{\sigma_{t\bar{t}Z}} \sim 10\%$

Recap: LEP/SLD Constraints



- Recall that if one modifies the fermion EW couplings the SM loops becomes UV divergent and this requires introducing a **cutoff $\Lambda \sim \text{TeV}$** to compute these contributions
- Given this cutoff the top EW **couplings anomalies** are limited by LEP/SLD measurements



Constraints due to Gauge Invariance

- Gauge invariance relates $ZtLtL$ to $WtLbL$ and $ZbLbL$

$$\kappa_{bL}^{NC} + \kappa_{tL}^{NC} \sim \kappa_{tL}^{NC} = 2\kappa_{tLbL}^{CC}$$

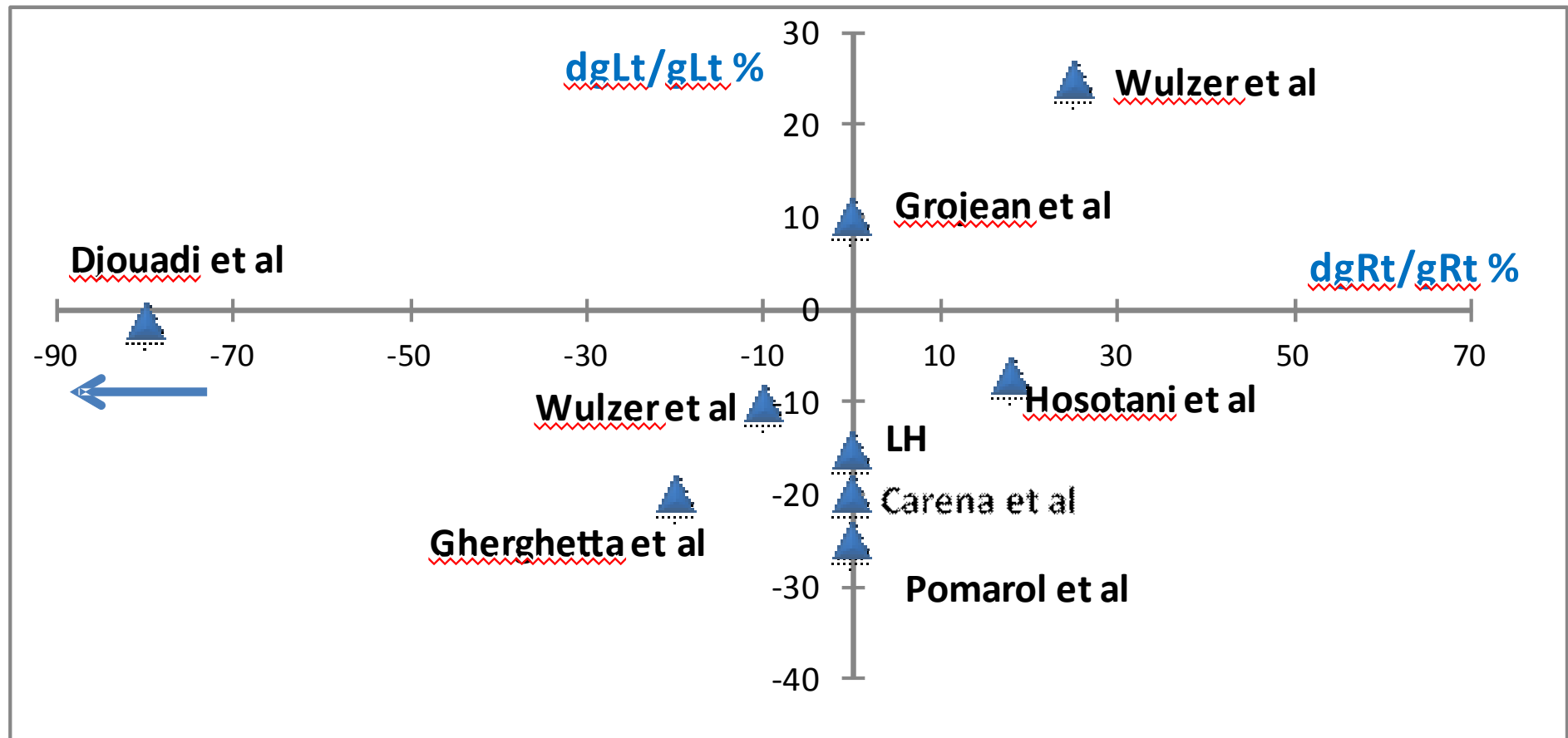
- From LEP1 we know that $ZbLbL$ has no anomaly meaning that

$$\frac{\delta WtLbL}{WtLbL} = 0.72 \frac{\delta ZtLtL}{ZtLtL}$$

- $d\epsilon_1$ and $d\epsilon_b$ only depend on neutral couplings $ZbLbL$ and $ZbRbR$
- Loop contributions therefore fully constrain $ZtLtL$ and $ZtRtR$ and the only freedom left comes from BSM compensating contributions to ϵ_1 and ϵ_b

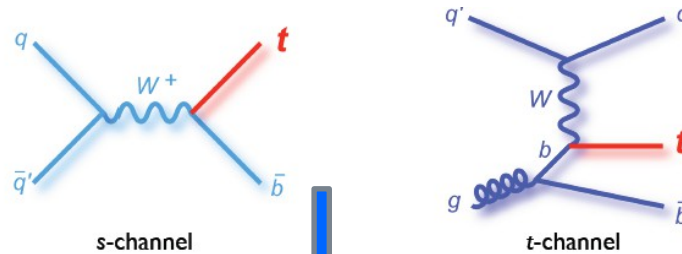
Discussion of precisions (IFIC/LAL [F. Richard])

Models realising Top/Higgs compositeness and/or extra dimensions



Variety of models predicting modifications to t_L and t_R due to couplings to new strong sector

Sensitivities and constraints



Model	dtR/tR %	dtL/tL %	dtLbL/tLbL %	dεb/εb	dε1/ε1	dsZtt/sZtt %
Carena	0	-20	-14	0.8	1.1	-30
Djouadi	-330	0	0	-1.4	1.1	70
Gherghetta	-20	-20	-14	0.7	2.1	-36
Grojean	0	10	7	-0.4	-1.0	17
Hosotani	18	-7	-5	-0.4	-0.8	-5
Little Higgs	0	-15	-10	0.6	1.0	-23
Pomarol	0	-25	-17	1.0	1.2	-37
Wulzer 1	25	25	17	-1.1	5.8	56
Wulzer 2	-10	-10	-7	0.4	1.3	-20

LEP constraints: $|\delta F_{1A}^Z| < 0.2$, $Q_{tL} \rightarrow Q_{tL}^{SM}$

=> LHC may see deviations but cannot distinguish Models
 => ILC will be able to distinguish at several sigma level

Summary and outlook

- A LC is **the** machine for precision top physics

First machine to produce top pairs in electroweak production!!!
Essential pillar of LC physics program

- Rich program of top quark physics with 'exciting' prospects

- Precision on top mass ~ 50 MeV =>

- 'Final word' on vacuum stability of the universe

- Test of models with extra dimensions and/or compositeness

- Top elw. Measurements are complementary to Higgs coupling Measurements

- Exploitation of potential requires huge experimental and theoretical efforts

- Theoretical uncertainty on top mass \gg Experimental uncertainty

- Uncertainty of theoretical prediction of AFB

- NNLO would be 10 years of work !!!

- Measurement of b quark charge still in infancy, may need revision of algorithms and detector

- In general experimentalists will have to make sure that systematic errors can be kept small

Backup

The solid pillars of the LC physics program

Top quark



Discovered 1995 at Tevatron

LHC and ILC are/would be
Top factories

W Boson



Discovered 1979 at SPS
Mass precisely at Tevatron
LHC and ILC are/would be
W factories

Higgs Boson



Discovered 2012 at LHC
ILC are/would be
Higgs factories
See talk by Klaus

Equations for cross section, A_{FB} and F_R

$$\sigma_I = 2\mathcal{A}N_c\beta \left[(1 + 0.5\gamma^{-2})(\mathcal{F}_{1V}^I)^2 + (\mathcal{F}_{1A}^{I'})^2 + 3\mathcal{F}_{1V}^I\mathcal{F}_{2V}^I \right],$$

$$(A_{FB}^t)_I = \frac{-3\mathcal{F}_{1A}^{I'}(\mathcal{F}_{1V}^I + \mathcal{F}_{2V}^I)}{2 \left[(1 + 0.5\gamma^{-2})(\mathcal{F}_{1V}^I)^2 + (\mathcal{F}_{1A}^{I'})^2 + 3\mathcal{F}_{1V}^I\mathcal{F}_{2V}^I \right]},$$

$$(F_R)_I = \frac{(\mathcal{F}_{1V}^I)^2(1 + 0.5\gamma^{-2}) + (\mathcal{F}_{1A}^{I'})^2 + 2\mathcal{F}_{1V}^I\mathcal{F}_{1A}^{I'} + \mathcal{F}_{2V}^I(3\mathcal{F}_{1V}^I + 2\mathcal{F}_{1A}^{I'}) - \beta\mathcal{F}_{1V}^I\Re(\mathcal{F}_{2A}^I)}{2 \left[(1 + 0.5\gamma^{-2})(\mathcal{F}_{1V}^I)^2 + (\mathcal{F}_{1A}^{I'})^2 + 3\mathcal{F}_{1V}^I\mathcal{F}_{2V}^I \right]}.$$