Cosmological Inflation and primordial non-Gaussianities

Sébastien Renaux-Petel

LPTHE - ILP

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Outline

1. Description of inflation

2. Beyond the simplest models

3. Primordial non-Gaussianities

4. Quasi-single-field inflation

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2. Beyond the simplest models

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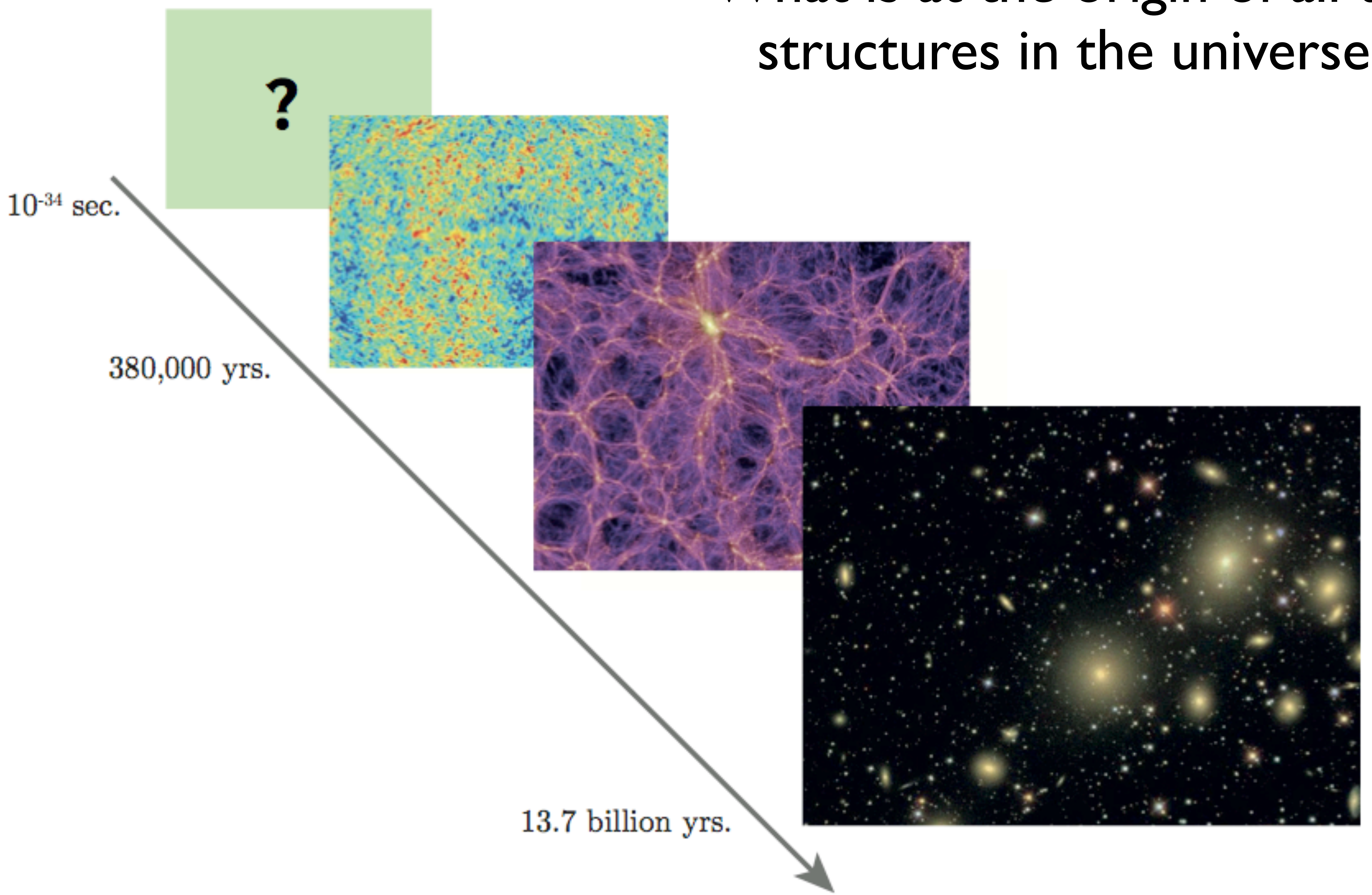
What is at the origin of all the structures in the universe?

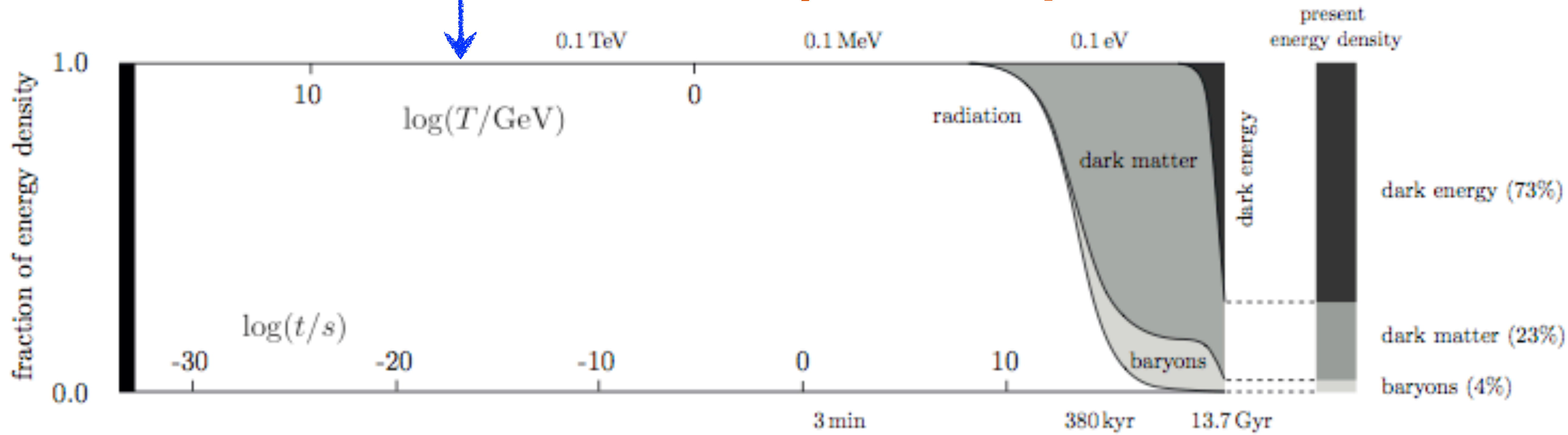


Cosmic history

LHC

Big Bang Nucleosynthesis

Structure formation



Inflation

Cosmic Microwave Background

3 main puzzles: Dark Matter, Dark Energy, Inflation:

a period of accelerated expansion before the radiation era that solves the problems of the Hot Big Bang model

The horizon problem of the Hot Big-Bang model ...

scale factor → a

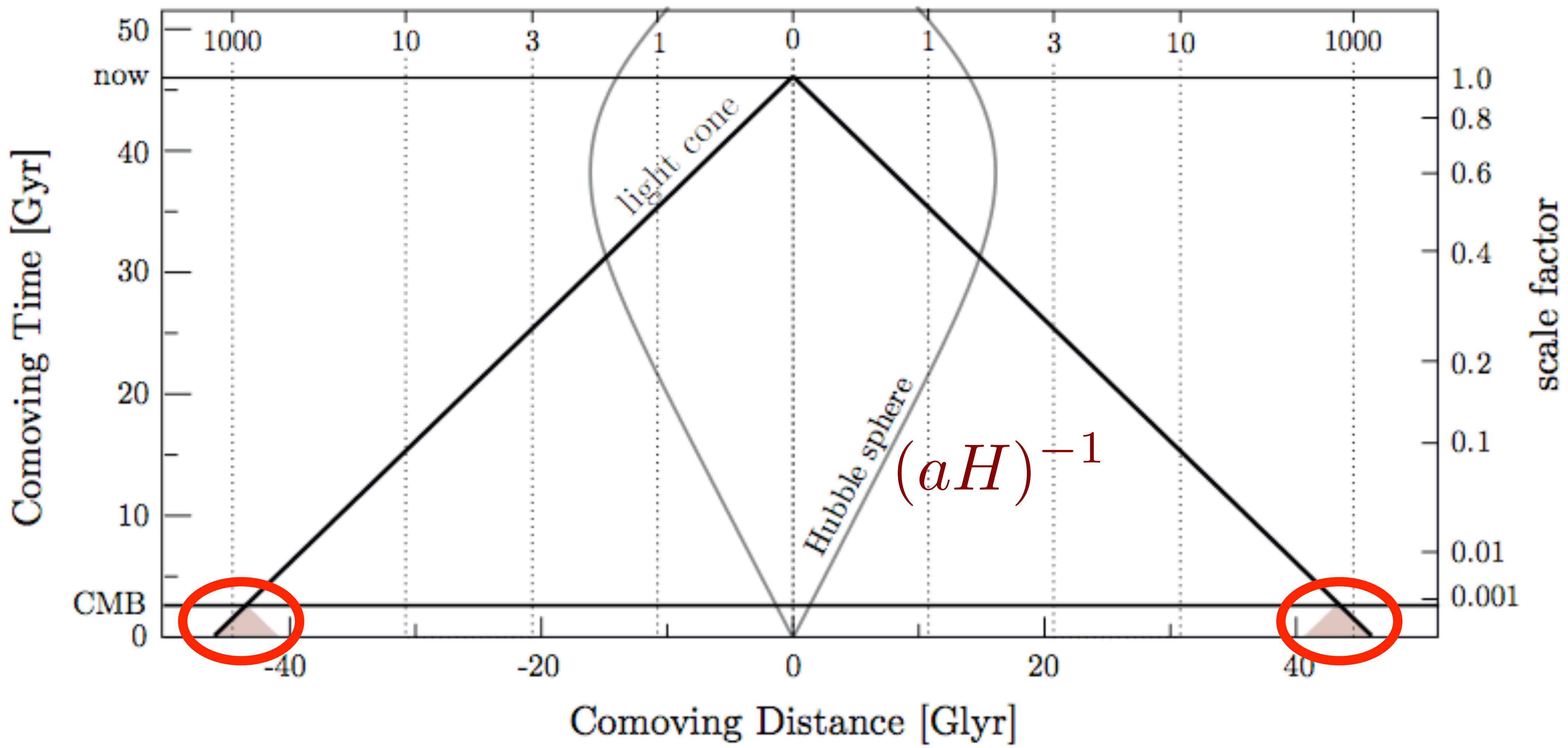
comoving (conformal) time → τ

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^{\,2} = a(\tau)^2\left(-d\tau^2 + d\vec{x}^{\,2}\right)$$

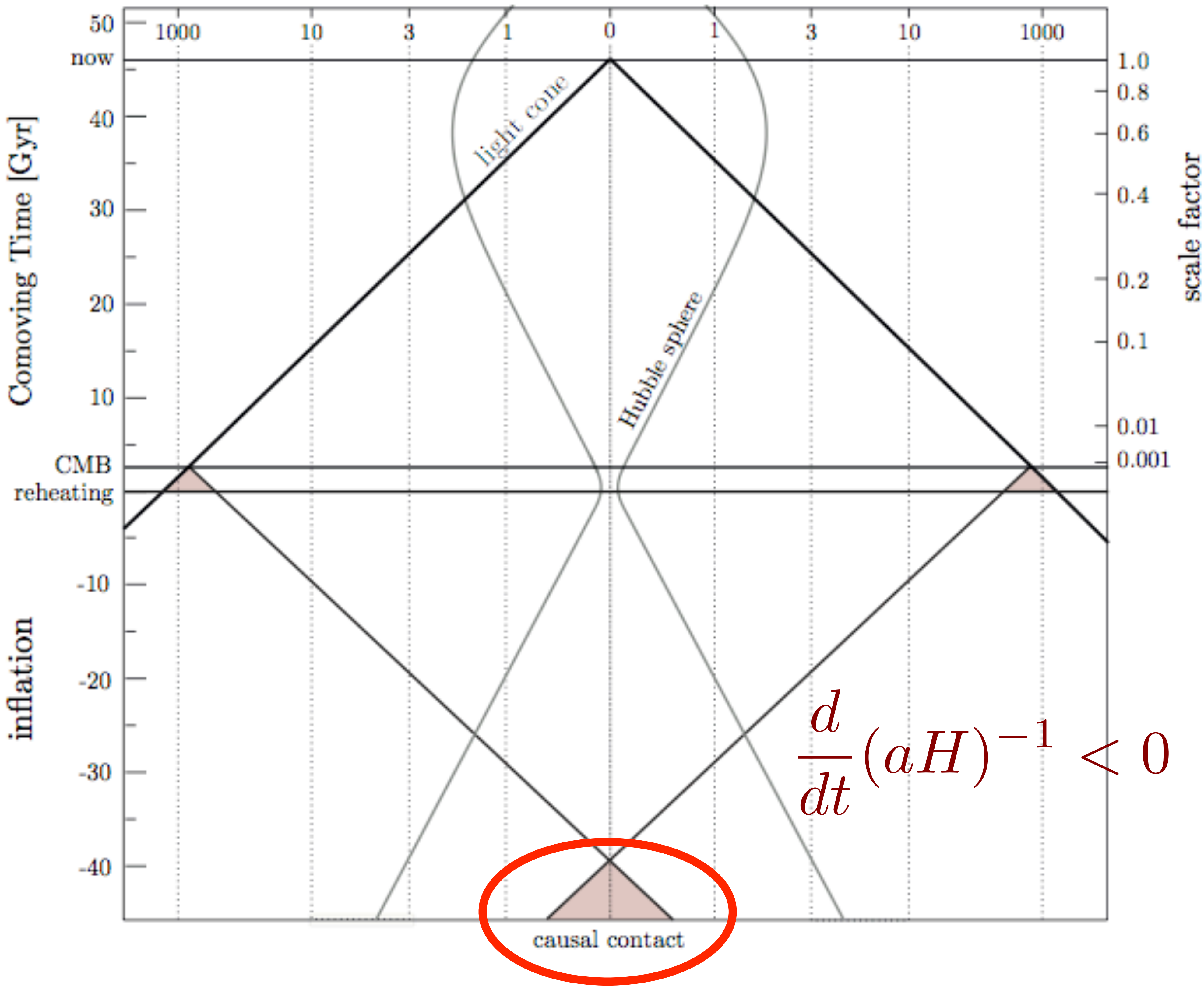
cosmic time → t

comoving distance → $\vec{x}$

Hubble scale $H = \frac{\dot{a}}{a}$



... solved by a schrinking comoving Hubble sphere



$$\frac{d}{dt}(aH)^{-1} < 0$$

Guth (80)

3 equivalent definitions of inflation

- Schrinking Hubble radius: $$\frac{d}{dt}(aH)^{-1} < 0$$
  (solving the horizon problem)

- **Accelerated expansion:** $$\frac{d}{dt}(aH)^{-1} = \frac{-\ddot{a}}{(aH)^2}$$

with $\frac{\ddot{a}}{a} = H^2(1-\epsilon)$ and $\epsilon \equiv -\frac{\dot{H}}{H^2}$ ⟷ $\epsilon < 1$

↘ $\epsilon \ll 1$ Almost de Sitter: $\mathrm{d}s^2 \simeq -\mathrm{d}t^2 + e^{2Ht}\mathrm{d}\vec{x}^{\,2}$

- Violation of strong energy condition:

$$p < -\frac{1}{3}\rho \Leftrightarrow w \equiv \frac{p}{\rho} < -\frac{1}{3}$$

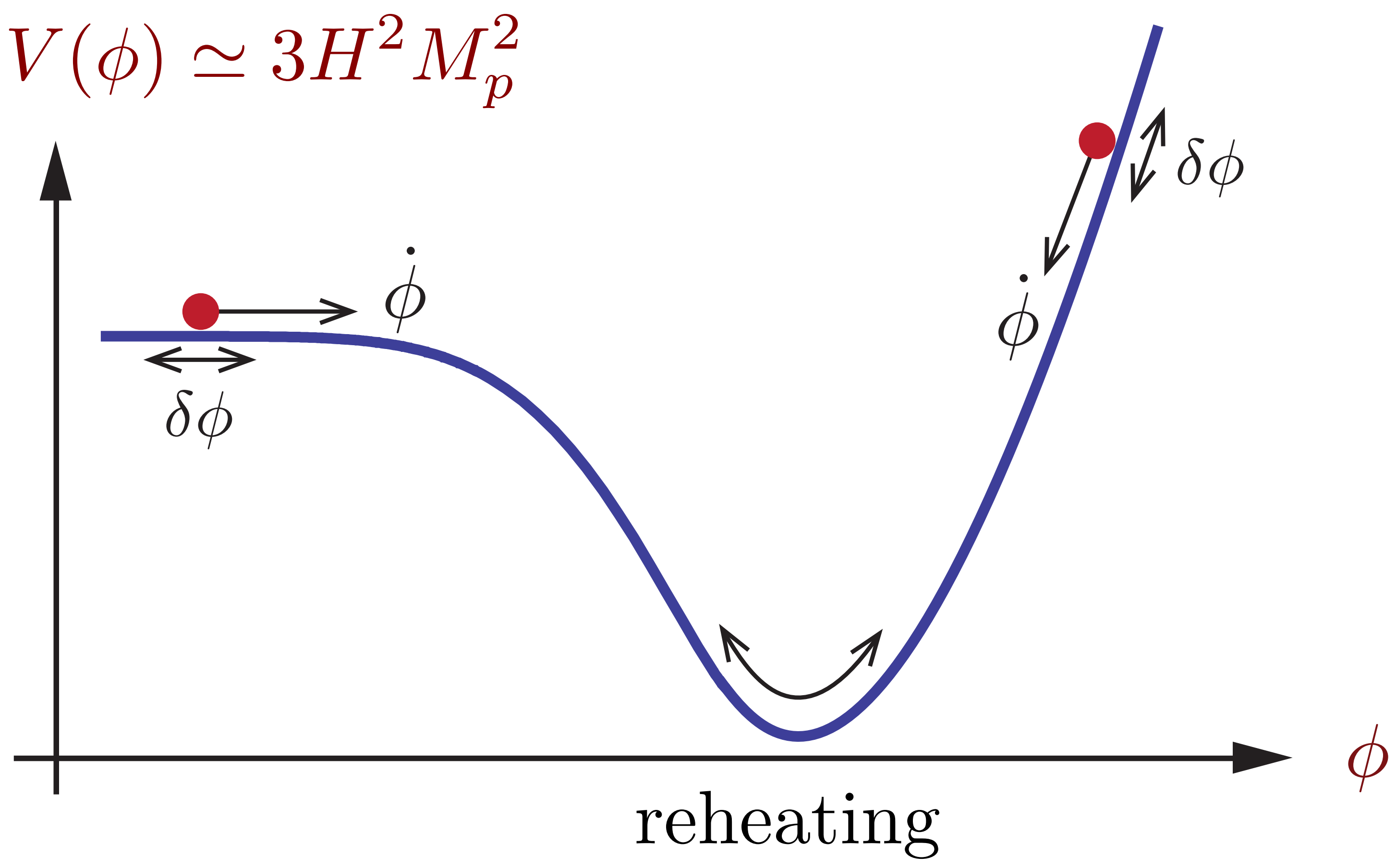
Big-Bang puzzles solved:

$$N_{\rm inf} \equiv \ln\left(\frac{a_{\rm f}}{a_{\rm i}}\right) \gtrsim 60$$

Slow-roll single field inflation

• Simplest implementation of the above mechanism: scalar field with flat potential in Planck units

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) \right]$$

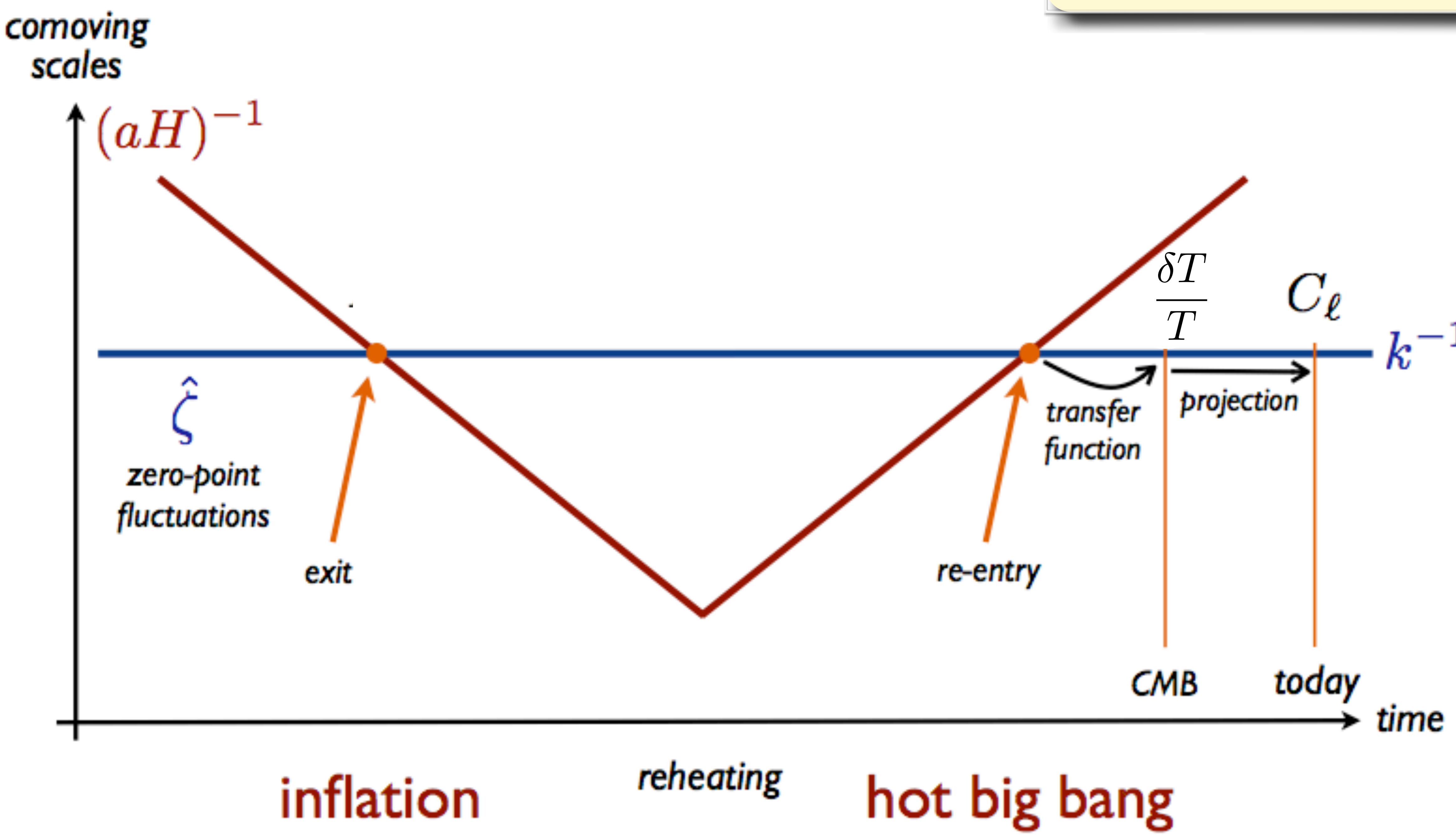


$$\frac{M_{\rm pl}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2 \ll 1$$

$$\eta \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

From quantum to temperature fluctuations

Gauge-invariant curvature perturbation $$\zeta = \psi + \frac{1}{\sqrt{2\epsilon}}\delta\phi$$



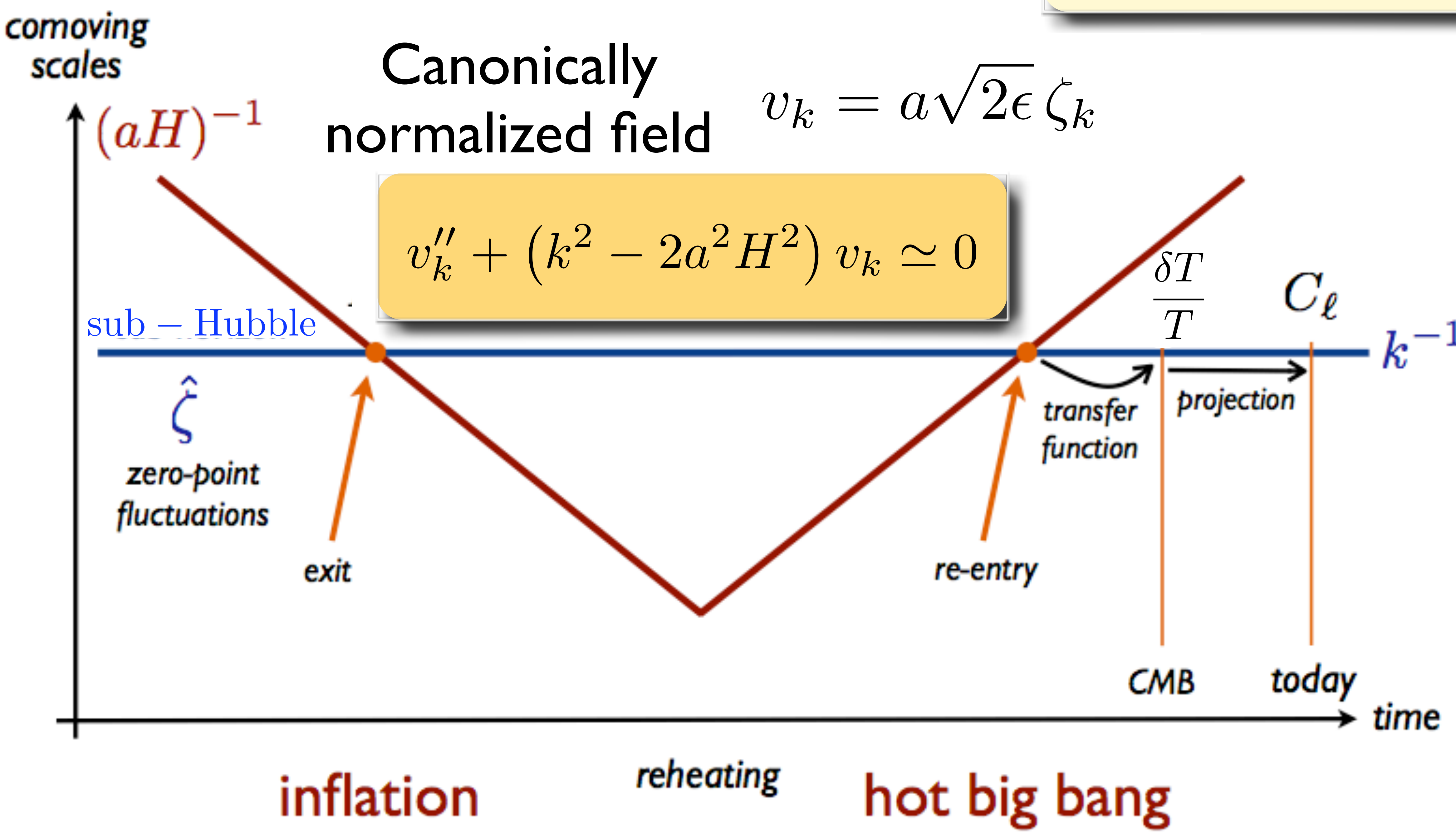
Tools: General Relativity and perturbative Quantum Field Theory in curved spacetime.

From quantum to temperature fluctuations

Gauge-invariant curvature perturbation $$\zeta = \psi + \frac{1}{\sqrt{2\epsilon}}\delta\phi$$

Canonically normalized field $$v_k = a\sqrt{2\epsilon}\,\zeta_k$$

$$v_k'' + \left(k^2 - 2a^2H^2\right) v_k \simeq 0$$



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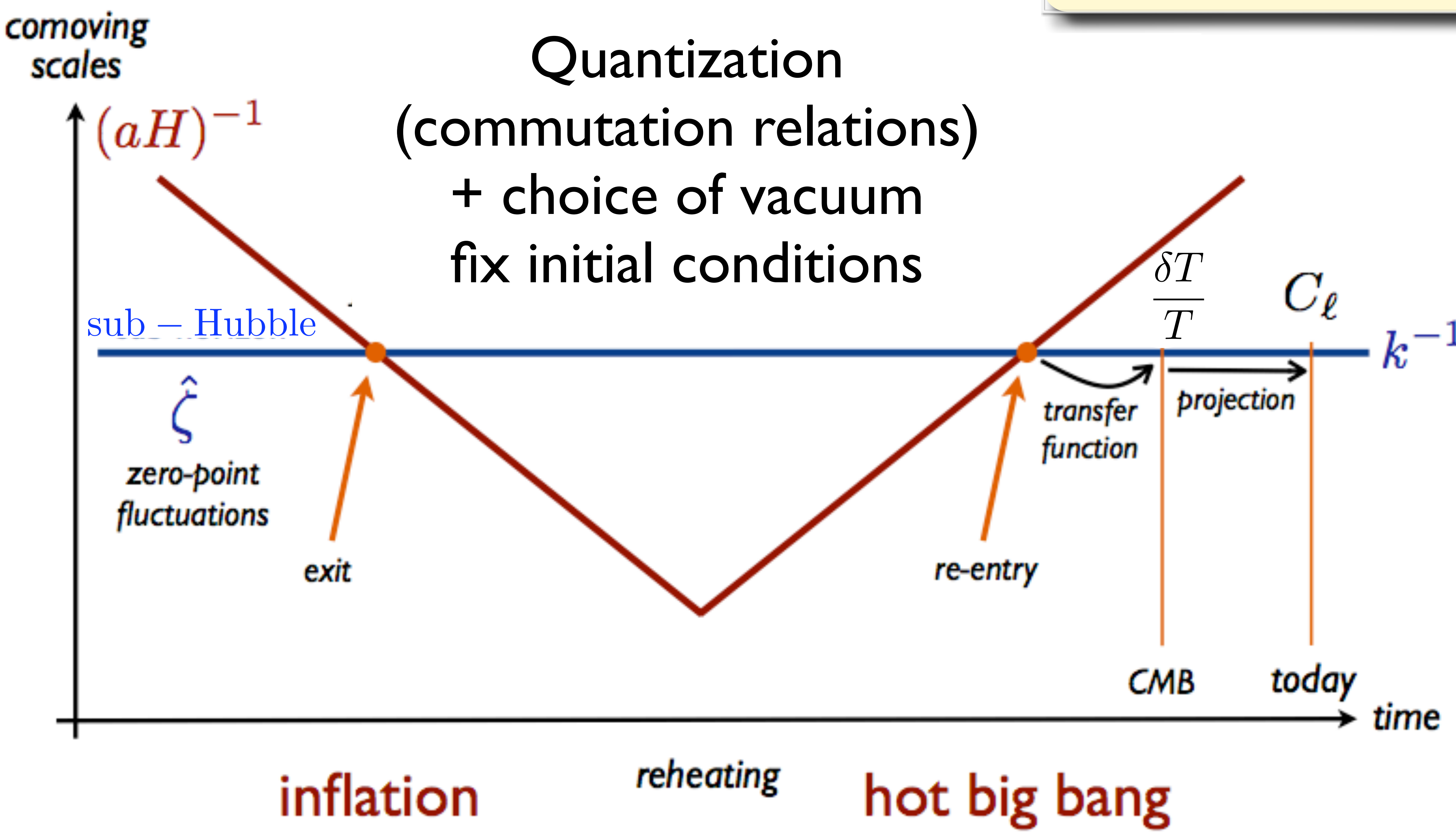
Gauge-invariant curvature perturbation $\zeta = \psi + \frac{1}{\sqrt{2\epsilon}}\delta\phi$

Quantization
(commutation relations)
+ choice of vacuum
fix initial conditions



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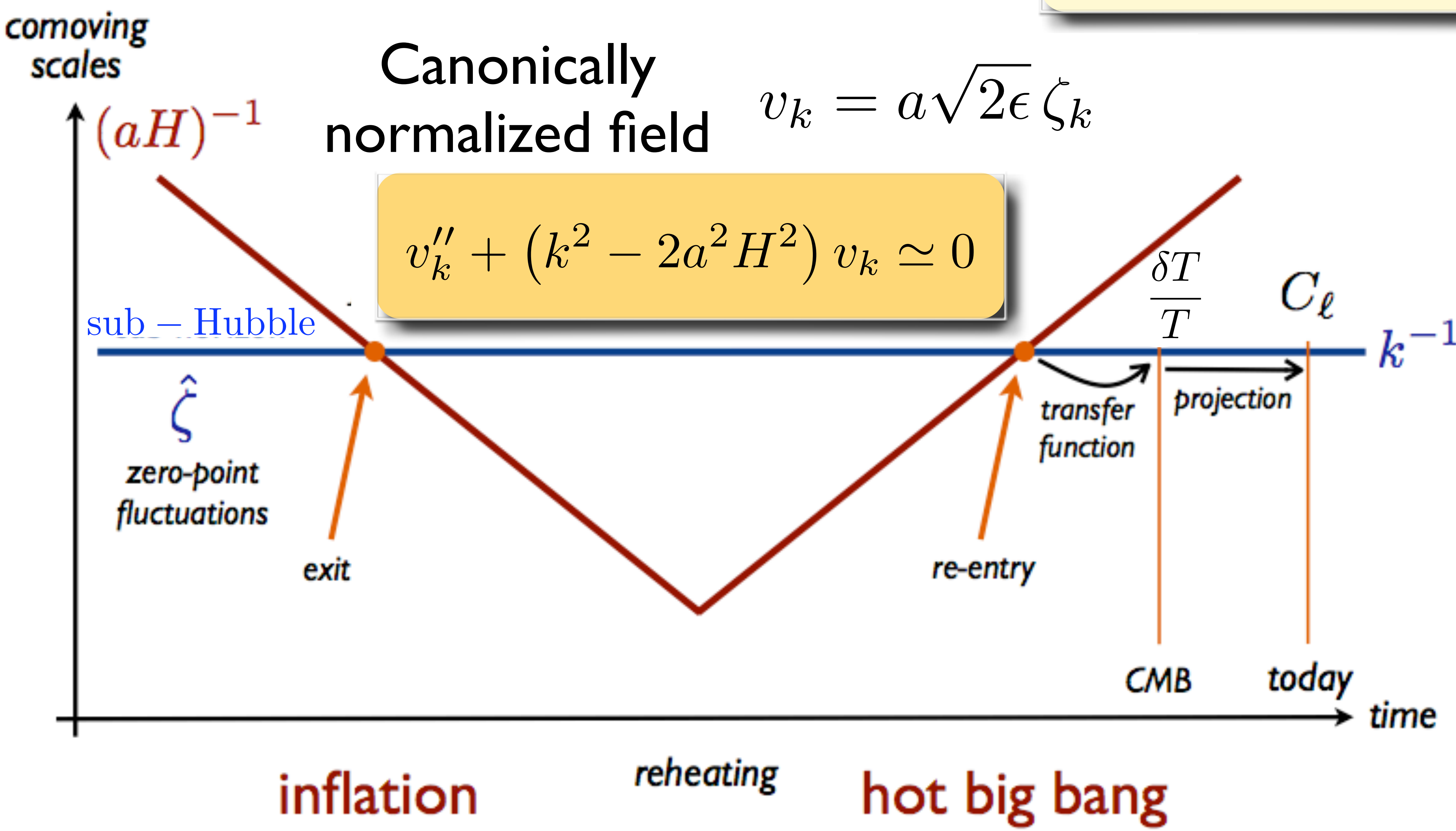

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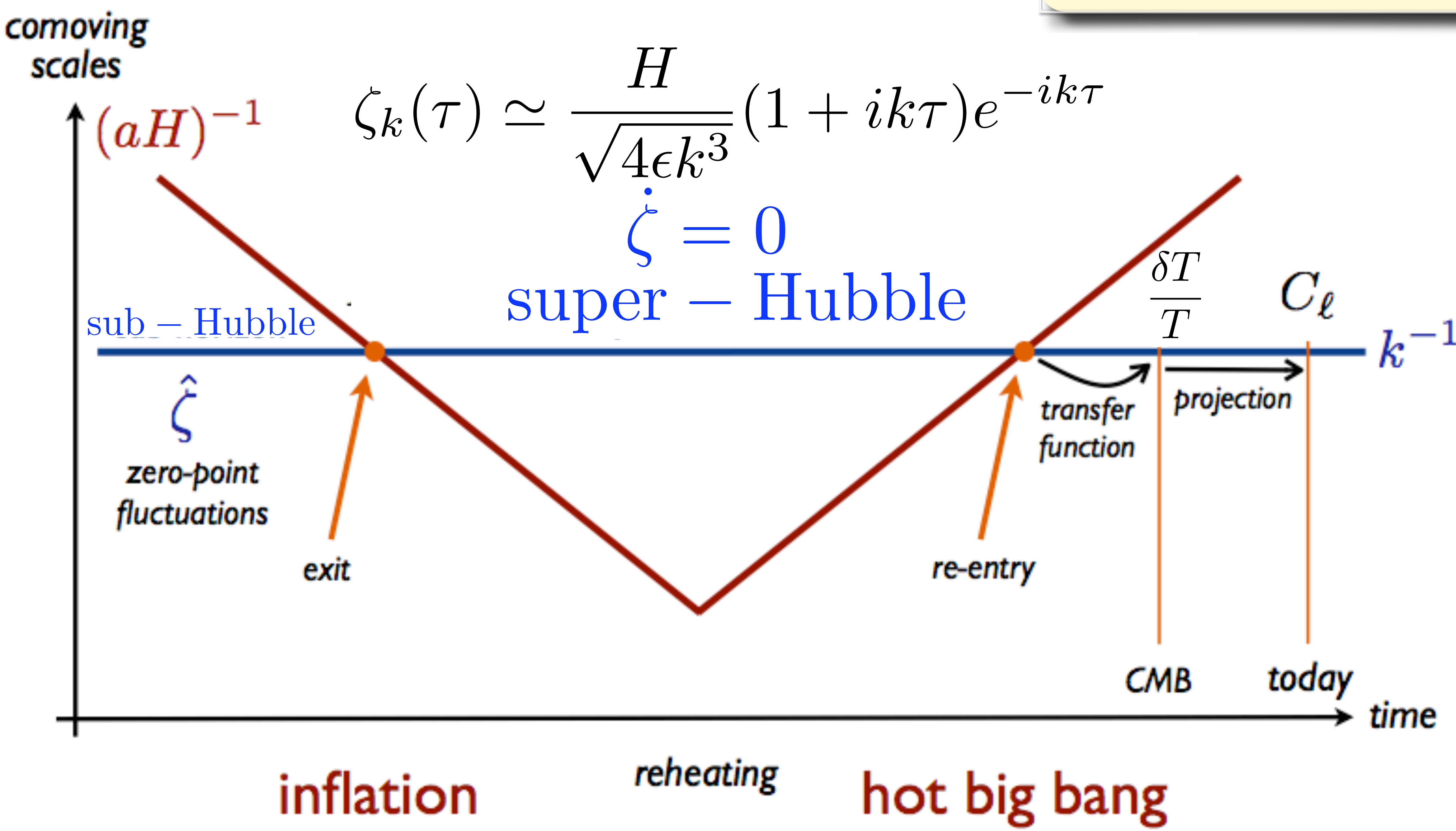

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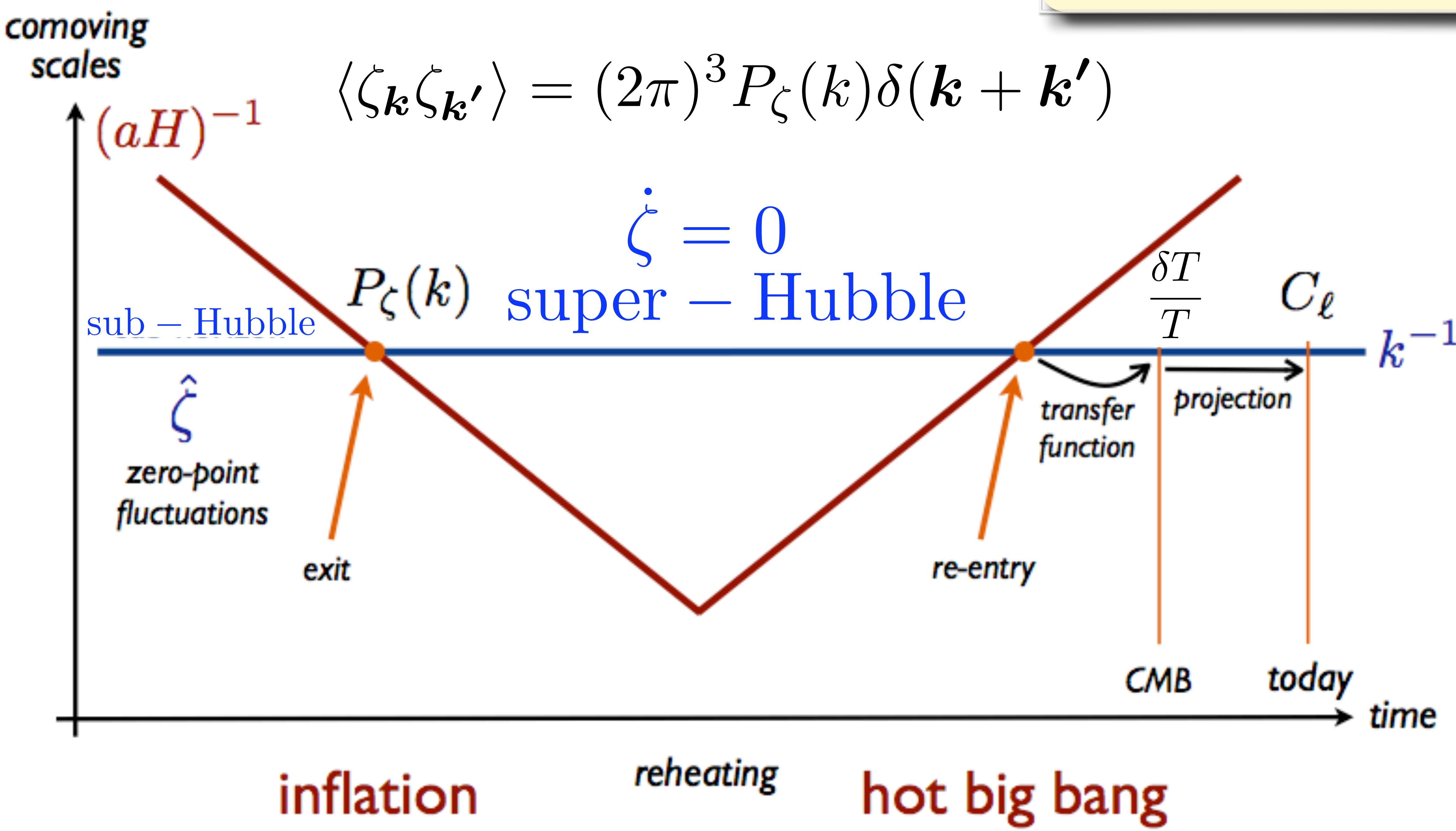

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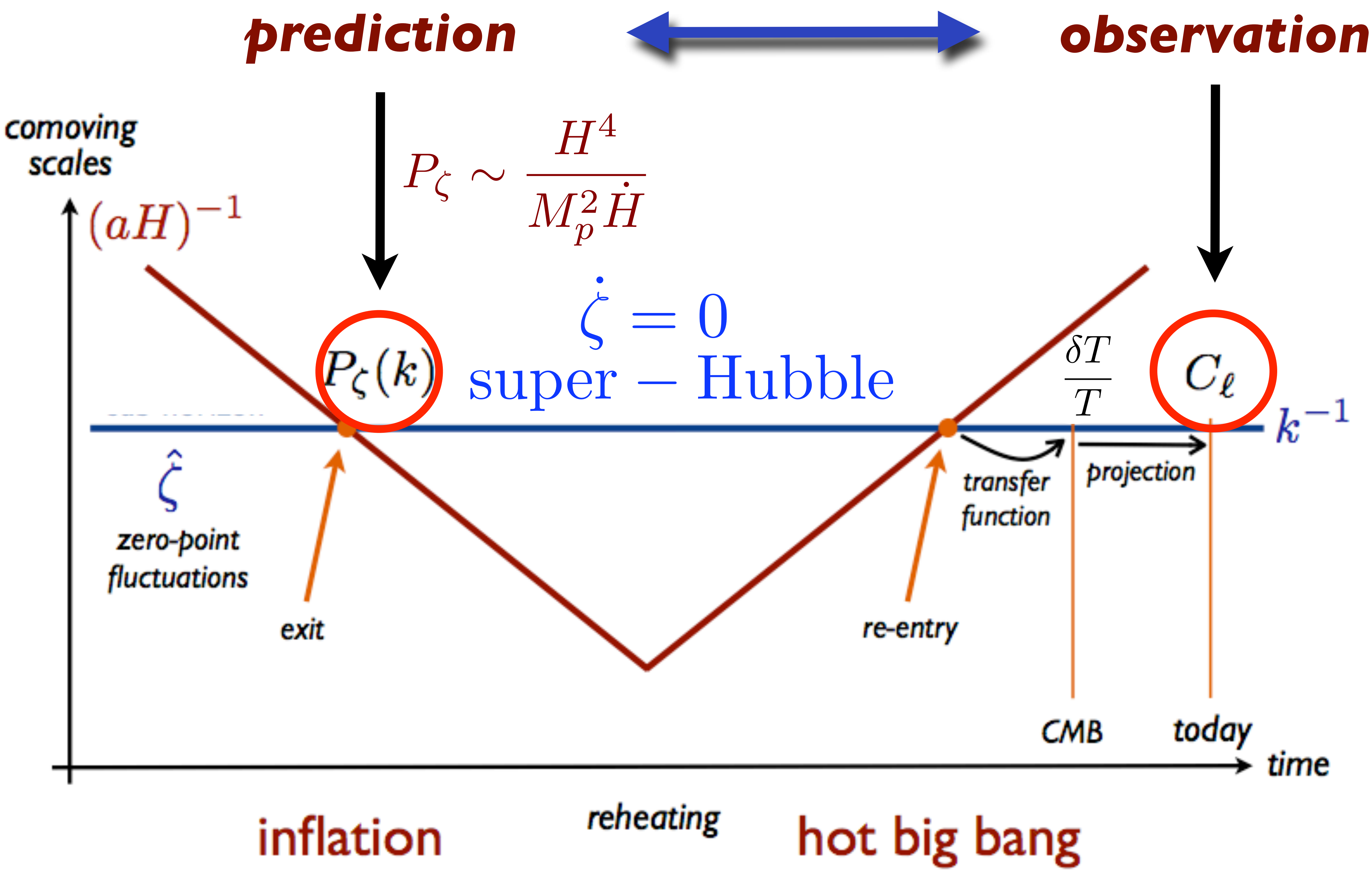


Tools: General Relativity and perturbative Quantum Field Theory in curved spacetime.

Inflation

predicts

universe on large scales is:

homogeneous

isotropic

flat

\+ density fluctuations are:

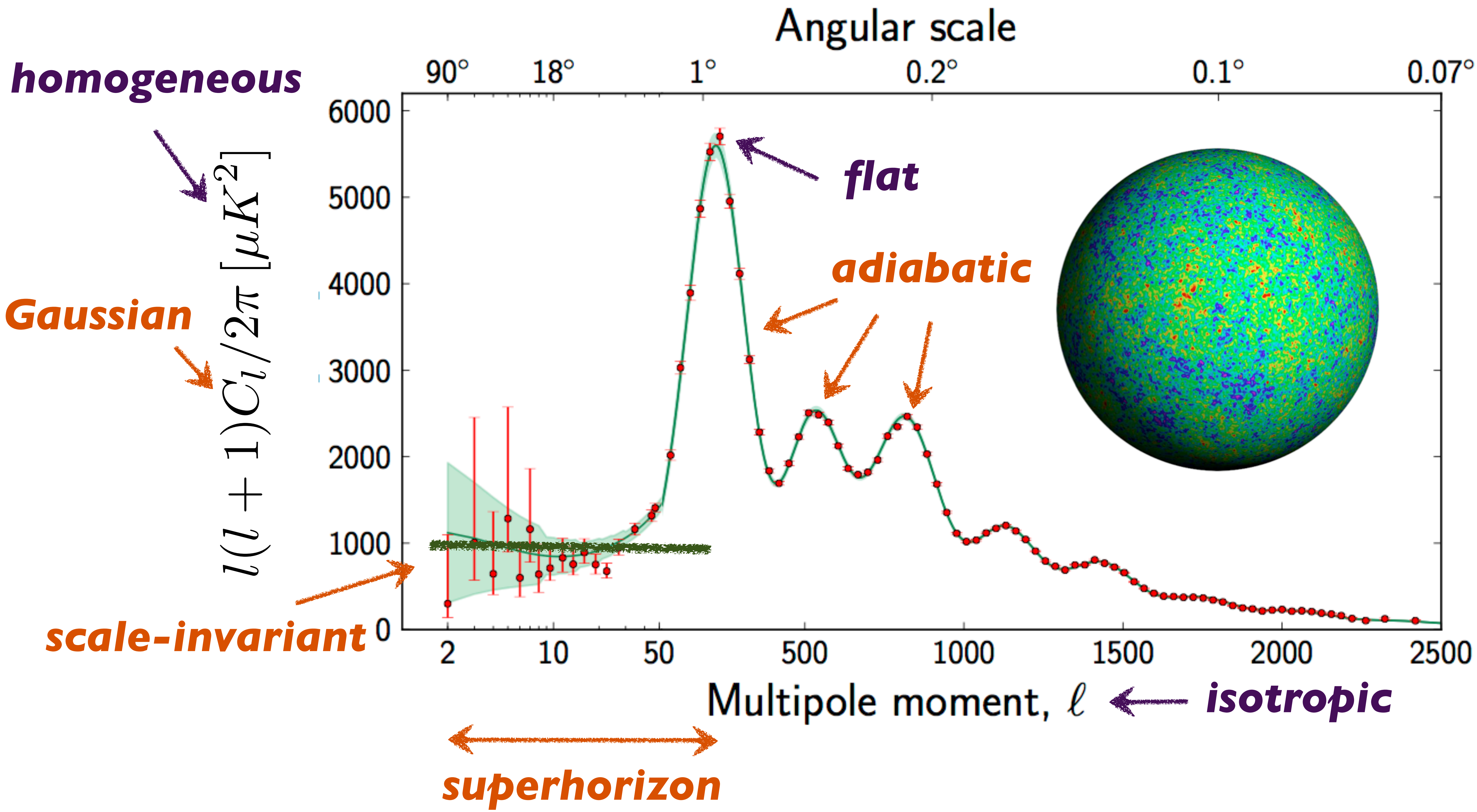
adiabatic (no spatial variation of composition of the cosmic fluid)

superhorizon at recombination

almost scale-invariant

almost Gaussian

Observations



The simplest inflationary models are in full agreement with data

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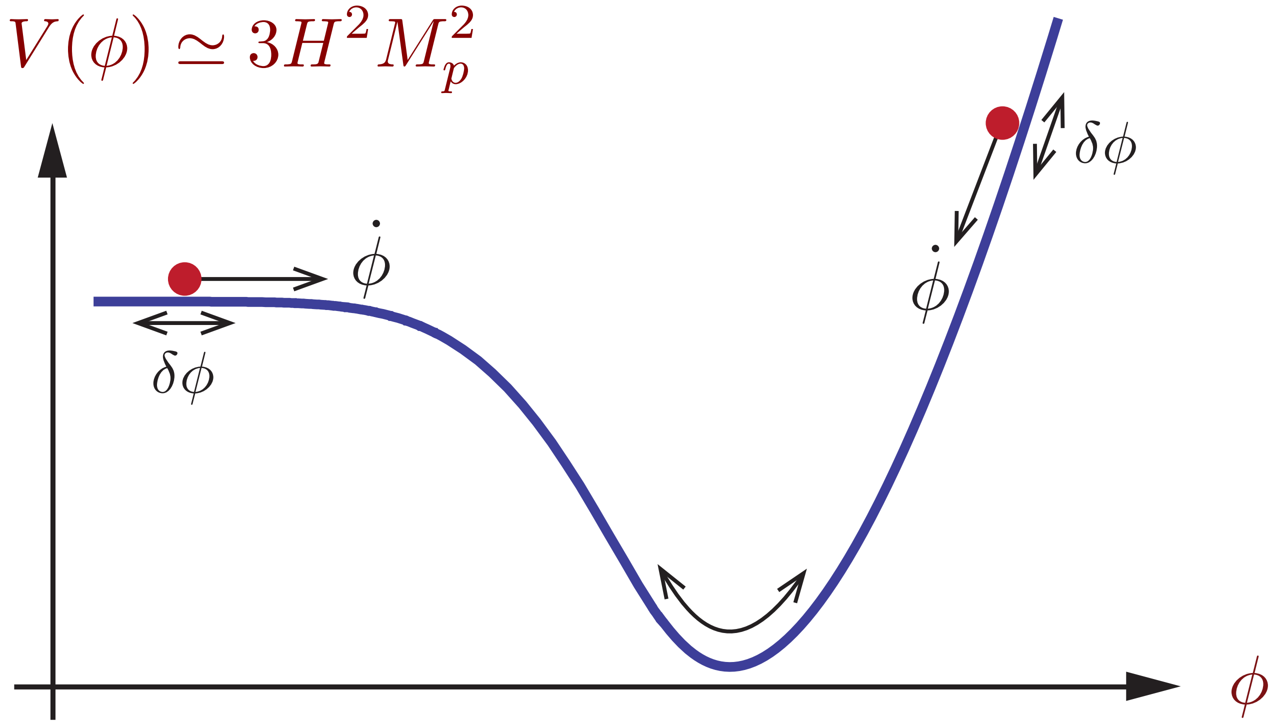
Microphysical origin of inflation?

- So far, merely phenomenological description

- Physics at the energy scale of inflation is unknown!
Observational probe of very high-energy physics

- Candidate physical theories motivate much more complicated dynamics than the simplest scenarios (toy models).

The Eta problem



$$\frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \ll 1$$

$$\eta \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

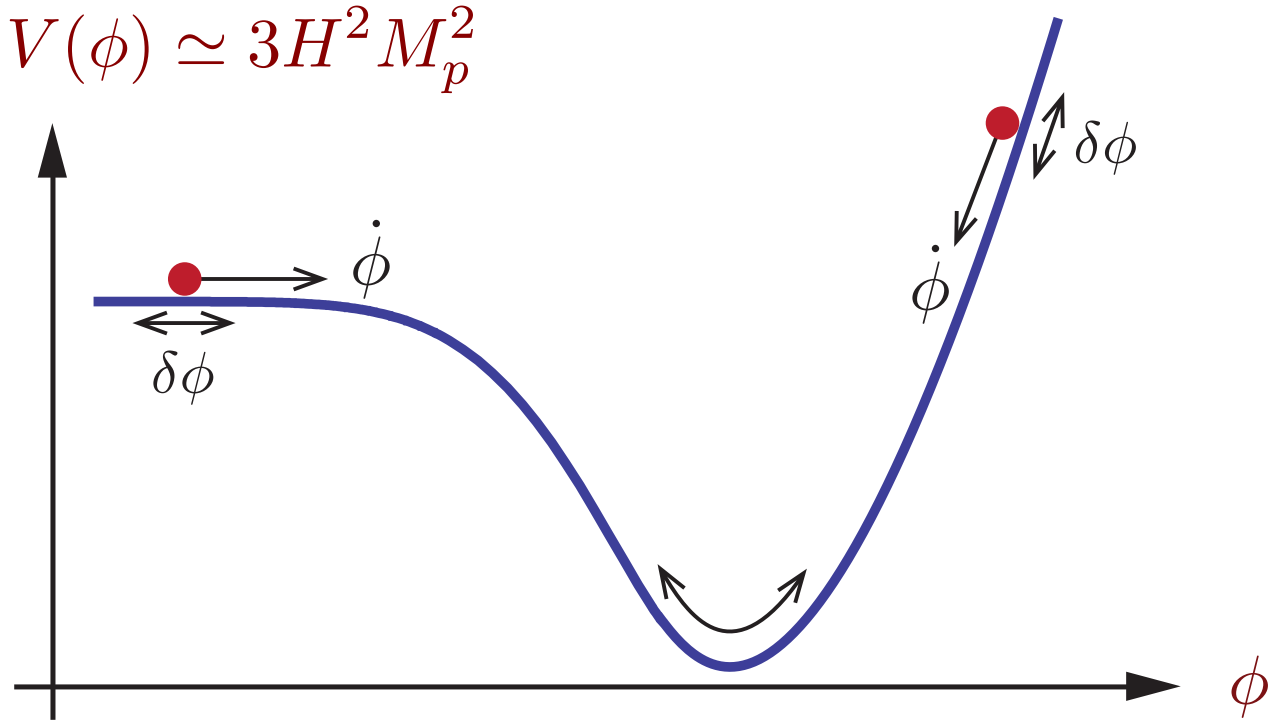
Why is the inflaton so light?

$$\eta \approx \frac{m_\phi^2}{H^2} \ll 1$$

like the Higgs
hierarchy problem

$$m_\phi^2 \sim \Lambda_{\rm uv}^2 \gg H^2$$

The Eta problem



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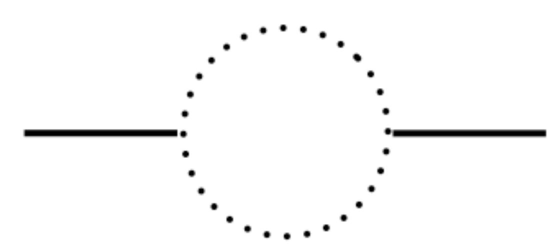
Supersymmetry
ameliorates the problem
but doesn't solve it.



$$m_\phi^2 \sim H^2$$

UV sensitivity of inflation

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_\delta \frac{\mathcal{O}_\delta(\phi)}{\Lambda^{\delta-4}}$$

Slow-roll action

Corrections to the low-energy effective action

Unless symmetry forbids it, presence of terms of the form

$$\Delta V = cV_0(\phi)\frac{\phi^2}{\Lambda^2}$$

$$\Rightarrow \quad \Delta m_\phi^2 \sim c\frac{V_0}{\Lambda^2} \sim c\, H^2 \left(\frac{M_P}{\Lambda}\right)^2$$

Wilson coefficient $c \sim \mathcal{O}(1)$

+

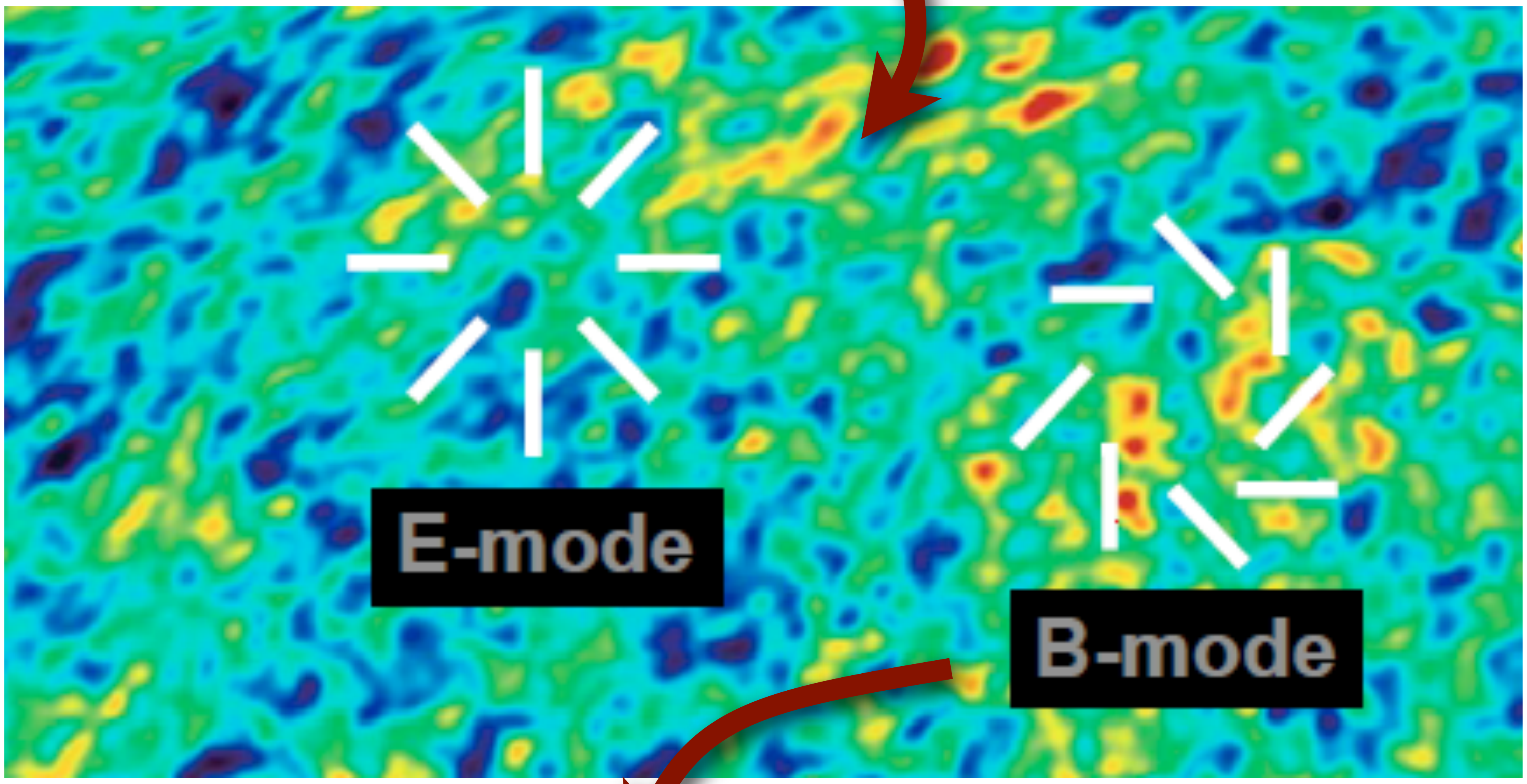
$\Lambda \lesssim M_P$

$$\Delta\eta \gtrsim 1$$

$\Rightarrow$ Sensitivity of slow-roll inflation to Planck-suppressed operators

Gravitational Waves

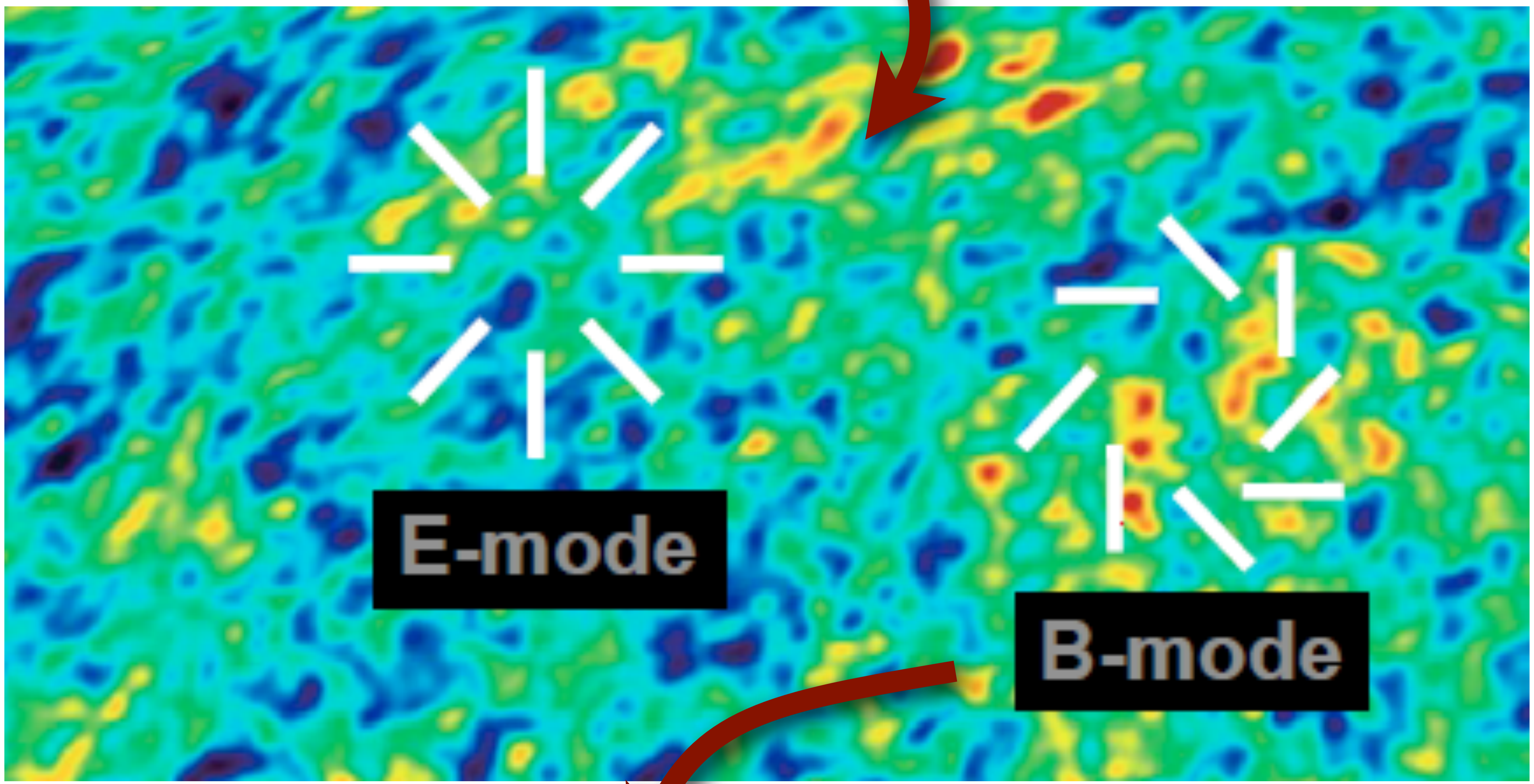
CMB polarization



measures: $$P_t \sim \frac{H^2}{M_p^2}$$ **Energy scale of inflation**

Gravitational Waves

CMB polarization



observable if:

tensor-to-scalar-ratio

$$r \equiv \frac{P_t}{P_\zeta} \gtrsim 0.01$$

Current constraints:

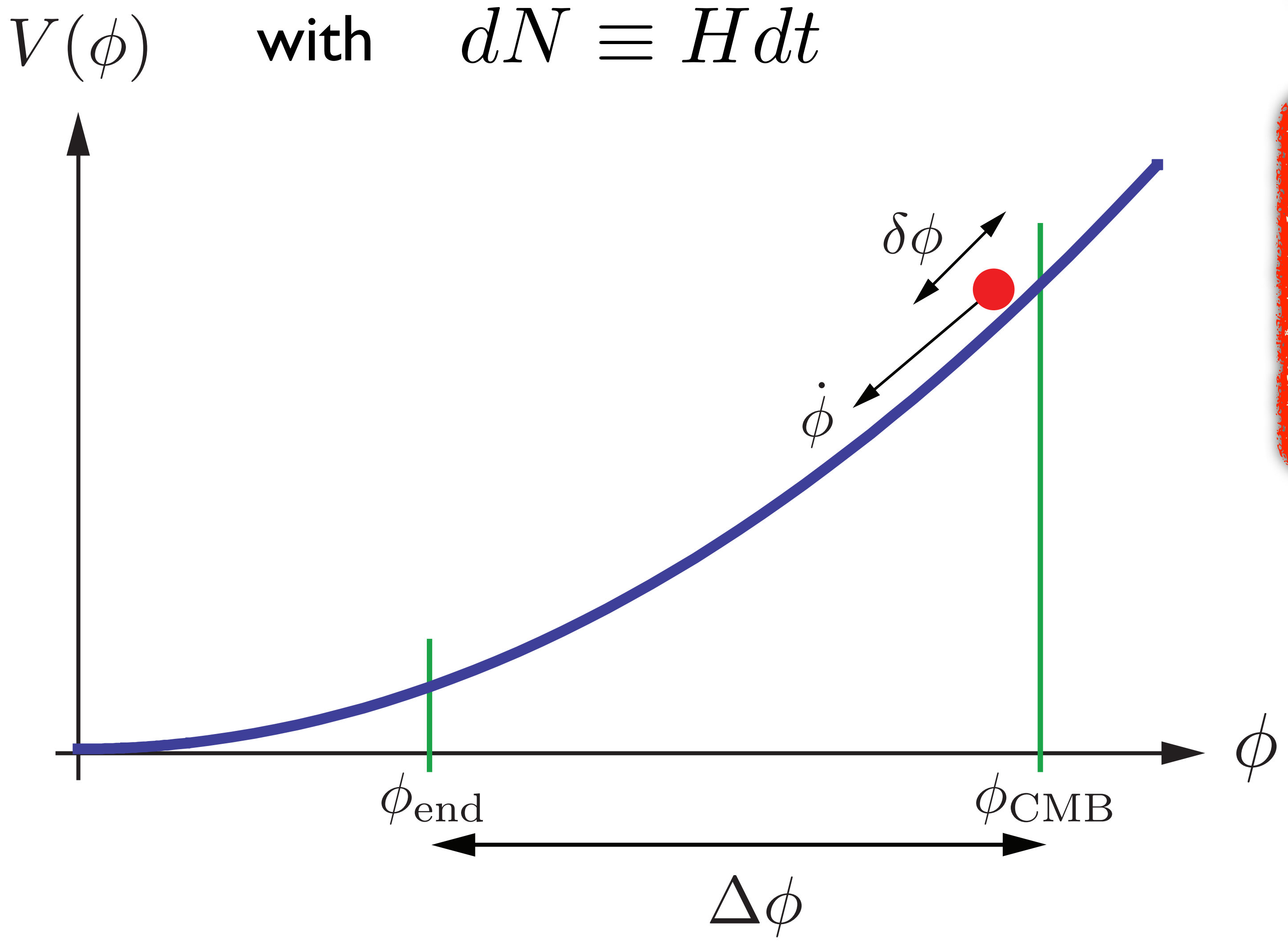
$$r < 0.11 \ (95\%\mathrm{CL})$$

The Lyth bound

$$r = 8\left(\frac{d\phi}{dN}\frac{1}{M_p}\right)^2$$

with $dN \equiv H dt$

Field evolution over 60 e-folds

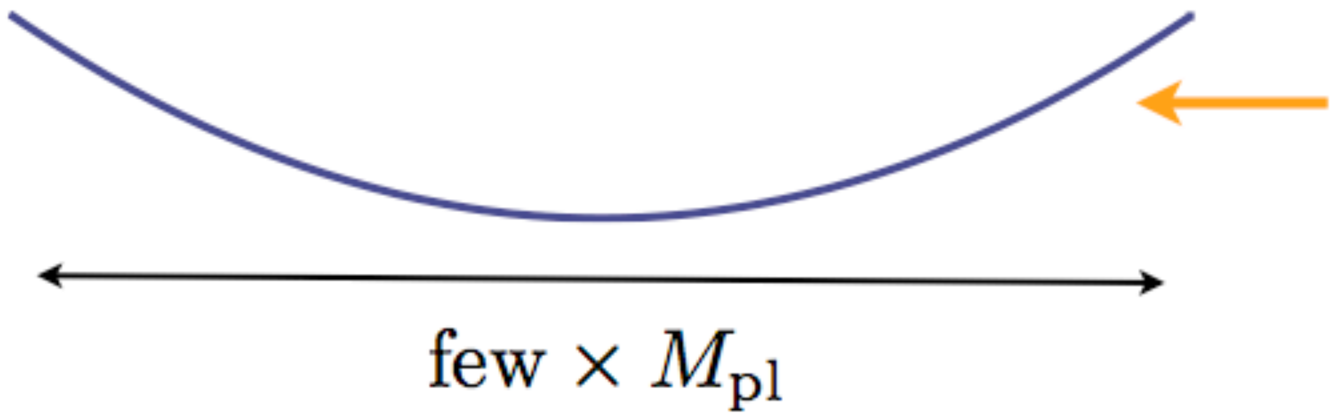


$$\frac{\Delta\phi}{M_p} \approx \left(\frac{r}{0.01}\right)^{1/2}$$

Lyth, 96

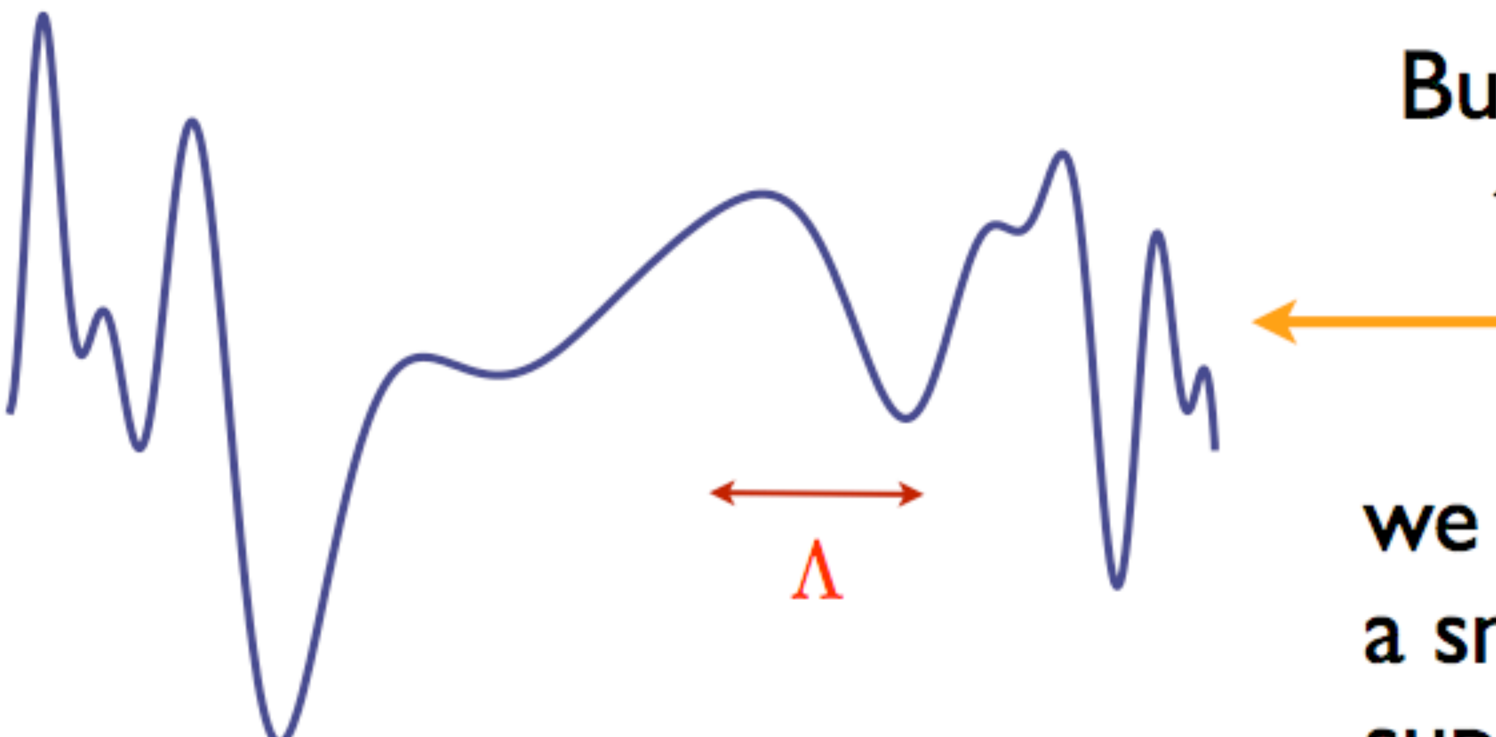
Observable gravitational waves require **super-Planckian field-variation**

The Lyth bound



Observable GWs require a smooth potential over a range

$$\Delta\phi \gtrsim M_p$$



But, in an effective field theory with cutoff

$$\Lambda < M_{\rm pl}$$

we generically don't expect a smooth potential over a super-Planckian range

Sensitivity to the UV-completion of large-field inflation

K-inflation $\mathcal{L}(X \equiv -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi, \phi)$

Prototypical example:
$$\mathcal{L}_{\rm DBI} = -\frac{1}{f(\phi)}\left(\sqrt{1-2f(\phi)X}-1\right) - V(\phi)$$

- Slow-roll regime: $f\dot{\phi}^2 \ll 1$ $\quad S = \int \mathrm{d}t\,\mathrm{d}^3x\,a^3\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right)$

- 'Relativistic' DBI regime: $c_s^2 \equiv 1 - f\dot{\phi}^2 \ll 1$

e.g: $f(\phi) = \frac{\lambda}{\phi^4}$ and $V(\phi) = \frac{m^2}{2}\phi^2$

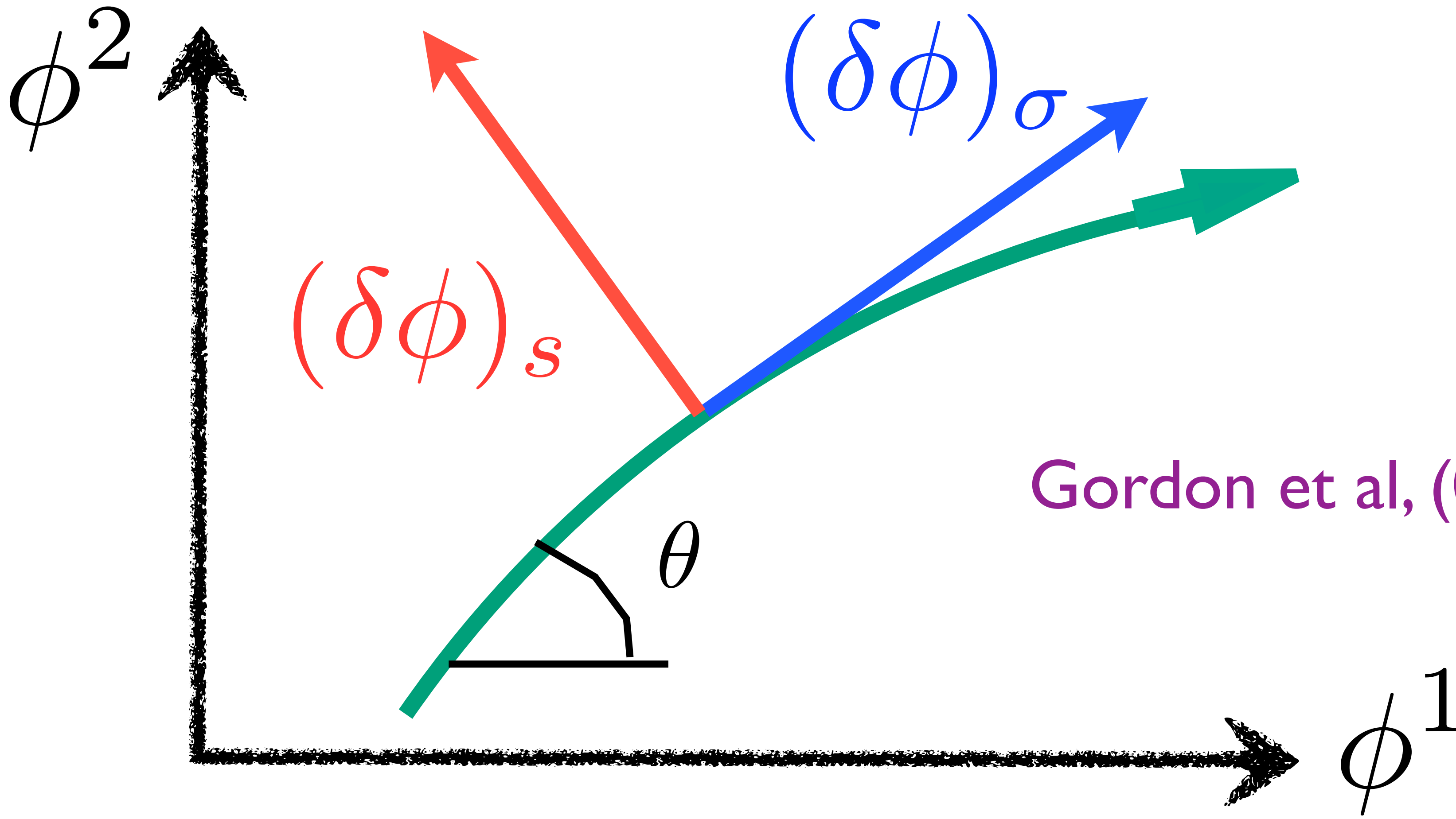
→ Condition for inflation: $\frac{m}{M_P}\sqrt{\lambda} \gg 1$

Inflation despite steep potential!
overcomes the eta-problem?

Silverstein, Tong (04)

Multifield inflation

$$\mathcal{L} = -\frac{1}{2} G_{IJ}(\phi^K)\partial_\mu \phi^I \partial^\mu \phi^J - V(\phi^I)$$



$$\zeta = \frac{(\delta\phi)_\sigma}{\sqrt{2\epsilon}}$$

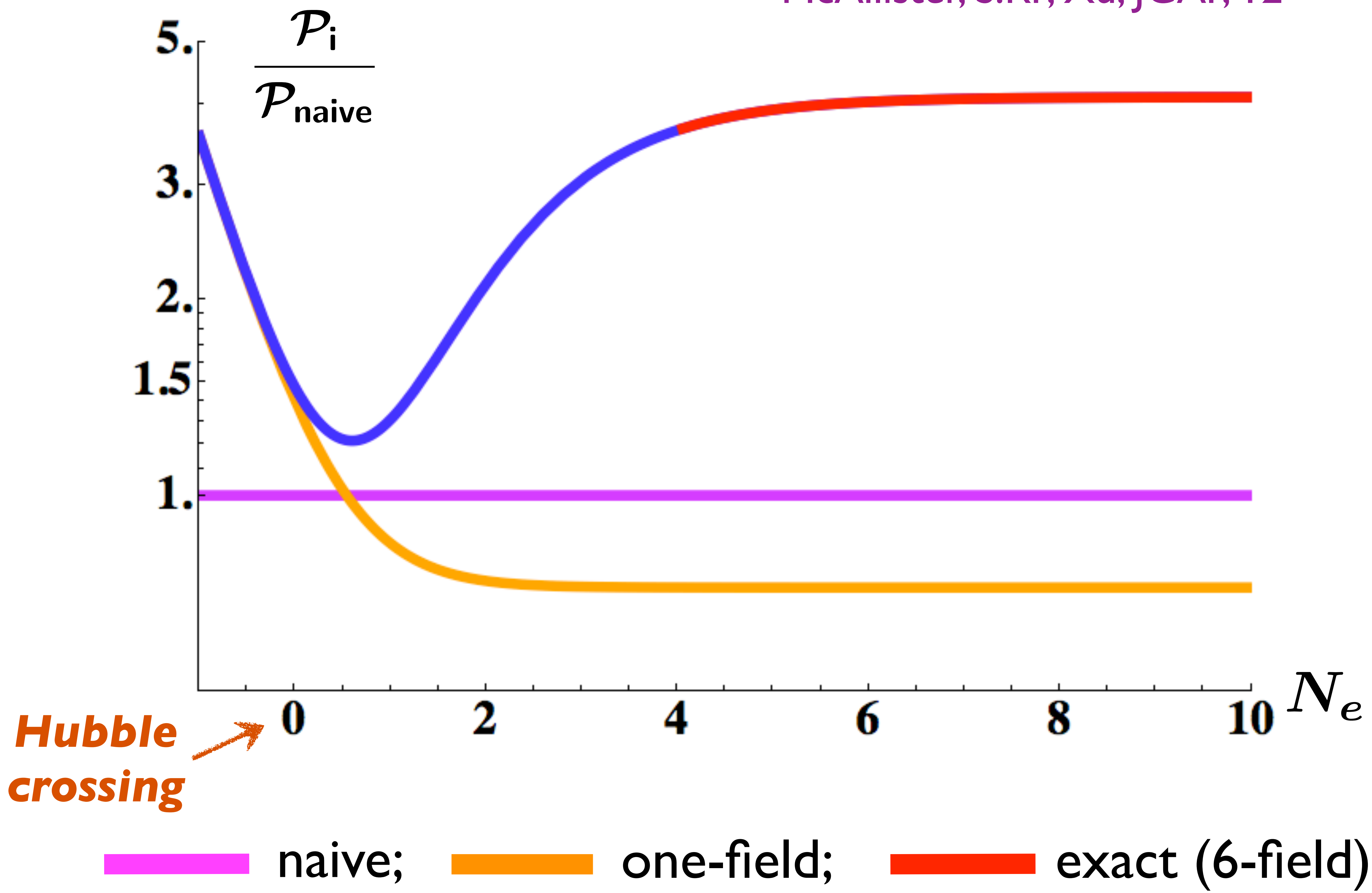
Gordon et al, (00)

$$\dot{\zeta} \propto \dot{\theta}(\delta\phi)_s + \mathcal{O}\left(\frac{k^2}{a^2 H^2}\right)$$

In general (bending trajectories):
super Hubble evolution of the curvature perturbation

An illustration

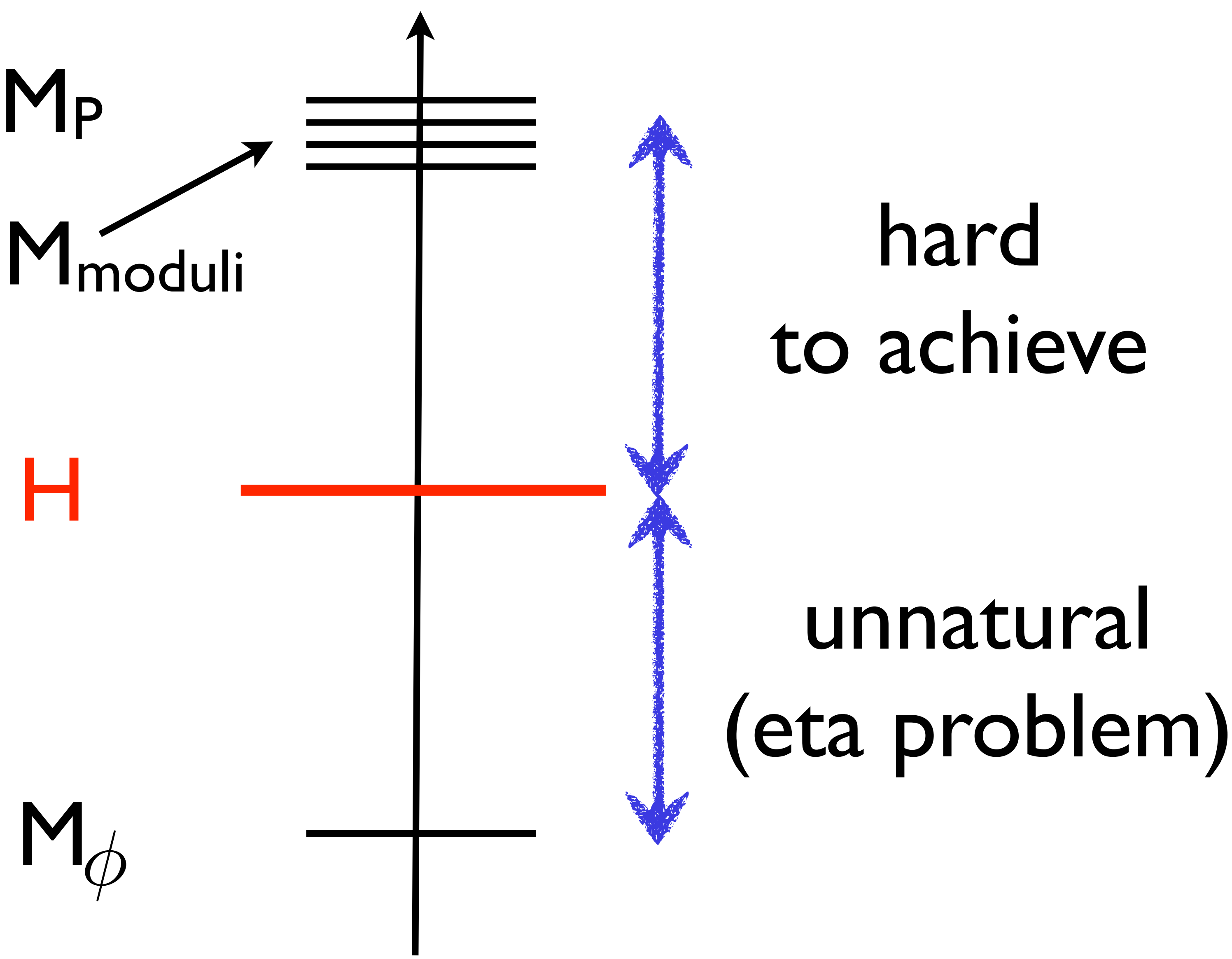
McAllister, S.RP, Xu, JCAP, 12

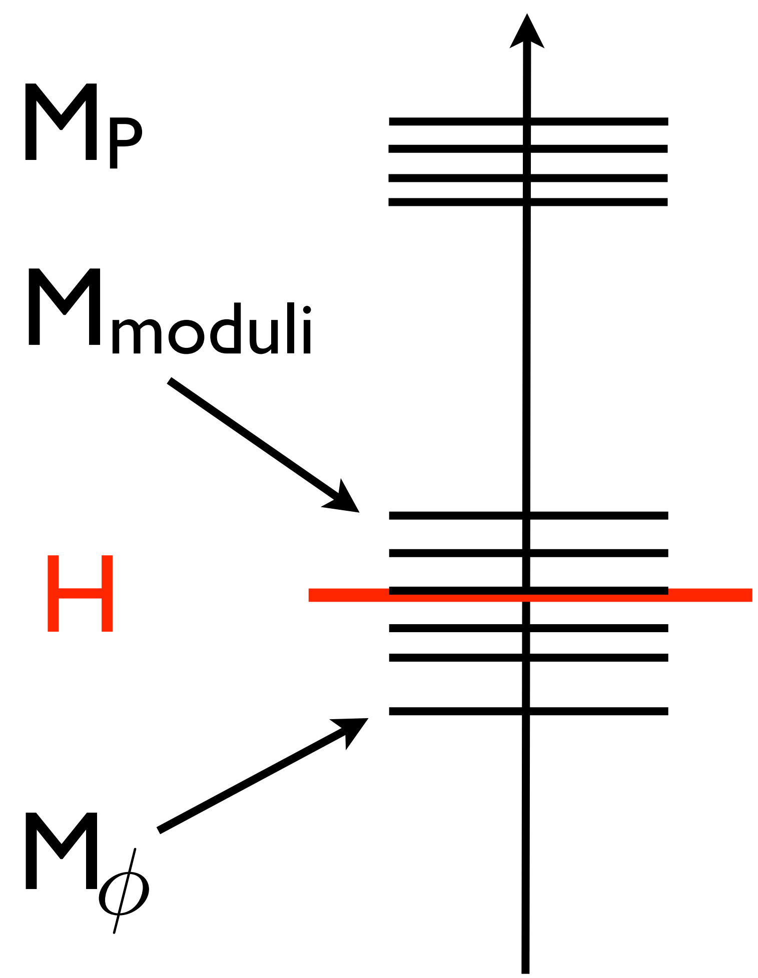


Mass scales in realistic set-up

Hope: light inflaton, Planck-mass moduli

Find: many masses of order H





Quasi-single-field

Chen, Wang 09

3 numbers to explain them all

- Plethora of inflationary models versus three numbers

$$\mathcal{P}_\zeta(k) = A_s(k_\star)\left(\frac{k}{k_\star}\right)^{n_s(k_\star)-1}$$

$$r < 0.11\ (95\%\mathrm{CL})$$

$$k_\star = 0.05\,\mathrm{Mpc}^{-1}$$

$$A_s = (2.441^{+0.088}_{-0.092}) \times 10^{-9}$$

Amplitude known since COBE

$$n_s = 0.9603 \pm 0.0073\ (68\%\mathrm{CL})$$

Planck 2013

Scale invariance ruled out at more than 5 sigma

How can we learn more?

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Primordial non-Gaussianities

- Gaussian approximation: freely propagating particles

- Non-Gaussianities measure the ***interactions*** of the field(s) driving inflation. ***Discrimination amongst models*** which are degenerate at the linear level

Particle physics

Cosmology

Beyond toy-models

• Embedding inflation into high-energy physics requires the understanding of the cosmological perturbations generated in much more complicated scenarios than the simplest models:

- ***multiple fields***
- ***non-standard kinetic terms***
- ***intermediate masses***
- ***modified gravity***

→ I have developped:

• General formalisms -- analytical, numerical -- to predict cosmological observables (in particular NGs) in a wide variety of situations.

• Applications to interesting early universe models.

Maldacena's 2003 result

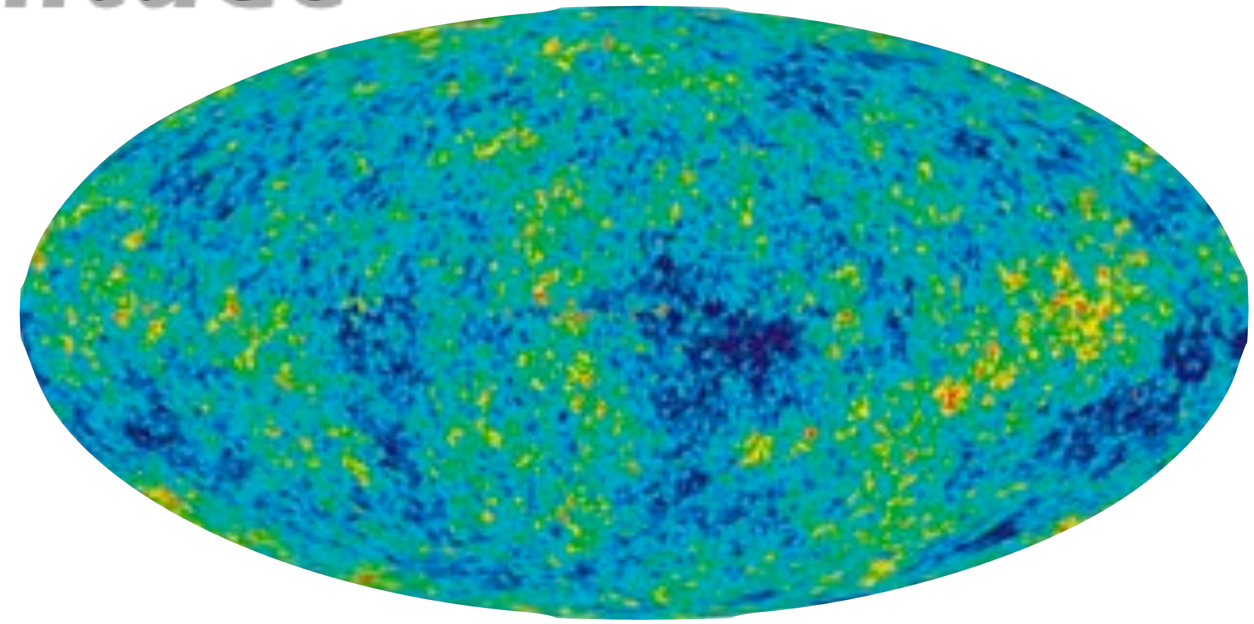
Very small non-Gaussianities (much more quantitative statement actually!)

UNDER HYPOTHESES

- Single field
- Standard kinetic term
- Slow-roll
- Initial vacuum state
- Einstein gravity

It is now clear that violating any of these assumptions might lead to observably large NGs.

A simple example and orders of magnitude



$$\frac{\delta T}{T} \sim \zeta \sim 10^{-5}$$

$$\zeta = \zeta_G + \frac{3}{5} f_{NL}^{loc} \zeta_G^2 \quad \text{(local)}$$

$f_{NL}^{loc} = 32 \pm 21 \, (68\% \, CL)$ WMAP, ApJS 10 (CMB)

$f_{NL}^{loc} = 28 \pm 23 \, (68\% \, CL)$ Slosar et al, JCAP 08 (LSS)

$f_{NL}^{loc} = 2.7 \pm 5.8 \; (68\,\% \, CL)$ Planck 2013

- Slow-roll single field prediction: $f_{NL}^{loc} \approx 10^{-2}$

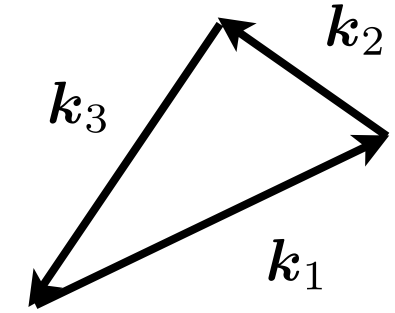
Primordial non-Gaussianities

- Beyond the power spectrum: $$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \rangle = P_\zeta(k_1)(2\pi)^3 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2)$$

- Higher-order connected, n-point functions:

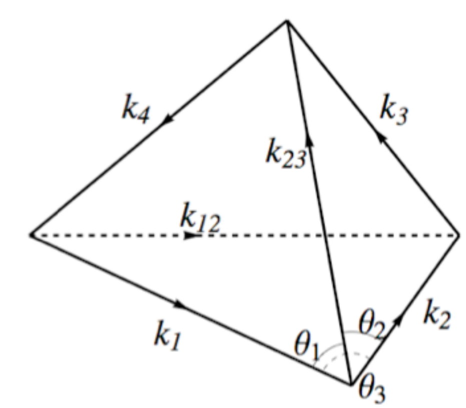
3 point: bispectrum

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = B_\zeta(k_1, k_2, k_3)(2\pi)^3 \delta^3(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3)$$



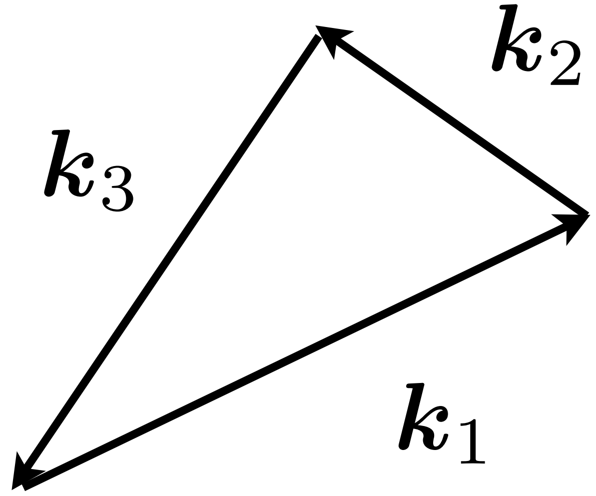
4 point: trispectrum

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \zeta_{\boldsymbol{k}_4} \rangle_c = T_\zeta(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4)(2\pi)^3 \delta^3(\sum_i \boldsymbol{k}_i)$$



The bispectrum

$$\langle\zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = (2\pi)^7\delta(\sum_{i=1}^{3}\boldsymbol{k}_i)\mathcal{P}_\zeta^2\frac{S(k_1,k_2,k_3)}{(k_1k_2k_3)^2}$$



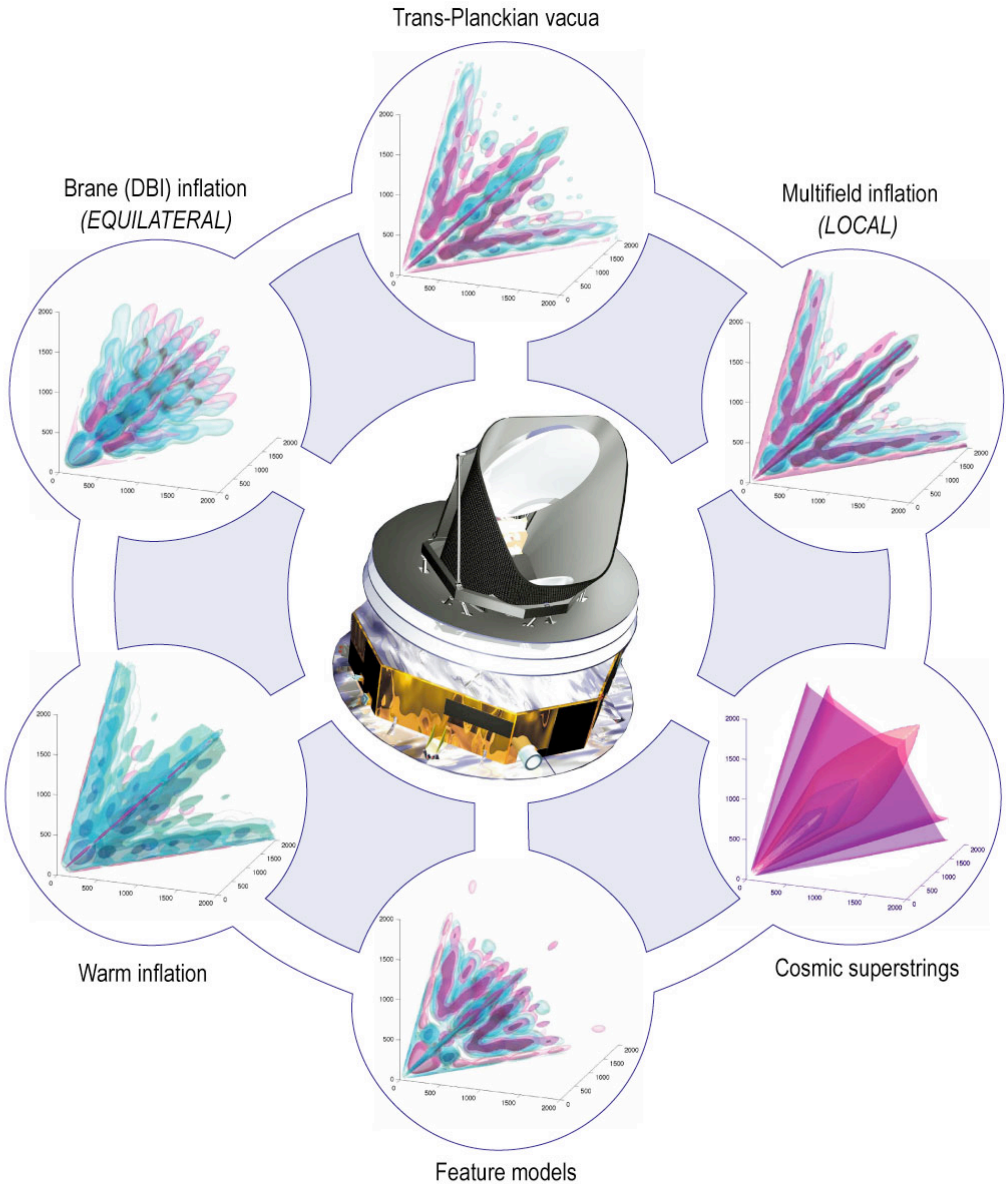
- $f_{NL} \sim S$ — dimensionless measure of the **amplitude** of the bispectrum
- **Scale-dependence** (growing or shrinking on small scales?)
- **Sign** (more or less cold spots?)
- **Shape** (dependence on the configuration of triangles)

Each of these features can rule out large classes of models

Primordial non-Gaussianities

'Happy families are all alike; every unhappy family is unhappy in its own way.'

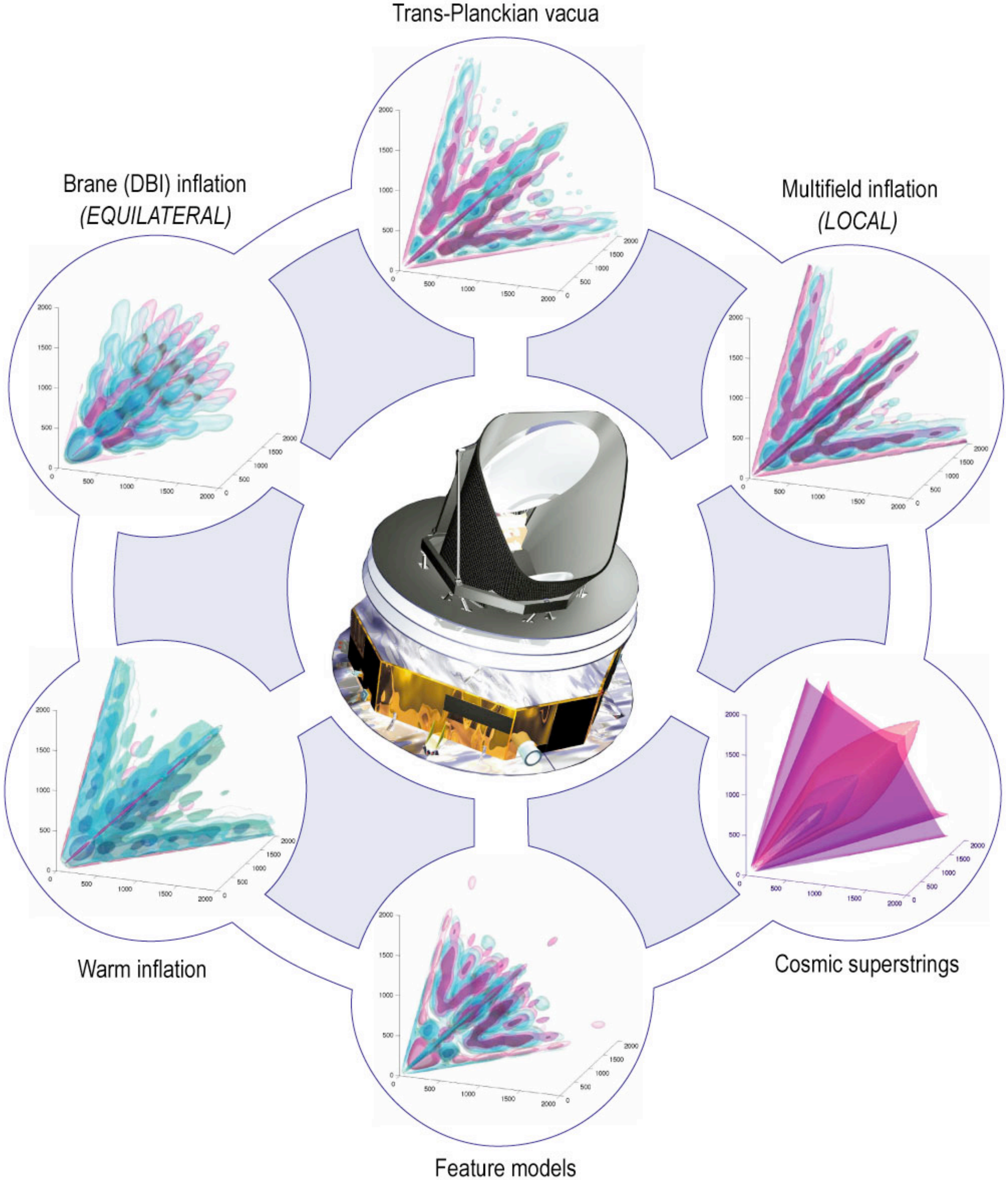
Anna Karénine, Tolstoï



Primordial non-Gaussianities

Gaussian distribution are all alike; every non-Gaussian distribution is non-Gaussian in its own way.

Cosmologist.



Multifield inflation with non-standard kinetic terms

• K-inflation and 'standard' multi-field inflation are subclasses of

$$\mathcal{L}(X^{IJ} \equiv -\frac{1}{2}\partial_\mu \phi^I \partial^\mu \phi^J, \phi^K)$$

The most general Lorentz invariant Lagrangian function of an arbitrary number of scalar fields and their first derivatives

• General study of bakground and fluctuations at first and second order. Reference formalism for many works on inflation and dark energy.

Langlois & S. RP JCAP 08

Langlois, S. RP, Steer, Tanaka PRD 08

General strategy

• Study of coupled fluctuations metric-scalar fields at non-linear level.

$$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + \delta g_{\mu\nu}(t, x^i)$$

$$\phi^I = \bar{\phi}^I(t) + \delta\phi^I(t, x^i)$$

• Tools: Gravitational Theory and perturbative Quantum Field Theory in curved spacetime.

$$S = \bar{S} + S^{(2)}(\delta g_{\mu\nu}, \delta\phi^I) + S^{(3)}(\delta g_{\mu\nu}, \delta\phi^I) + S^{(4)}(\delta g_{\mu\nu}, \delta\phi^I) + \dots$$

• Identification of the gauge-invariant physical dofs. Calculations done in the ADM formalism. Quantization of the linear theory.

• Higher-order correlation functions in the Schwinger-Keldysh, or in-in, formalism:

Schwinger (61), Keldysh (64), Weinberg (05)

$$\langle Q(t)\rangle = \langle 0| \left[\bar{T}\exp\left(i\int_{-\infty(1+i\epsilon)}^{t} H_I(t')dt'\right)\right] Q^I(t) \left[T\exp\left(-i\int_{-\infty(1-i\epsilon)}^{t} H_I(t'')dt''\right)\right]|0\rangle$$

In practice, accurate analytically only until a few e-folds after Hubble crossing

The delta-N formalism

Light fields acquire vacuum quantum fluctuations during inflation $\bar{\phi}_*^A \to \bar{\phi}_*^A + Q^A$

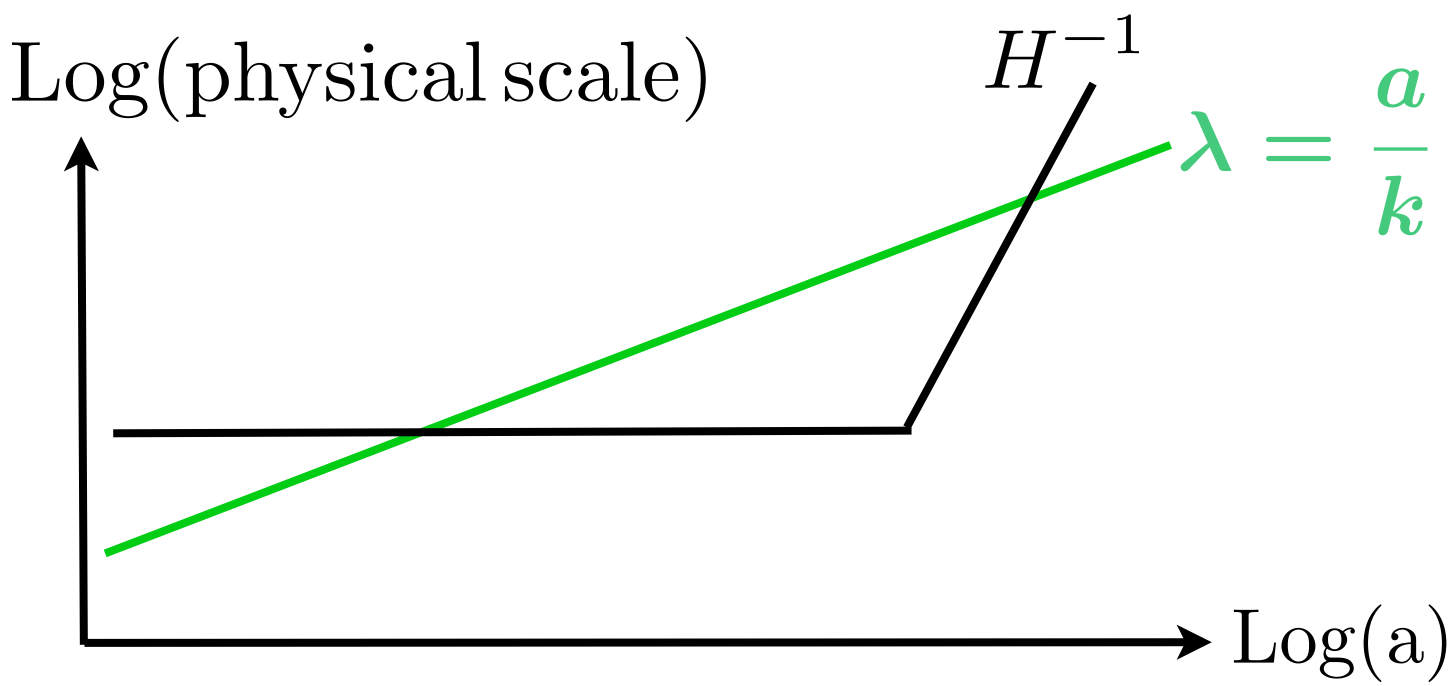
Delta-N formalism: Taylor expansion of the curvature perturbation in terms of the field fluctuations at Hubble crossing

$$\zeta = N_A Q^A + \frac{1}{2} N_{AB} Q^A Q^B + \ldots$$

Sasaki, Stewart (96)
Lyth et al (95)

Origin of the bispectrum

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = N_A N_B N_C \langle Q^A(\boldsymbol{k}_1) Q^B(\boldsymbol{k}_2) Q^C(\boldsymbol{k}_3)\rangle$$



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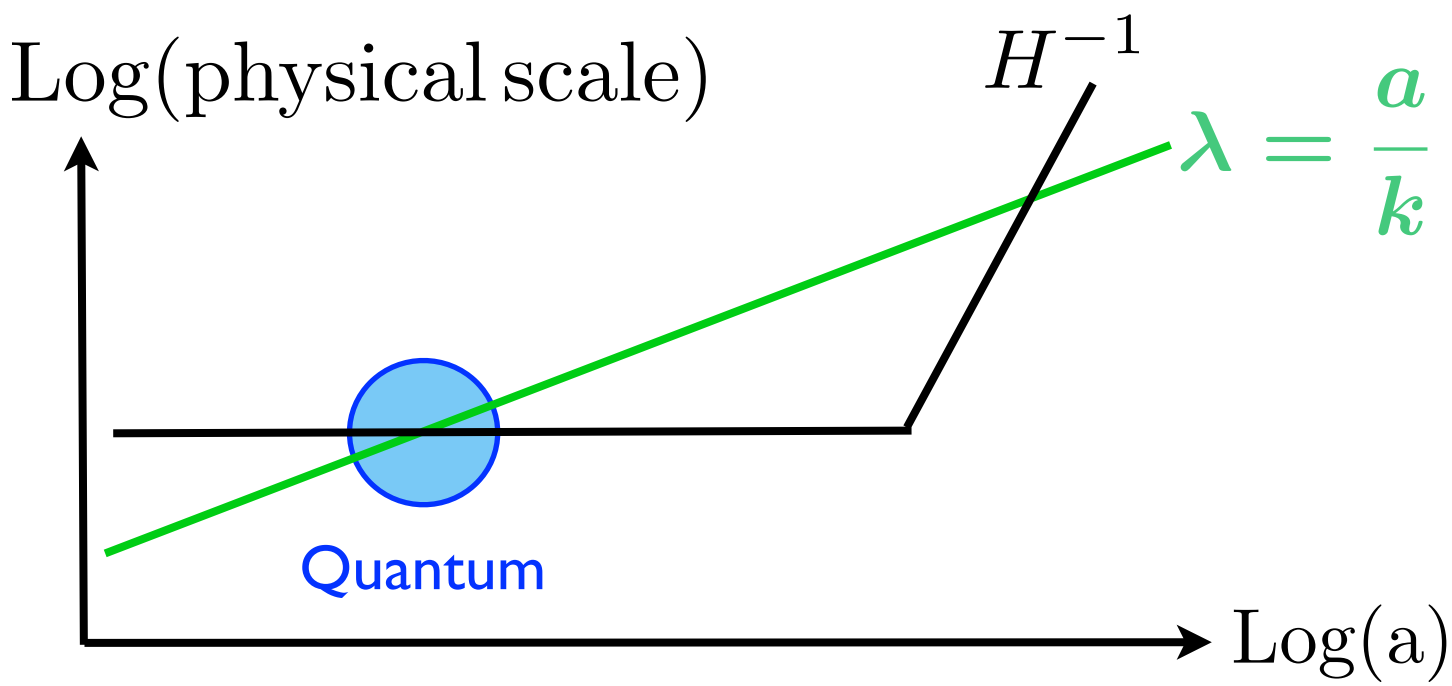
Quantum NGs of the fields around Hubble crossing $k_1 \sim k_2 \sim k_3$

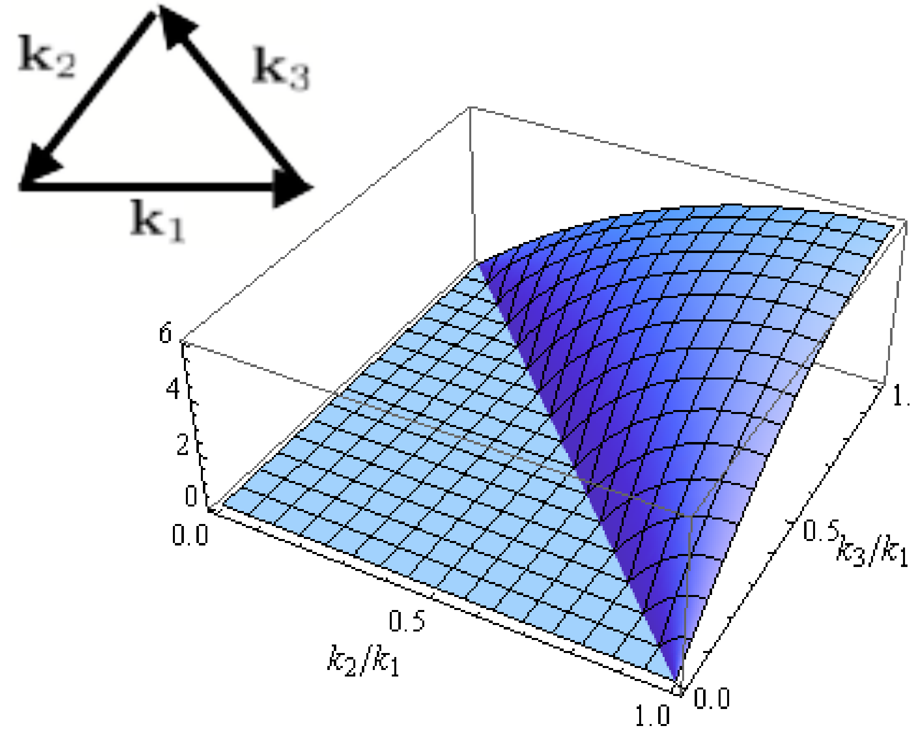
Suppressed by the flatness of the potential in standard slow-roll single and multifield models

Maldacena (03)
Lidsey, Seery (05)

Important for models with non-standard kinetic terms

Chen et al (06)
Langlois, S. RP, Steer, Tanaka 08





K-inflation $\mathcal{L}(X \equiv -\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi, \phi)$

Prototypical example:

$$\mathcal{L}_{\mathrm{DBI}} = -\frac{1}{f(\phi)}\left(\sqrt{1-2f(\phi)X}-1\right) - V(\phi)$$

Key quantity:

$$\frac{1}{c_s^2} - 1 = \frac{2X\mathcal{L}_{,XX}}{\mathcal{L}_{,X}}$$

$$\mathcal{L} \supset \frac{\epsilon}{c_s^2}\left(\dot{\zeta}^2 - c_s^2\frac{(\partial\zeta)^2}{a^2}\right)$$

Reduced 'speed of sound' of fluctuations ...

$$+\left(\frac{1-c_s^2}{H}\right)\dot{\zeta}\frac{(\partial\zeta)^2}{a^2}$$

... comes with non-trivial derivative interactions

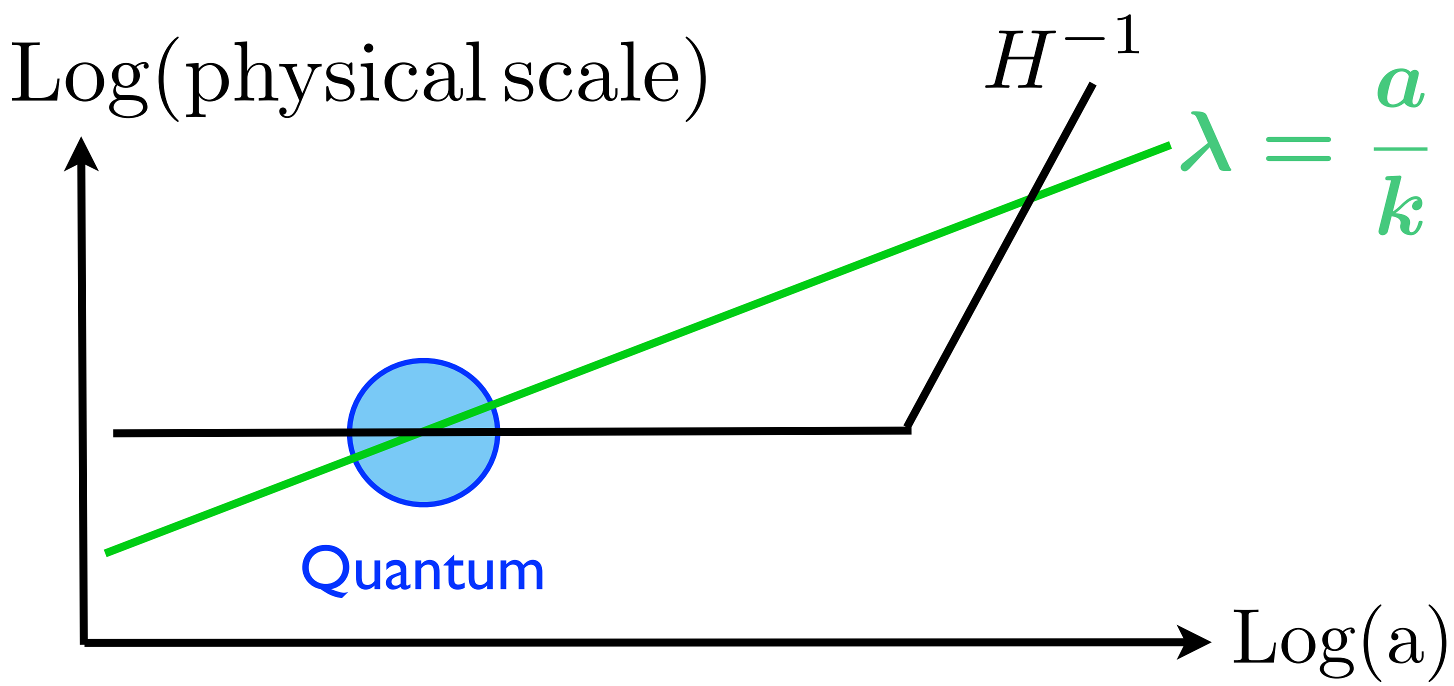
$$f_{NL}^{eq} \sim \frac{1}{c_s^2}$$

Origin of the bispectrum

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle = N_A N_B N_C \langle Q^A(\boldsymbol{k}_1) Q^B(\boldsymbol{k}_2) Q^C(\boldsymbol{k}_3)\rangle$$

Quantum NGs of the fields around Hubble crossing $k_1 \sim k_2 \sim k_3$

$$+\frac{1}{2} N_A N_B N_{CD} \langle Q^A(\boldsymbol{k}_1) Q^B(\boldsymbol{k}_2)(Q^C \star Q^D)(\boldsymbol{k}_3)\rangle + 2\,\text{perms.}$$



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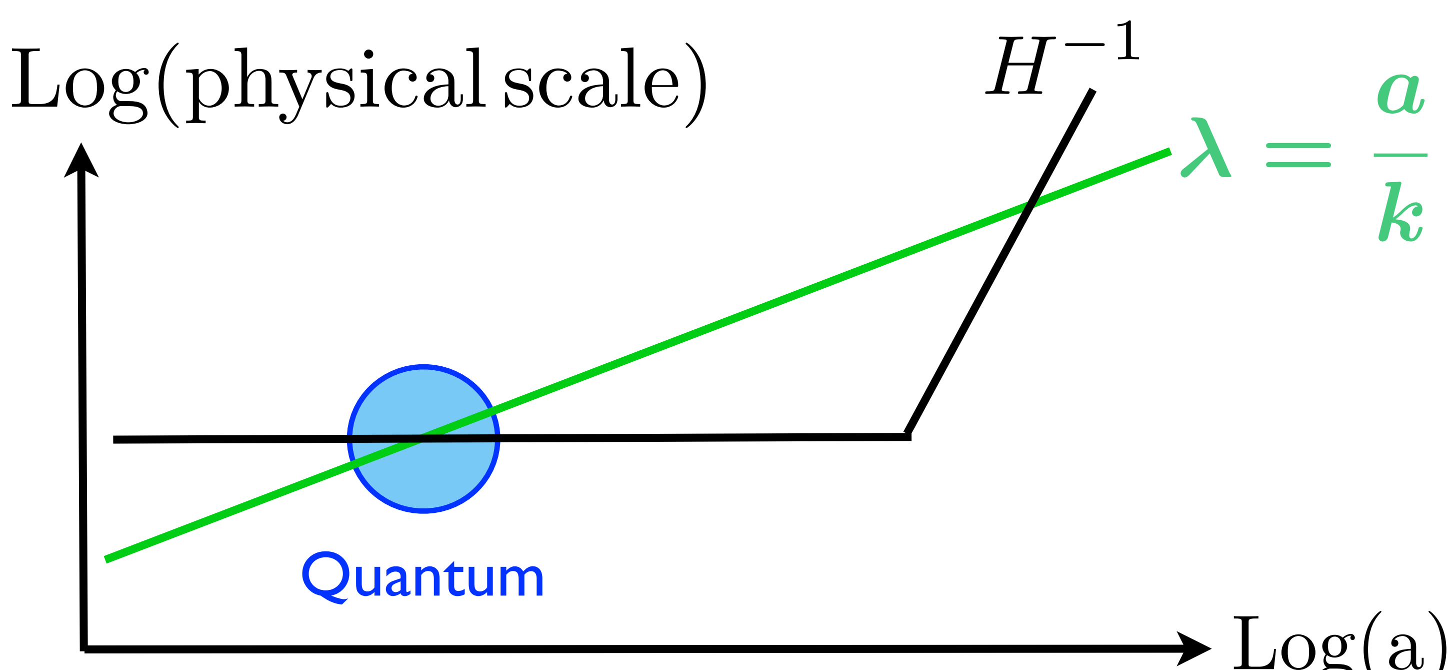
Non-zero even for Gaussian fields (Wick)



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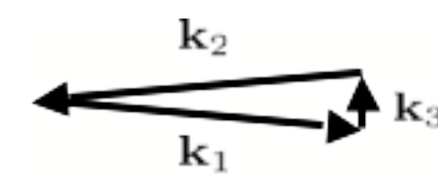
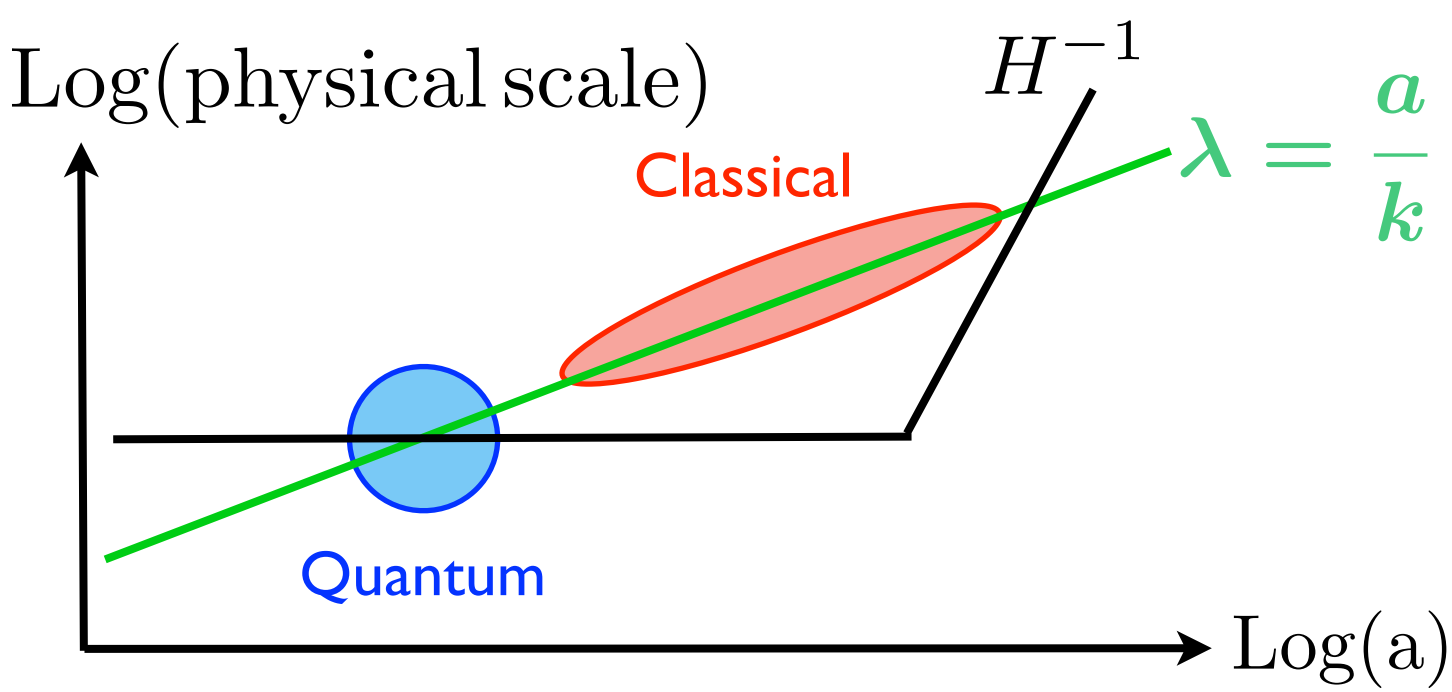
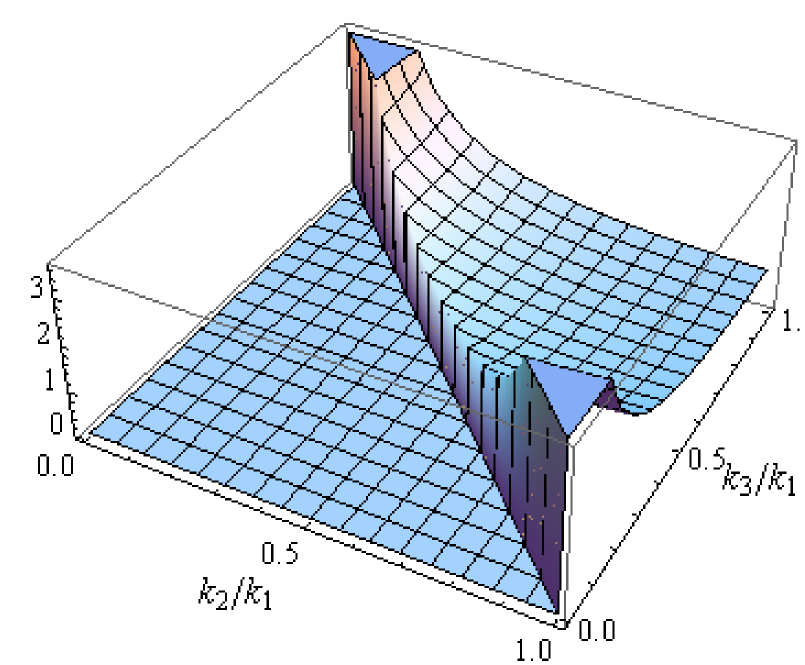
Super-Hubble nonlinear relation between zeta and the fields

$k_3 \ll k_1, k_2$



Local non-Gaussianities





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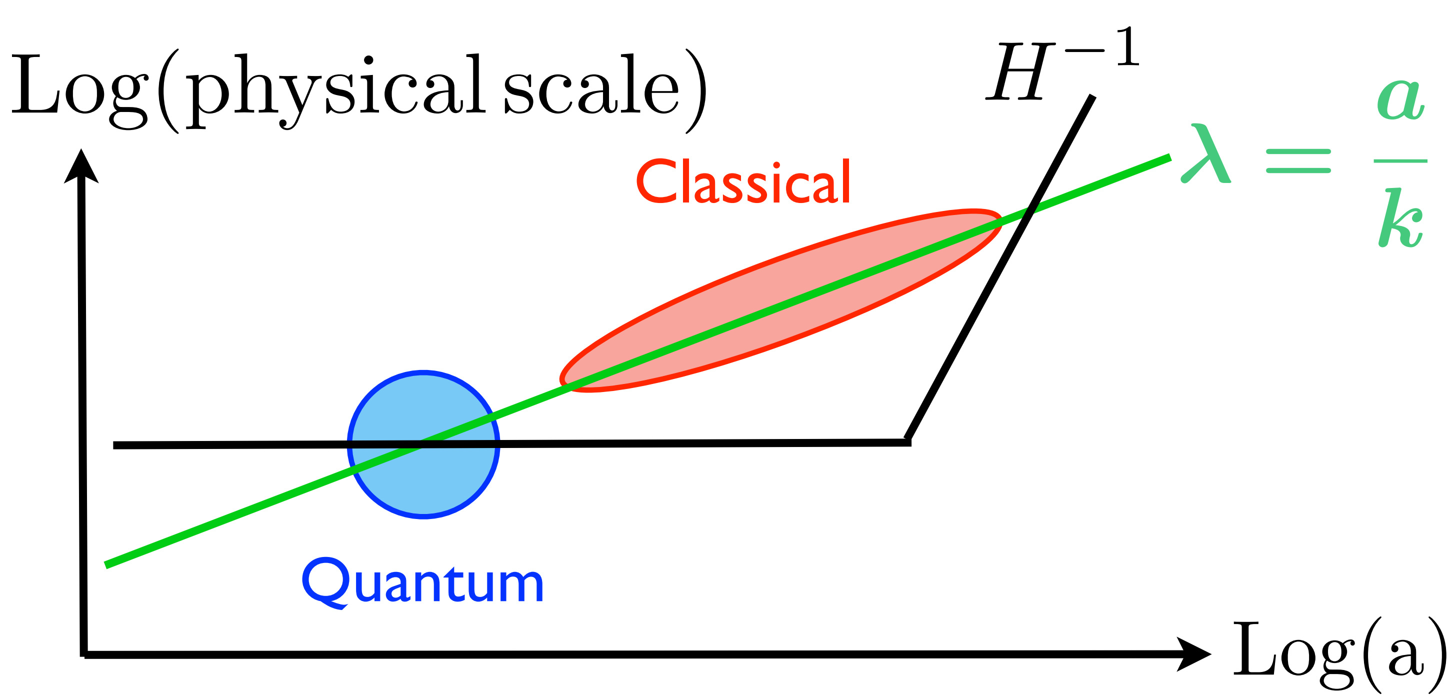
Because $\zeta = \mathrm{cte}$ on super-Hubble scales in single-field inflation, important only for multiple field models



$$f_{NL}^{loc} = \frac{5}{6} \frac{N_{AB} N^A N^B}{(N_C N^C)^2}$$

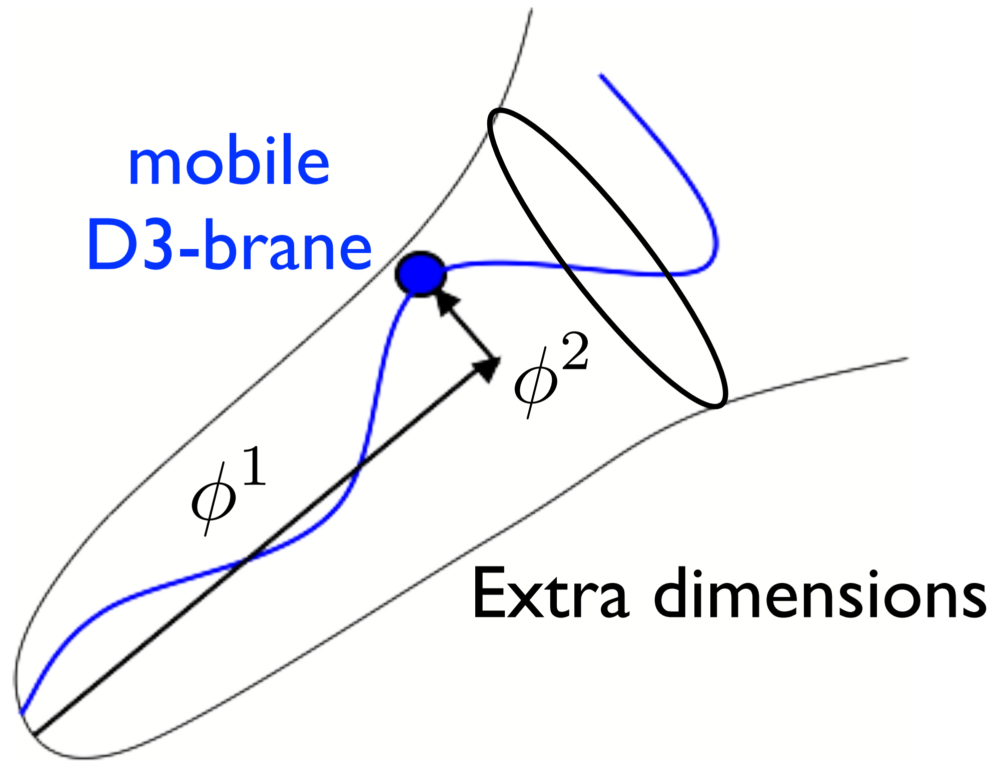
Example of multifield brane inflation

• Brane inflation: moving D3-brane in higher dimensions, non-standard kinetic terms

$\rightarrow \quad f_{NL}^{eq}$

• Inflaton: position of the brane, multifeld.

• Multifield effects reduce the amplitude of equilateral non-Gaussianities



D.Langlois, SRP, D.Steer, T.Tanaka, PRL 08

Super-Hubble non-linear evolution in a simple model $\rightarrow \quad f_{NL}^{loc}$

SRP, JCAP 09

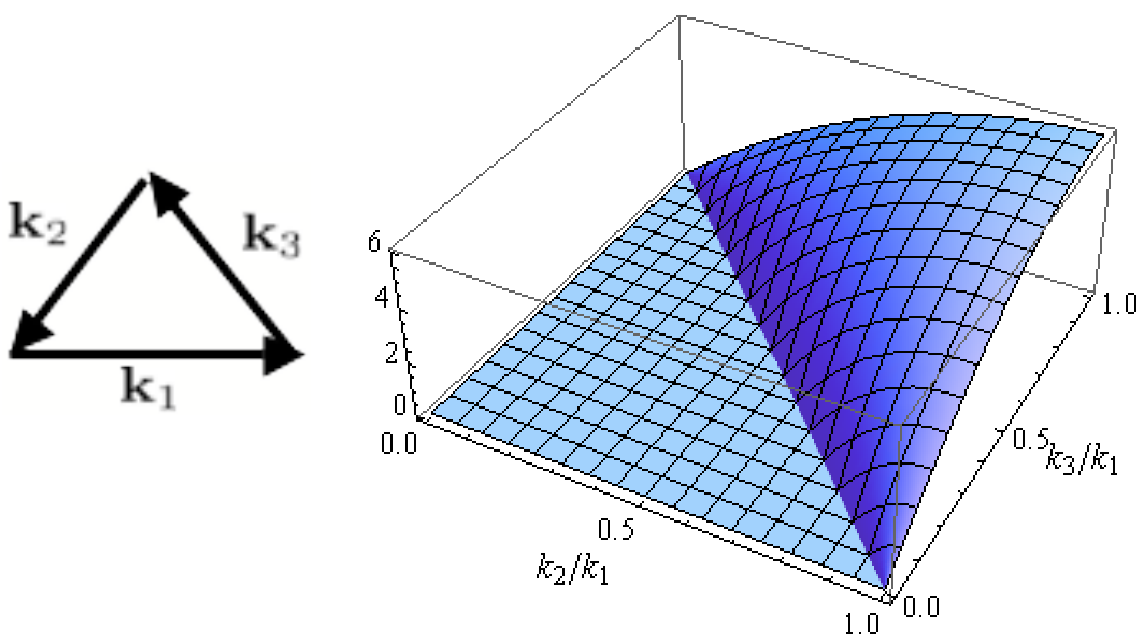
Combined local and equilateral non-Gaussianities

• Unique signature in the 4pf function: new shape with a consistency relation

$$s_{NL} = f_{NL}^{eq} f_{NL}^{loc}$$

Inflationary physics and shapes of non-Gaussianities

Equilateral type (quantum)

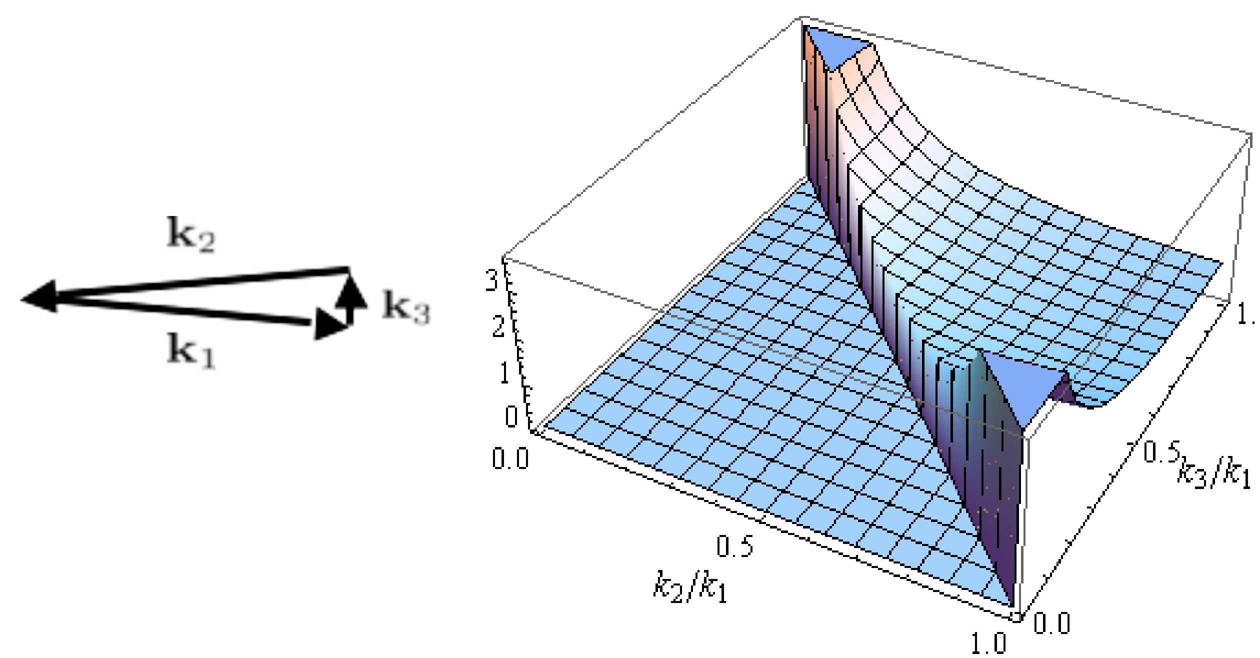


$$f_{NL}^{eq} = -42 \pm 75 \; (68\% \, CL)$$

Planck 13

Non-standard kinetic terms:
DBI, low sound speed models.

Local type (classical)



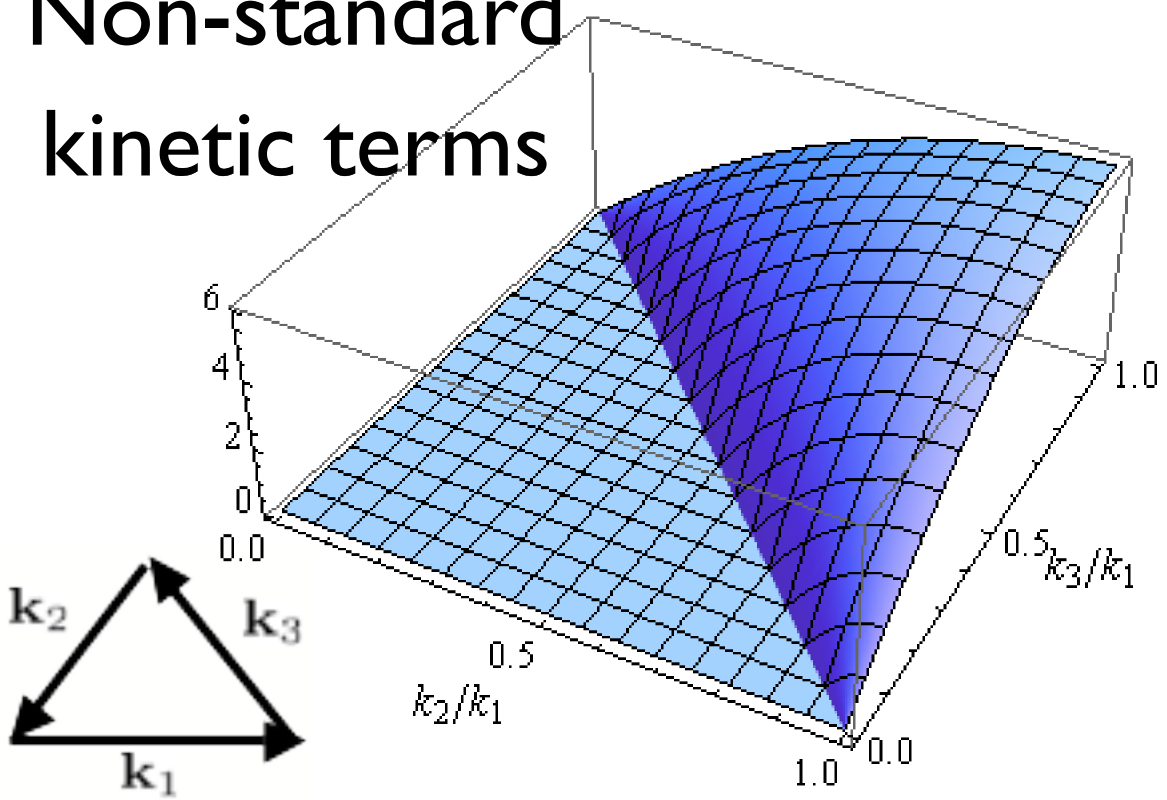
$$f_{NL}^{loc} = 2.7 \pm 5.8 \; (68\,\% \, CL)$$

Planck 13

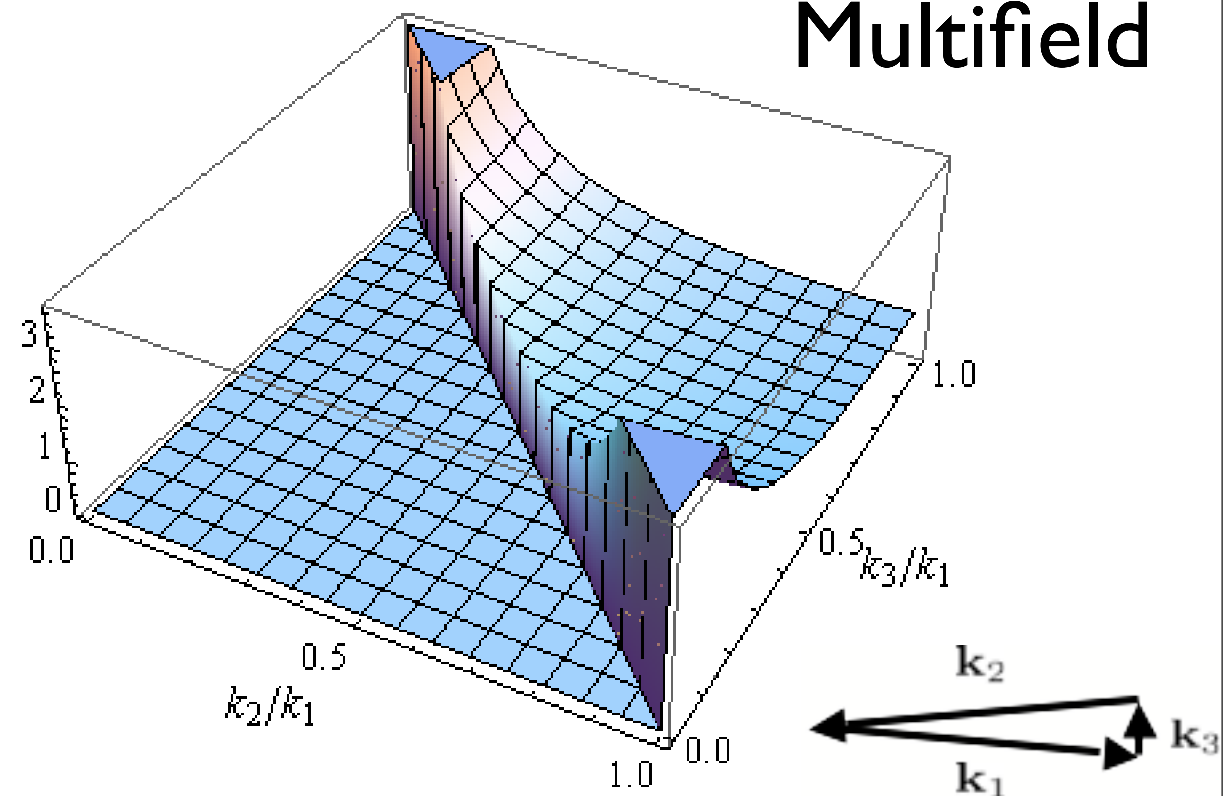
Multiple degrees of freedom:
Multified inflation, curvaton...

Inflationary physics and shapes of non-Gaussianities

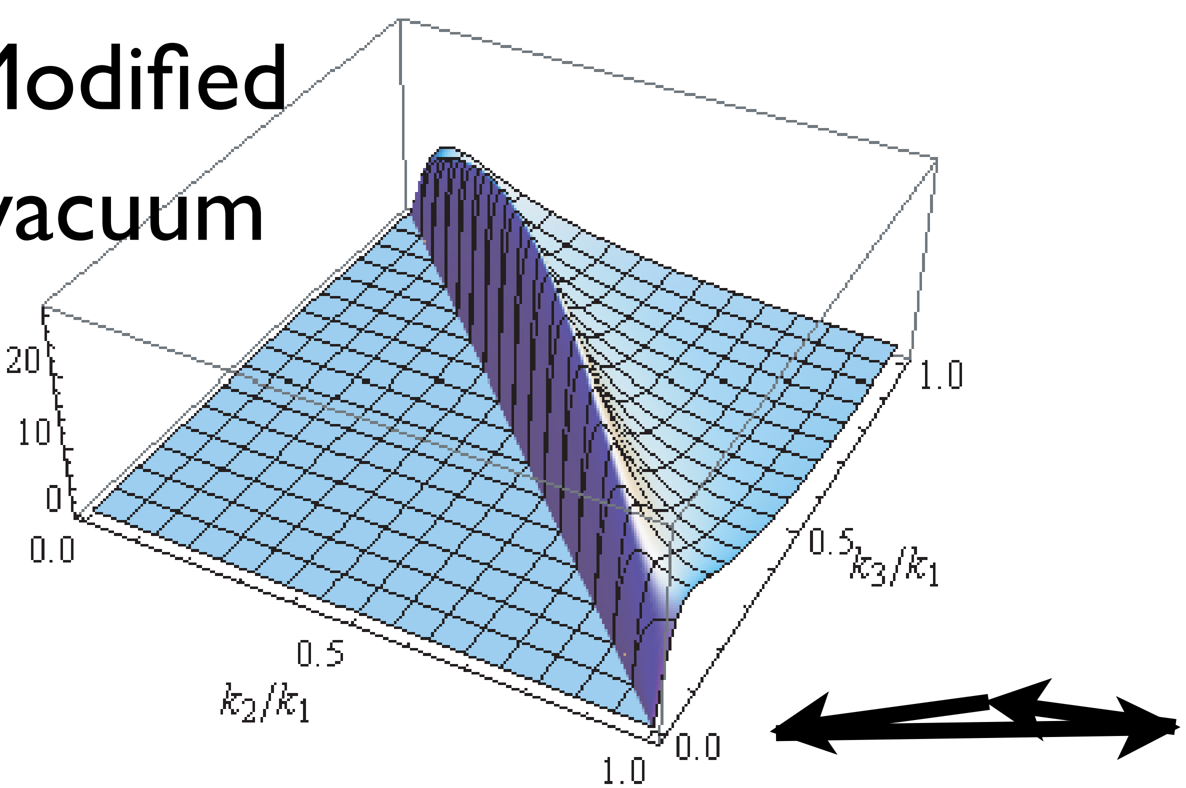
Non-standard kinetic terms



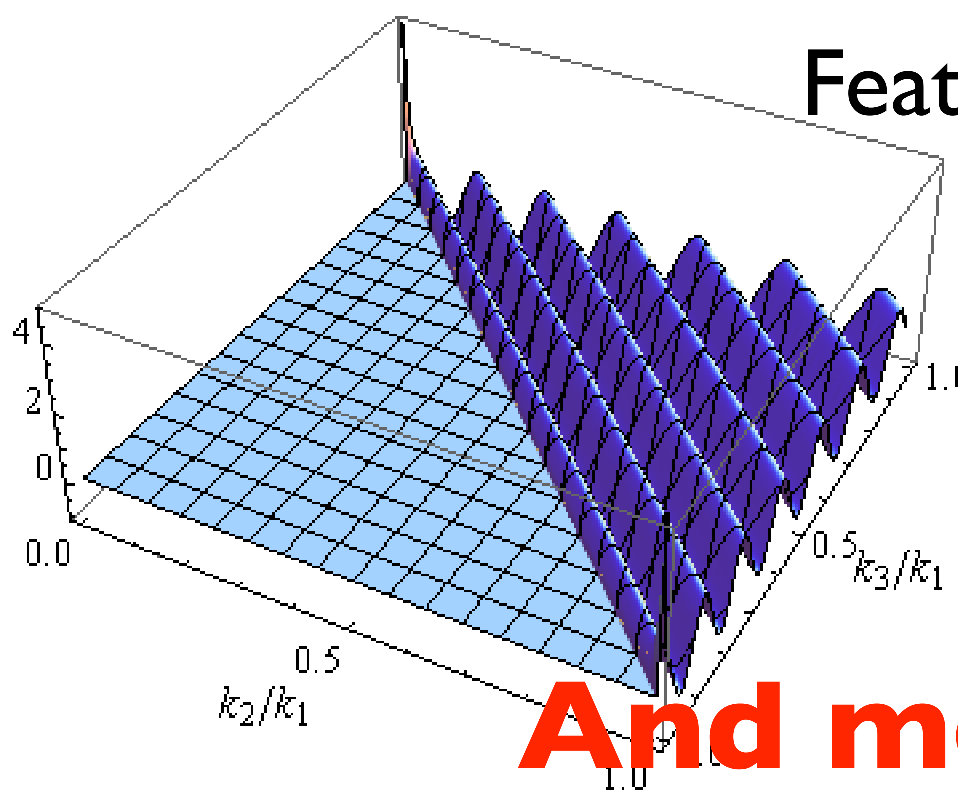
Multifield



Modified vacuum



Features



And more!

Single field consistency relation

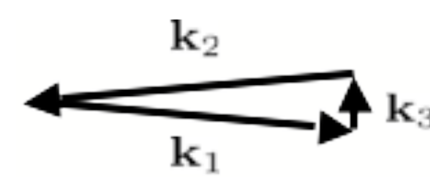
Any single-clock inflation (irrespective of kinetic terms, potential etc)

Maldacena (03), Creminelli & Zaldarriaga (04), SRP (10)

$$f_{NL}^{sq}(k_1) = \frac{5}{12}(1 - n_s(k_1))$$

with

$$f_{NL}^{sq}(k_1) \equiv \lim_{k_3 \to 0} f_{NL}(k_1, k_2, k_3)$$



With $n_s = 0.9603 \pm 0.0073 \ (68\%\mathrm{CL})$

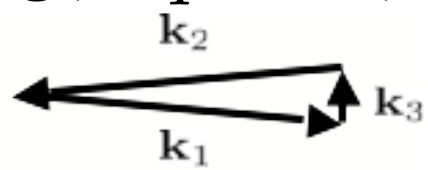
If $f_{NL}^{sq} \gtrsim 1$ of primordial origin is robustly detected,

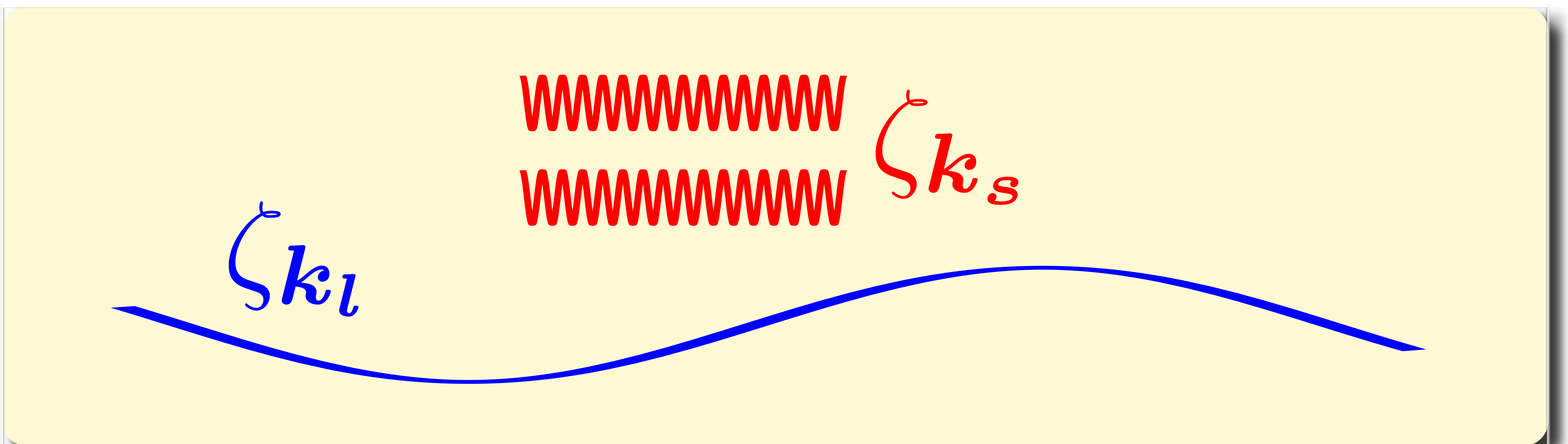
all single field models would be ruled out!

Understanding the theorem (I)

In the squeezed limit, one correlates one very long wavelength mode with two shorter wavelength modes

$$\langle \zeta_{\boldsymbol{k_1}} \zeta_{\boldsymbol{k_2}} \zeta_{\boldsymbol{k_3}} \rangle_{sq} \simeq \langle (\zeta_{\boldsymbol{k_s}})^2 \zeta_{\boldsymbol{k_l}} \rangle$$



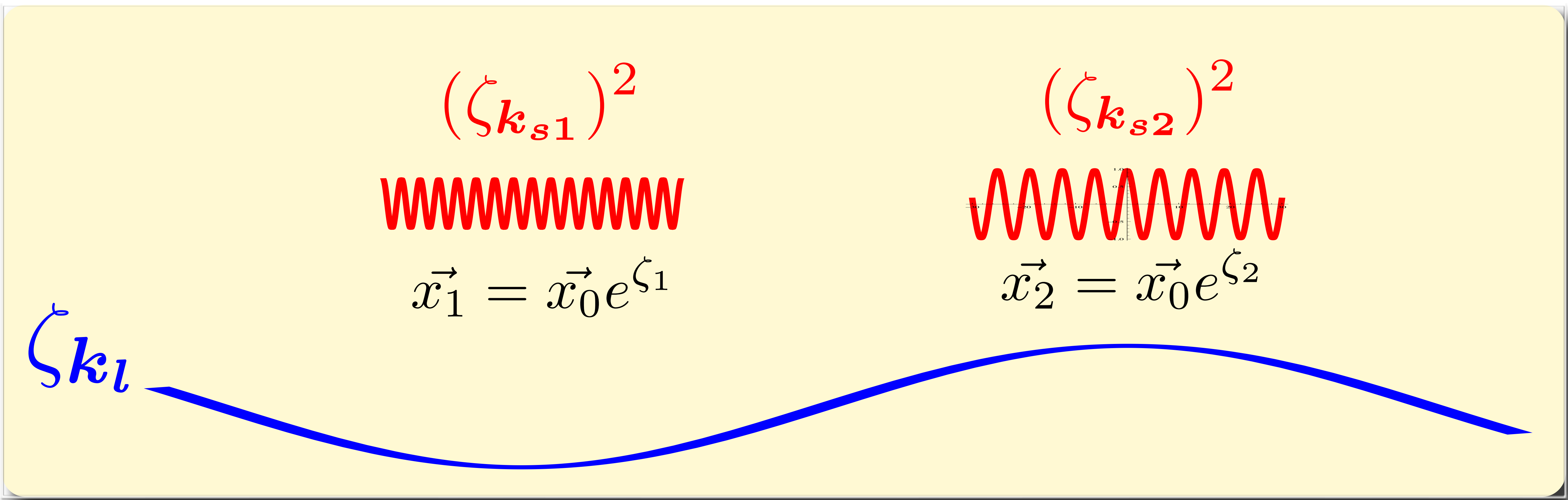


The theorem says $(\zeta_{\boldsymbol{k_s}})^2$ does not care about $\zeta_{\boldsymbol{k_l}}$ if $\zeta_{\boldsymbol{k}}$ is exactly scale-invariant.

Understanding the theorem (II)

A very long wavelength mode acts as a local rescaling of the spatial coordinates (equivalently, of the scale factor)

$$\mathrm{d}s^2 \simeq -\mathrm{d}t^2 + a(t)^2 e^{2\zeta_l} \mathrm{d}\vec{x}^{\,2}$$



The effect of the **long-wavelength modulation**

is proportional to $n_s - 1 = \dfrac{d \ln \mathcal{P}_\zeta(k)}{d \ln k}$

Computing fNL local (and beyond)

• Origin of local type non-Gaussianities is purely classical: non-linear evolution of perturbations on super-Hubble scales.

• Pure General Relativistic calculations (covariant formalism).

→

• Derivation of the exact super-Hubble equations of motion for gauge-invariant variables at second- and third-order in 2-field models! SRP, Tasinato JCAP 08, Lehners, SRP PRD 09

• Efficient numerical method. Alternative to the deltaN formalism.

Computing fNL local (and beyond): an idea...

$$\zeta^{(3)\prime} \approx \frac{2H}{\bar\sigma'^2}\left(\bar V_{,s}\delta s^{(3)} - \frac{1}{2\bar\sigma'}\bar V_{,\sigma}(\delta s \delta s^{(2)})' - \frac{\bar\theta'}{6\bar\sigma'^2}\bar V_{,\sigma}\delta s^2\delta s' + \bar V_{,ss}\delta s\delta s^{(2)} - \frac{1}{2\bar\sigma'}\bar V_{,s\sigma}\delta s^2\delta s' + \frac{1}{6}\bar V_{,sss}\delta s^3\right)$$
$$+\frac{8H}{\bar\sigma'^4}\left(\bar V_{,s}^2\delta s\delta s^{(2)} - \frac{1}{2\bar\sigma'}\bar V_{,s}\bar V_{,\sigma}\delta s^2\delta s' + \frac{1}{2}\bar V_{,s}\bar V_{,ss}\delta s^3\right) + \frac{8H}{\bar\sigma'^6}\bar V_{,s}^3\delta s^3 .$$

$$\delta s^{(3)\prime\prime} + 3H\delta s^{(3)\prime} + (\bar V_{,ss} + 3\bar\theta'^2)\delta s^{(3)} + 2\frac{\bar\theta'}{\bar\sigma'}\delta s^{(2)\prime}\delta s'$$
$$+\left(2\frac{\bar\theta''}{\bar\sigma'} + 2\frac{\bar\theta'\bar V_{,\sigma}}{\bar\sigma'^2} - 3H\frac{\bar\theta'}{\bar\sigma'}\right)(\delta s^{(2)}\delta s)' + \left(\bar V_{,sss} - 10\frac{\bar\theta'\bar V_{,ss}}{\bar\sigma'} - 18\frac{\bar\theta'^3}{\bar\sigma'}\right)\delta s^{(2)}\delta s$$
$$+\frac{\bar V_{,\sigma}}{\bar\sigma'^3}\delta s'^3 + \left(\frac{\bar V_{,\sigma\sigma}}{\bar\sigma'^2} + 3\frac{\bar V_{,\sigma}^2}{\bar\sigma'^4} + 3H\frac{\bar V_{,\sigma}}{\bar\sigma'^3} - 2\frac{\bar V_{,ss}}{\bar\sigma'^2} - 6\frac{\bar\theta'^2}{\bar\sigma'^2}\right)\delta s'^2\delta s$$
$$+\left(-10\frac{\bar\theta'\bar\theta''}{\bar\sigma'^2} - \frac{3}{2\bar\sigma'}\bar V_{,ss\sigma} - 5\frac{\bar V_{,\sigma}\bar V_{,ss}}{\bar\sigma'^3} - 7\frac{\bar\theta'^2\bar V_{,\sigma}}{\bar\sigma'^3} - 3H\frac{\bar V_{,ss}}{\bar\sigma'^2} + 14H\frac{\bar\theta'^2}{\bar\sigma'^2}\right)\delta s'\delta s^2$$
$$+\left(\frac{1}{6}\bar V_{,ssss} - \frac{7}{3}\frac{\bar\theta'}{\bar\sigma'}\bar V_{,sss} + 2\frac{\bar V_{,ss}^2}{\bar\sigma'^2} + 21\frac{\bar\theta'^2\bar V_{,ss}}{\bar\sigma'^2} + 27\frac{\bar\theta'^4}{\bar\sigma'^2}\right)\delta s^3 = 0 .$$

Lehners, SRP PRD 09

Planck implications

$f_{NL}^{loc} = 2.7 \pm 5.8$ → Constrain multi-field effects

$f_{NL}^{eq} = -42 \pm 75$

$f_{NL}^{orth} = -25 \pm 39$

→ Lower bound on the inflaton speed of sound

$c_s \geq 0.02 \, (95\% \, \mathrm{CL})$

Strong constraints on light hidden sector fields coupled to the inflaton via operators suppressed by a high mass scale.

$\Lambda > 10^5 H$ $\Lambda > 10^2 H$ Assassi et al, 2013.

depending on assumptions on the hidden sector

Outline

1. Description of inflation

2. Beyond the simplest models

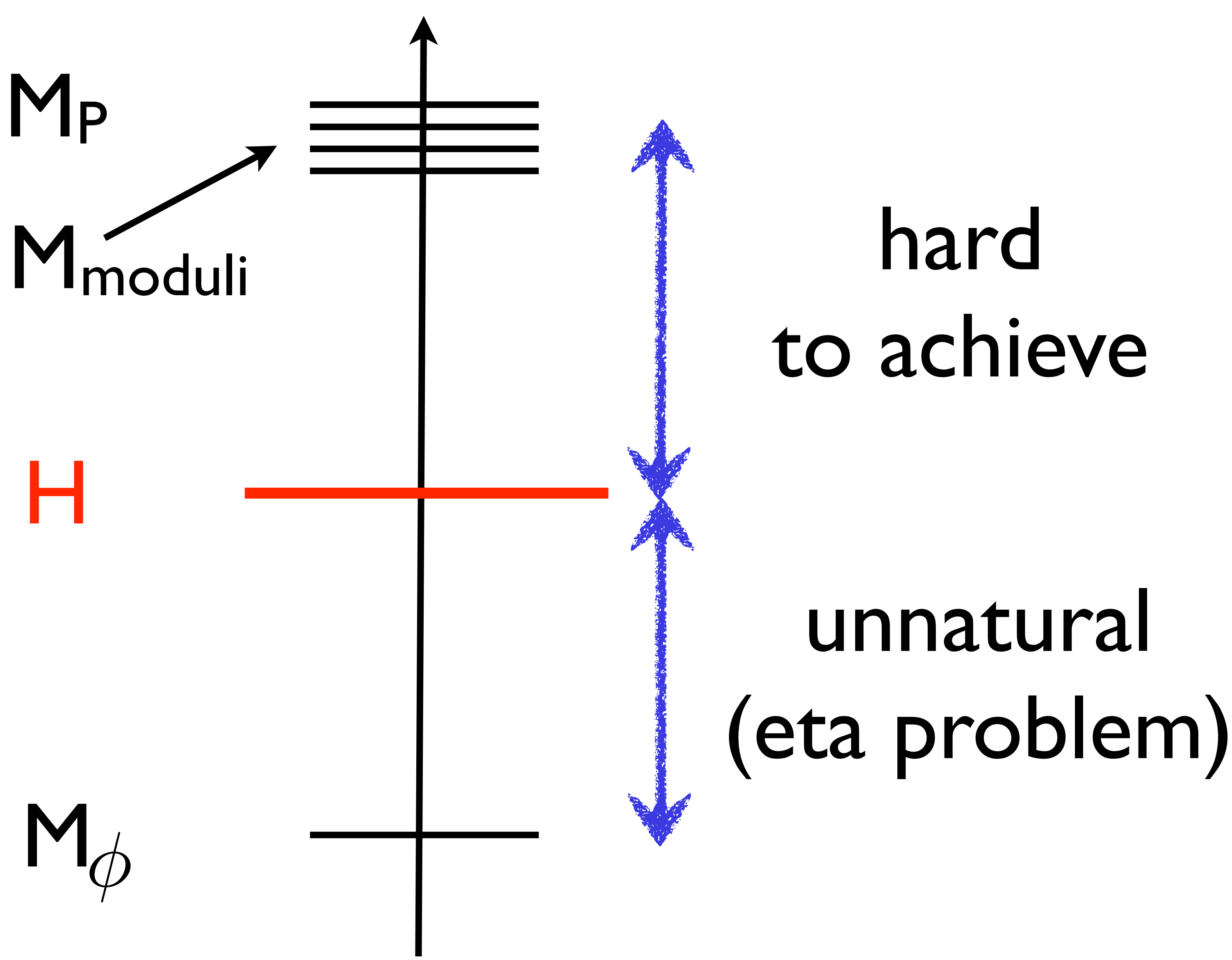
3. Primordial non-Gaussianities

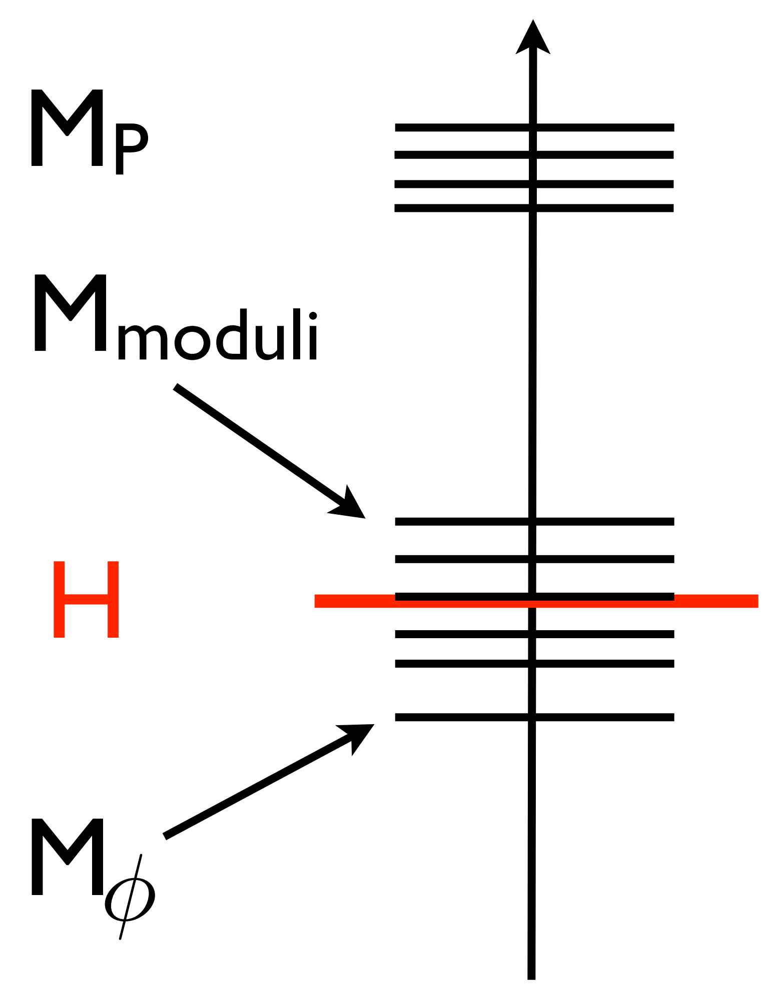
4. Quasi-single-field inflation

Mass scales in realistic set-up

<u>Hope</u>: light inflaton, Planck-mass moduli

<u>Find</u>: many masses of order H





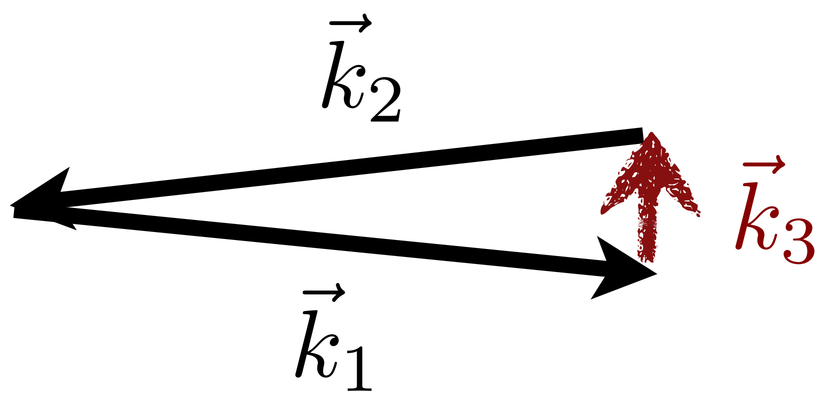
Quasi-single-field

Chen, Wang 09

Non-Gaussianity as a particle detector with the soft limits

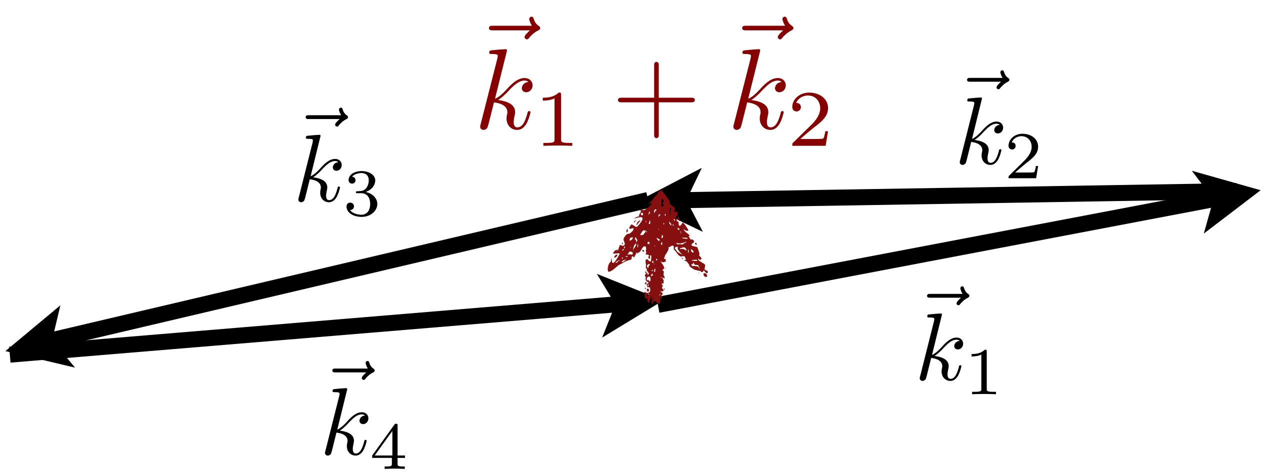
- Squeezed limit of the bispectrum:

cf soft limits in QCD



$$\hat{f}_{NL} \equiv \frac{5}{12} \lim_{k_3 \to 0} \frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle'}{P_2 P_3}$$

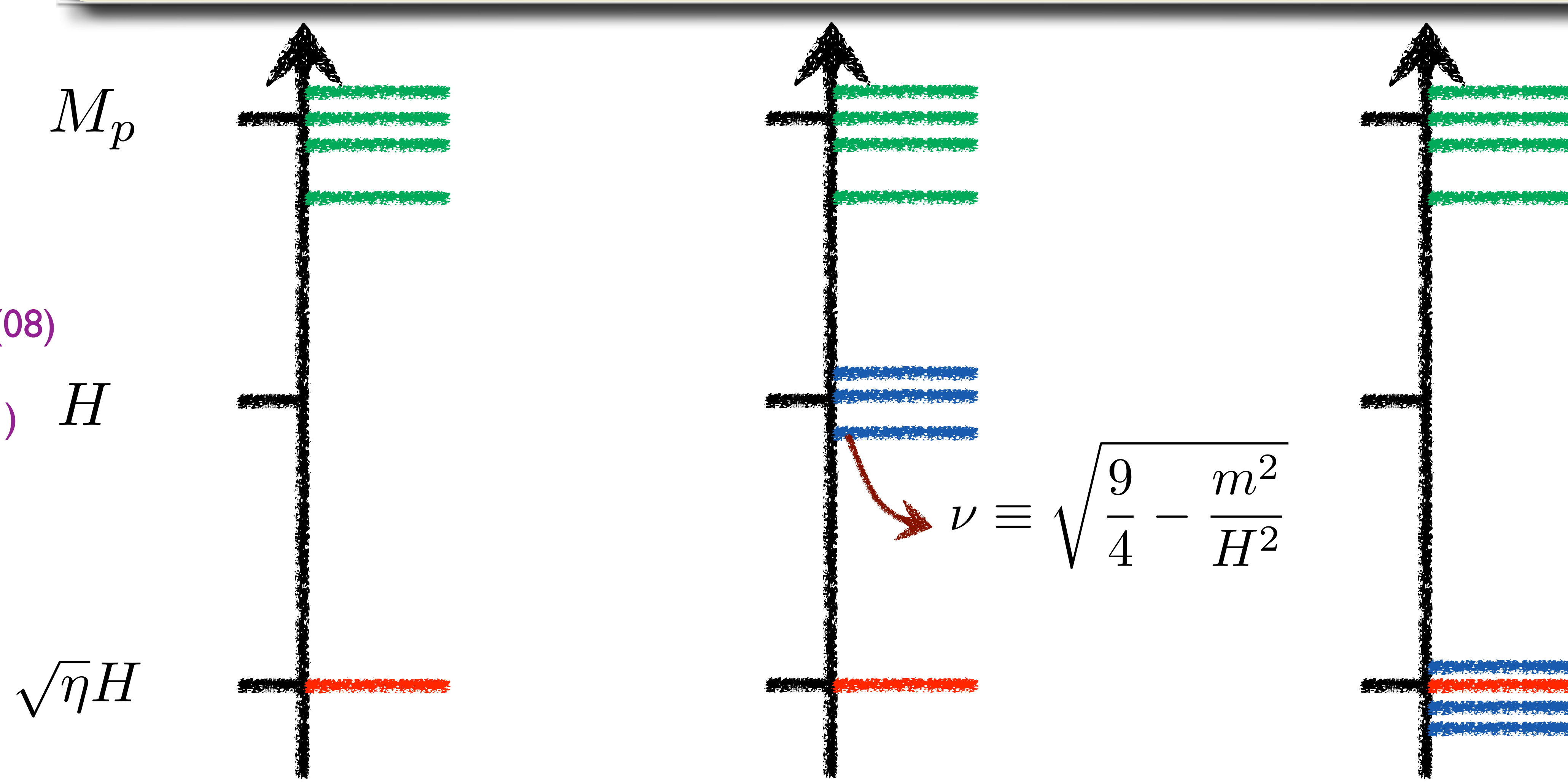
- Collapsed limit of the trispectrum:



$$\hat{\tau}_{NL} \equiv \frac{1}{4} \lim_{k_{12} \to 0} \frac{\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle'}{P_1 P_3 P_{12}}$$

Non-Gaussianity as a particle detector

	Single-field inflation	Quasi-single-field inflation	Multi-field inflation



Suyama-Yamaguchi (08)
Chen-Wang (09)
Baumann-Green (11)
Assassi et al (12)

$\lim_{k\to 0} k^3 \langle \zeta_{\vec{k}} \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle$	k^2	$k^{\frac{3}{2}-\nu}$	k^0
$\hat{\tau}_{NL}$	$\left(\frac{6}{5}\hat{f}_{NL}\right)^2$	$\gg \left(\frac{6}{5}\hat{f}_{NL}\right)^2$	$\geq \left(\frac{6}{5}\hat{f}_{NL}\right)^2$

Random potentials from Planck-suppressed interactions

• High energy physics motivates considering many fields of intermediate masses governed by a complicated potential induced by Planck-suppressed couplings:

$$V(\phi_1, ..., \phi_N) = \sum_{J=1}^{\infty} c^{(J)}_{i1...i_J} \frac{\phi^{i_1} ... \phi^{i_J}}{\Lambda^{J-4}}$$

• The form of the potential can be computed with some effort, but computing the coefficients is hopeless in general.

• Key question: when inflation arises in this context, what are its characteristic properties? **What universal properties can we learn without knowing the details of the Wilson coefficients?**

(motivation/analogy: Random Matrix Theory)

Quasi-single-field inflation

• I studied in detail the first microphysical realizations of quasi-single-field inflation (and in EFT framework). McAllister, S.RP, Xu, JCAP, 12

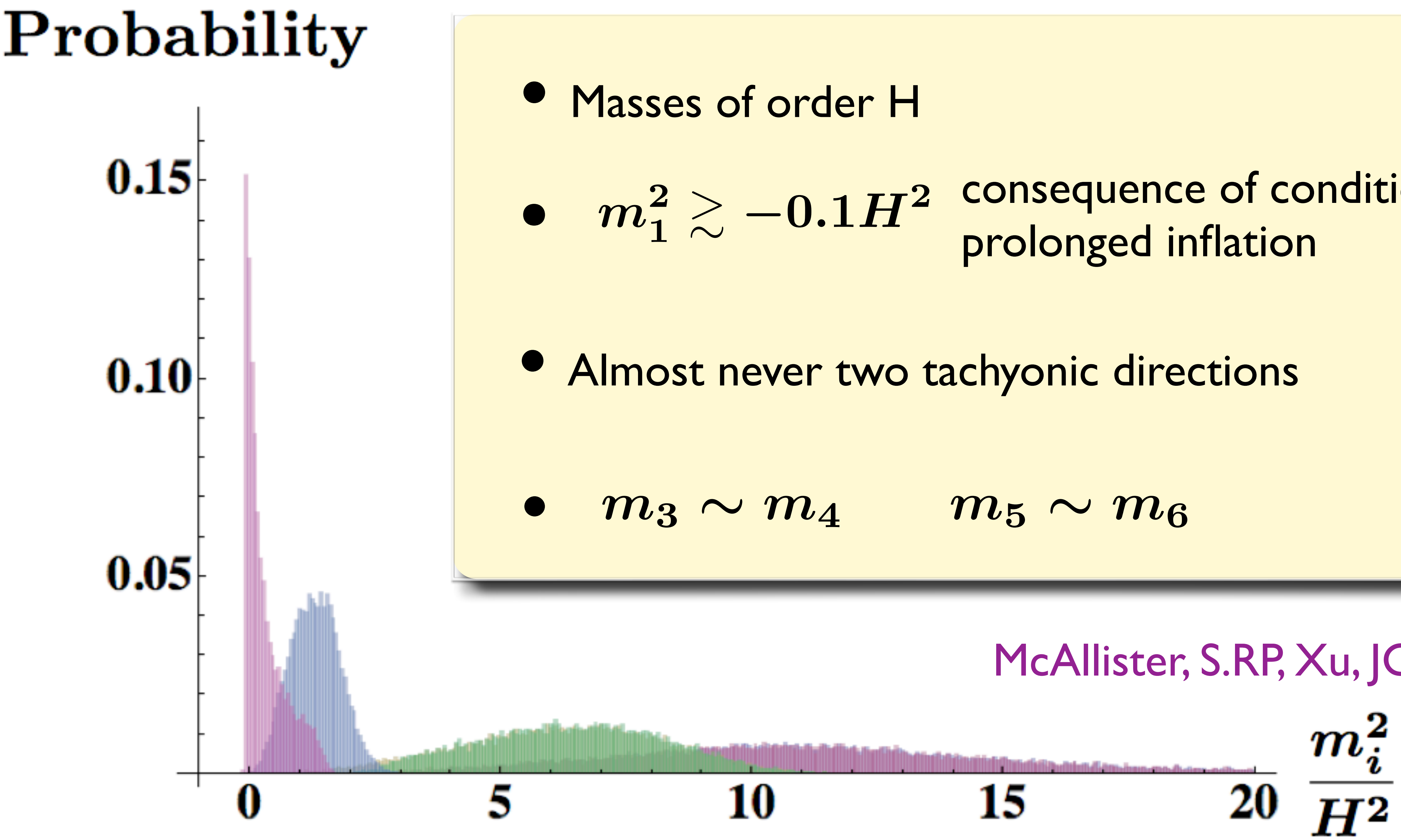
• Precise set-up is warped D-brane inflation (6 fields) but **methods and findings have much broader applicability**:

- statistical study of a large ensemble of potentials
- mass spectrum predicted by Random Matrix Theory
- reveal physics by comparing exact numerical results and truncated models of the perturbations.

• Rich phenomenology is natural (not put by hand): slow-roll violation, bending trajectories and 'many-field' effects are commonplace. New unexpected effects.

Mass spectrum and random matrix theory

$m_1^2 \leq \ldots \leq m_6^2 =$ eigenvalues of the mass matrix at Hubble crossing



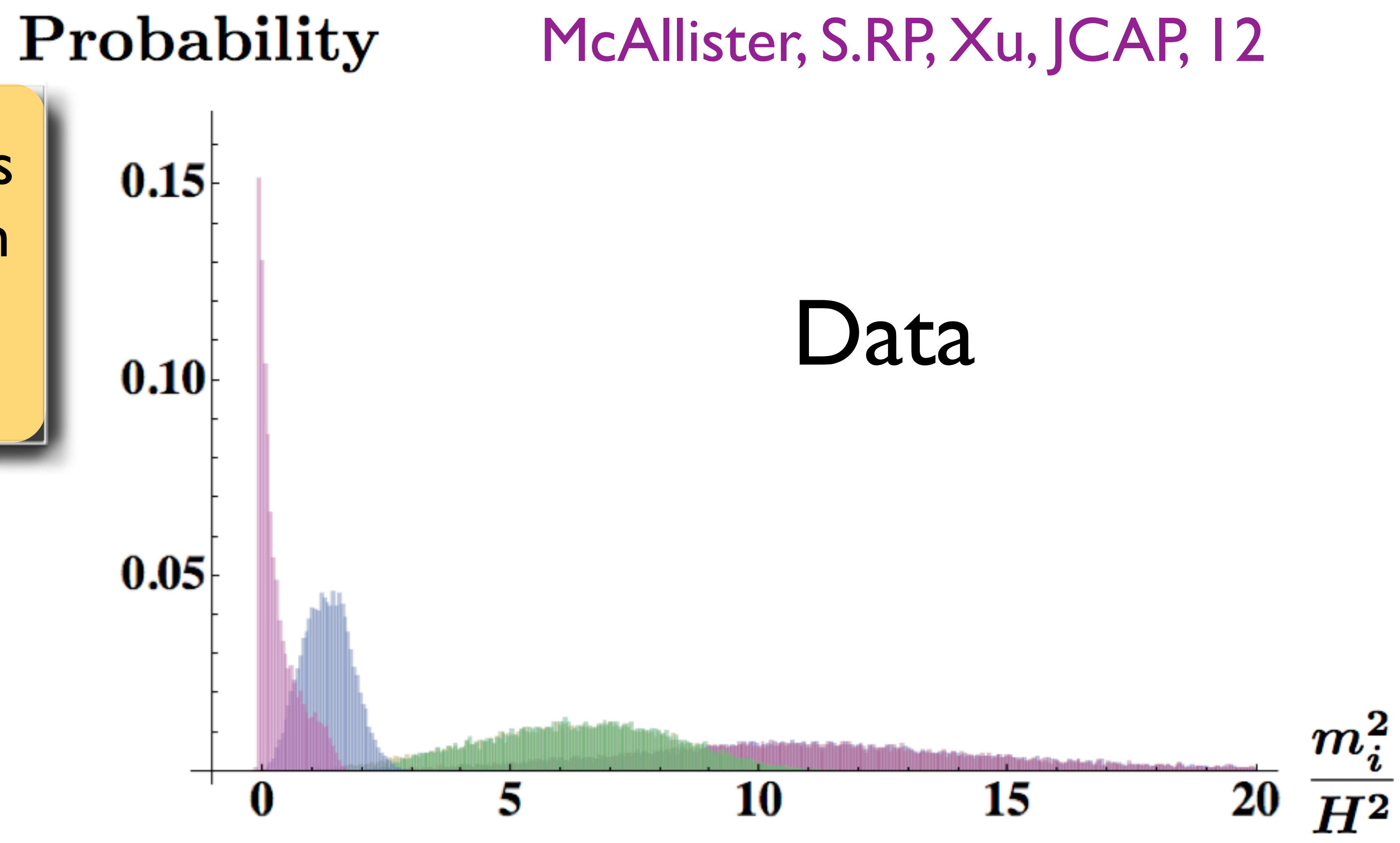
- Masses of order H
- $m_1^2 \gtrsim -0.1 H^2$ consequence of conditioning on prolonged inflation
- Almost never two tachyonic directions
- $m_3 \sim m_4 \qquad m_5 \sim m_6$

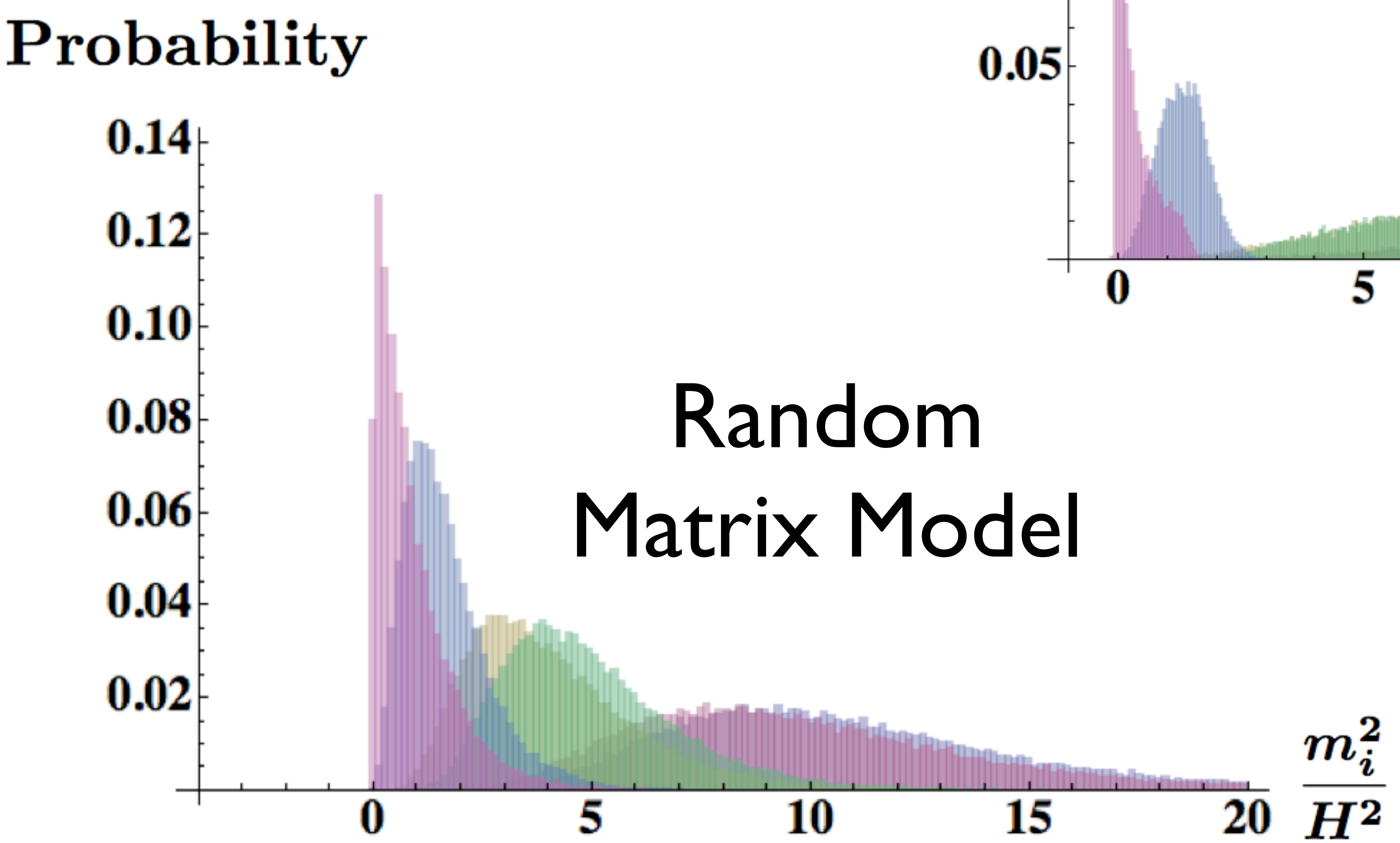
McAllister, S.RP, Xu, JCAP, 12

Mass spectrum and random matrix theory

McAllister, S.RP, Xu, JCAP, 12

Qualitative features of the mass spectrum can be reproduced in a random matrix model of supergravity



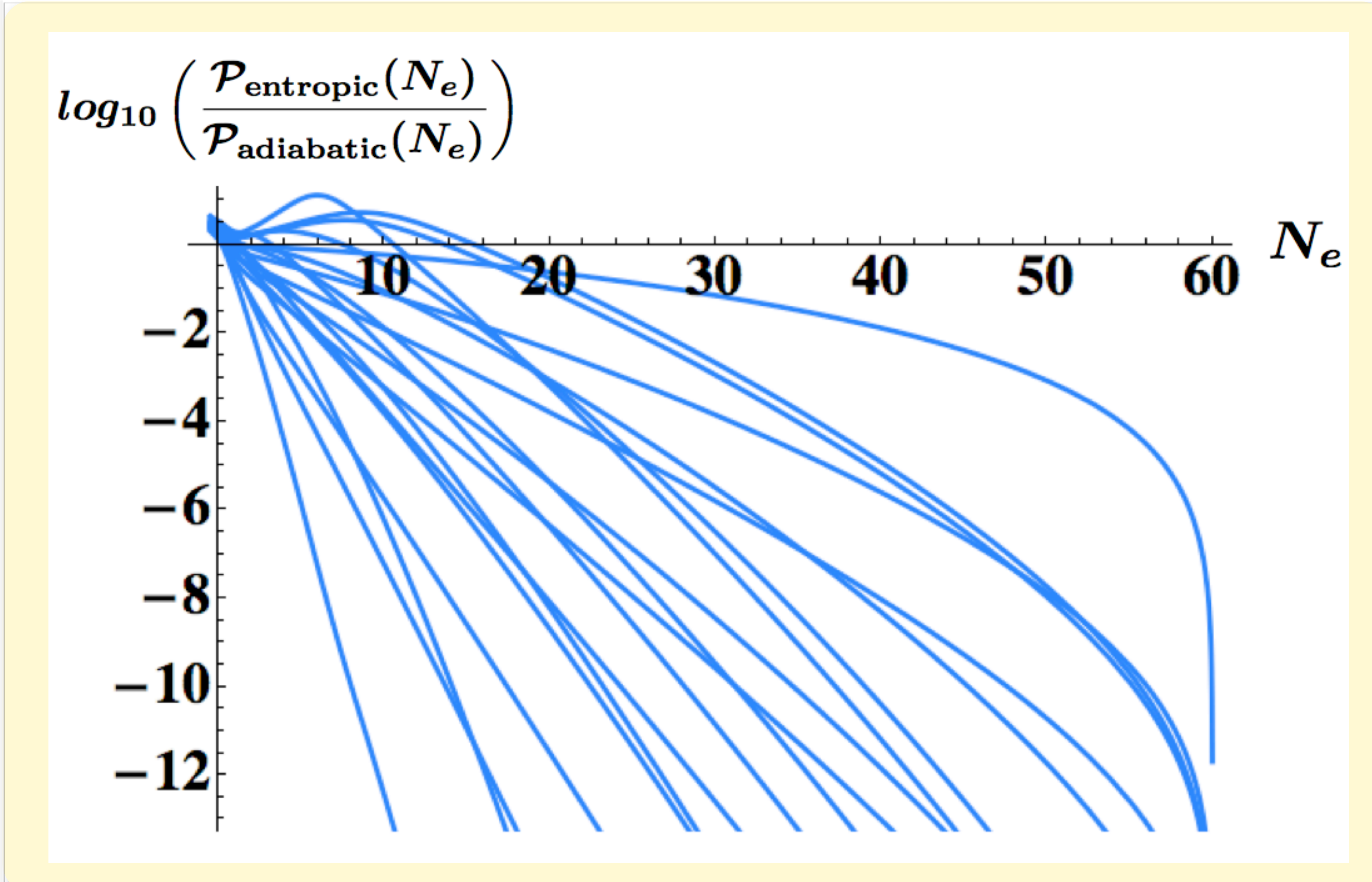


$$\mathcal{H} = \begin{pmatrix} A\bar{A} + B\bar{B} & C \\ \bar{C} & \bar{A}A + \bar{B}B \end{pmatrix}$$

Marsh, McAllister, Wrase, 2011

Reaching the adiabatic limit

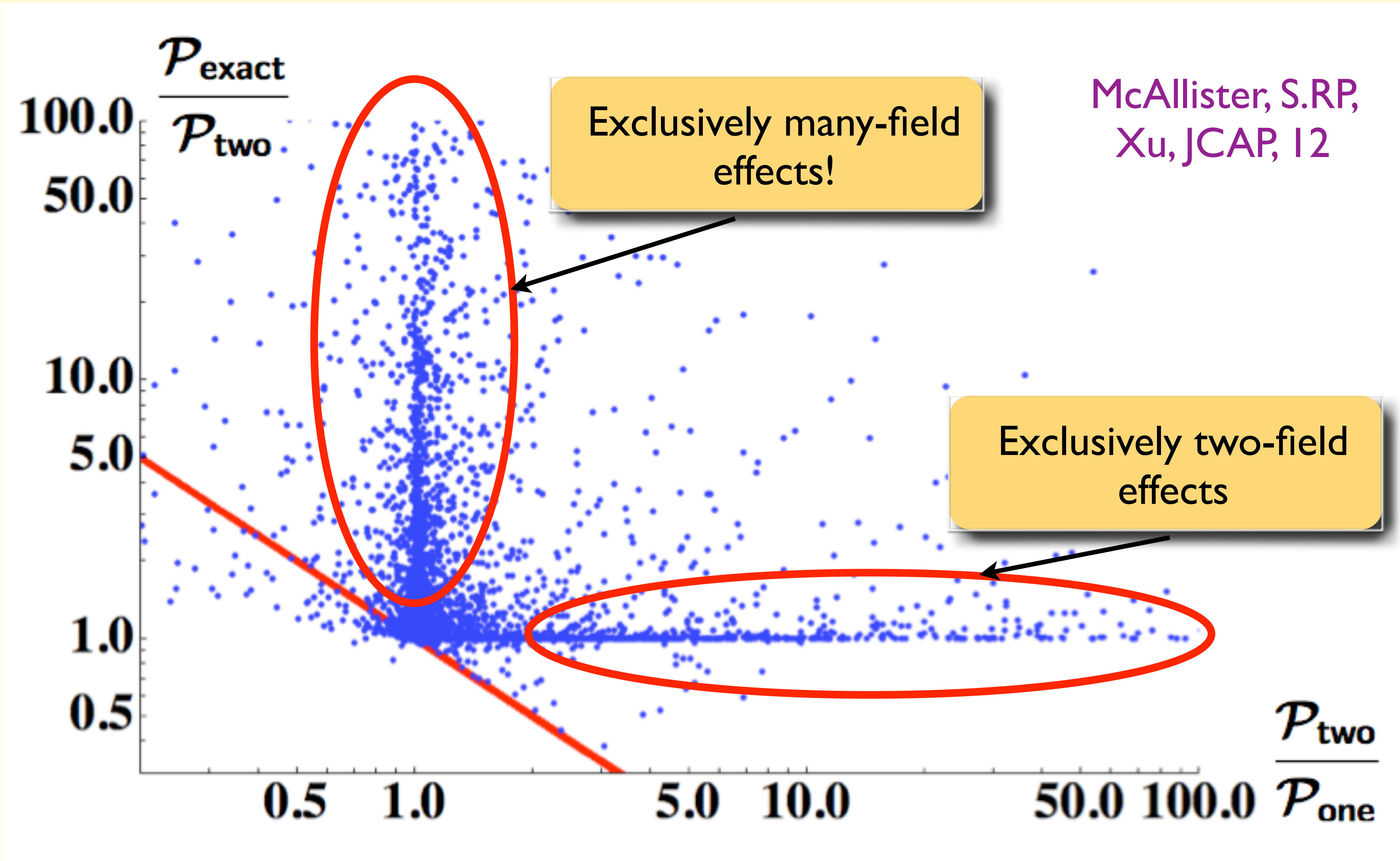
Masses of order H → Suppression of the entropic modes by the end of inflation



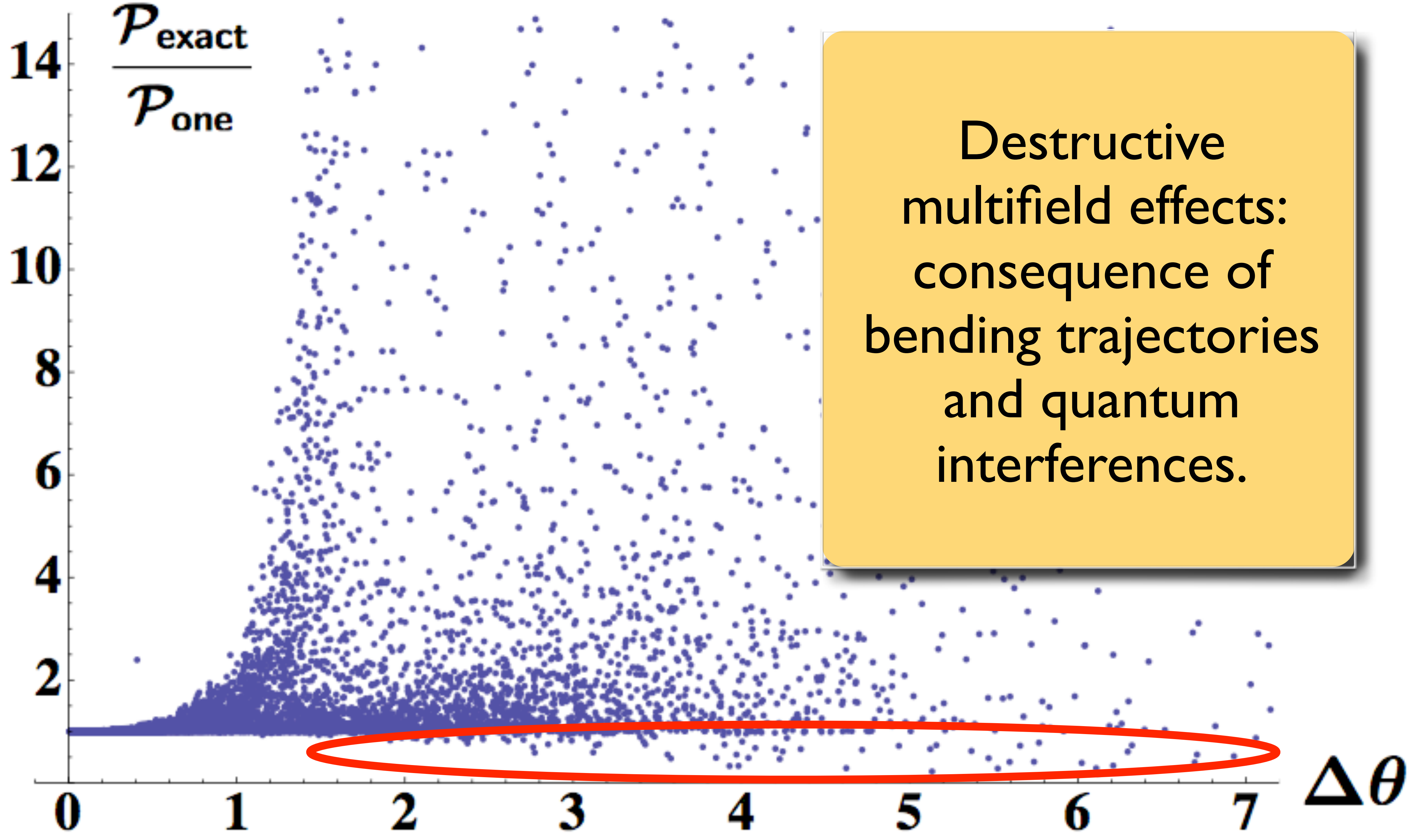
McAllister, S.RP, Xu, JCAP, 12

Multifield effects and definite predictions without a description of (p)reheating

Two-field versus many-field



Destructive multifield effects



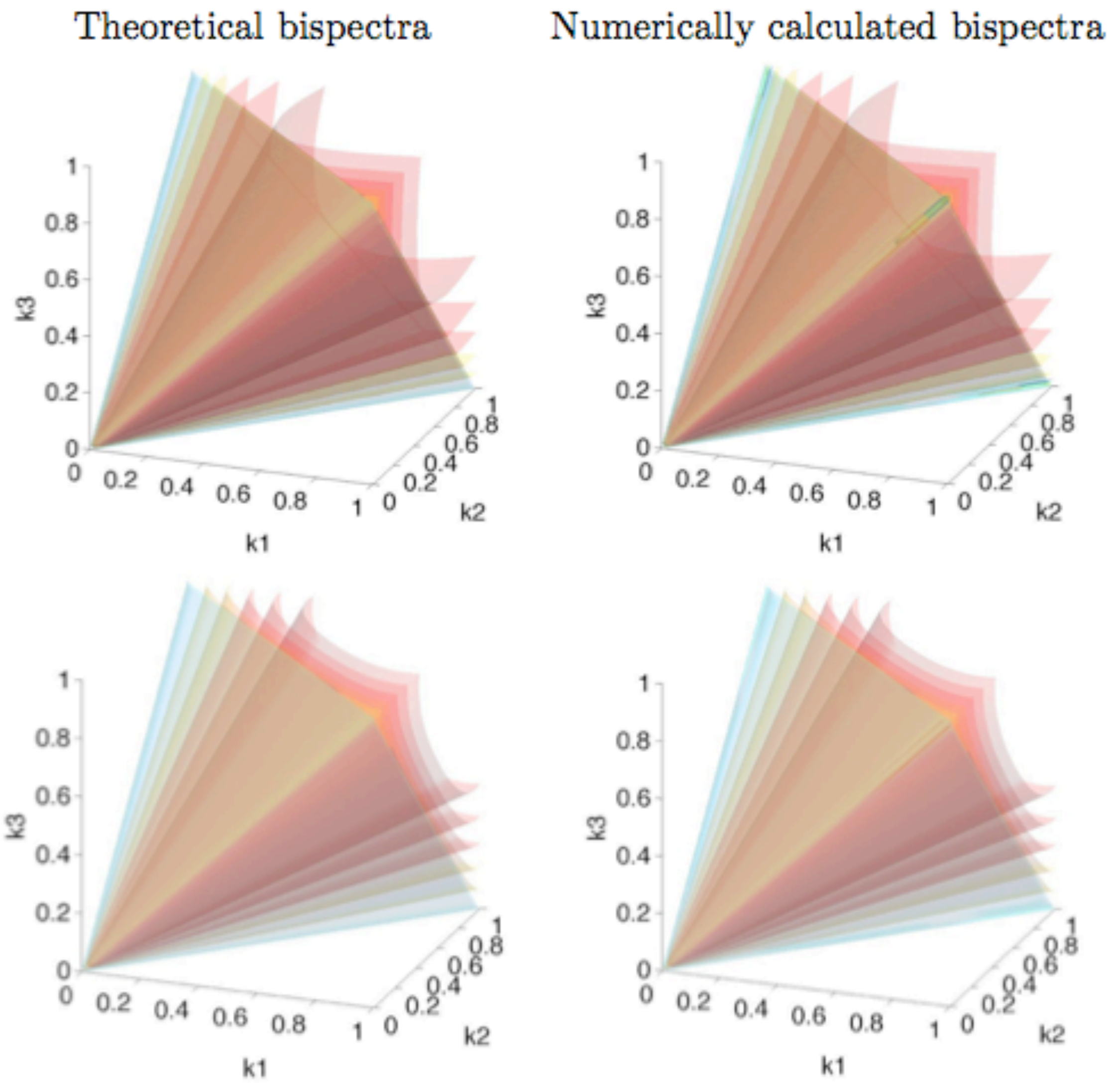
Destructive multifield effects: consequence of bending trajectories and quantum interferences.

Numerical calculation of non-Gaussianities

Efficient numerical method to calculate primordial non-Gaussianities based on spectral methods. Predictions in form ready for data-analysis.

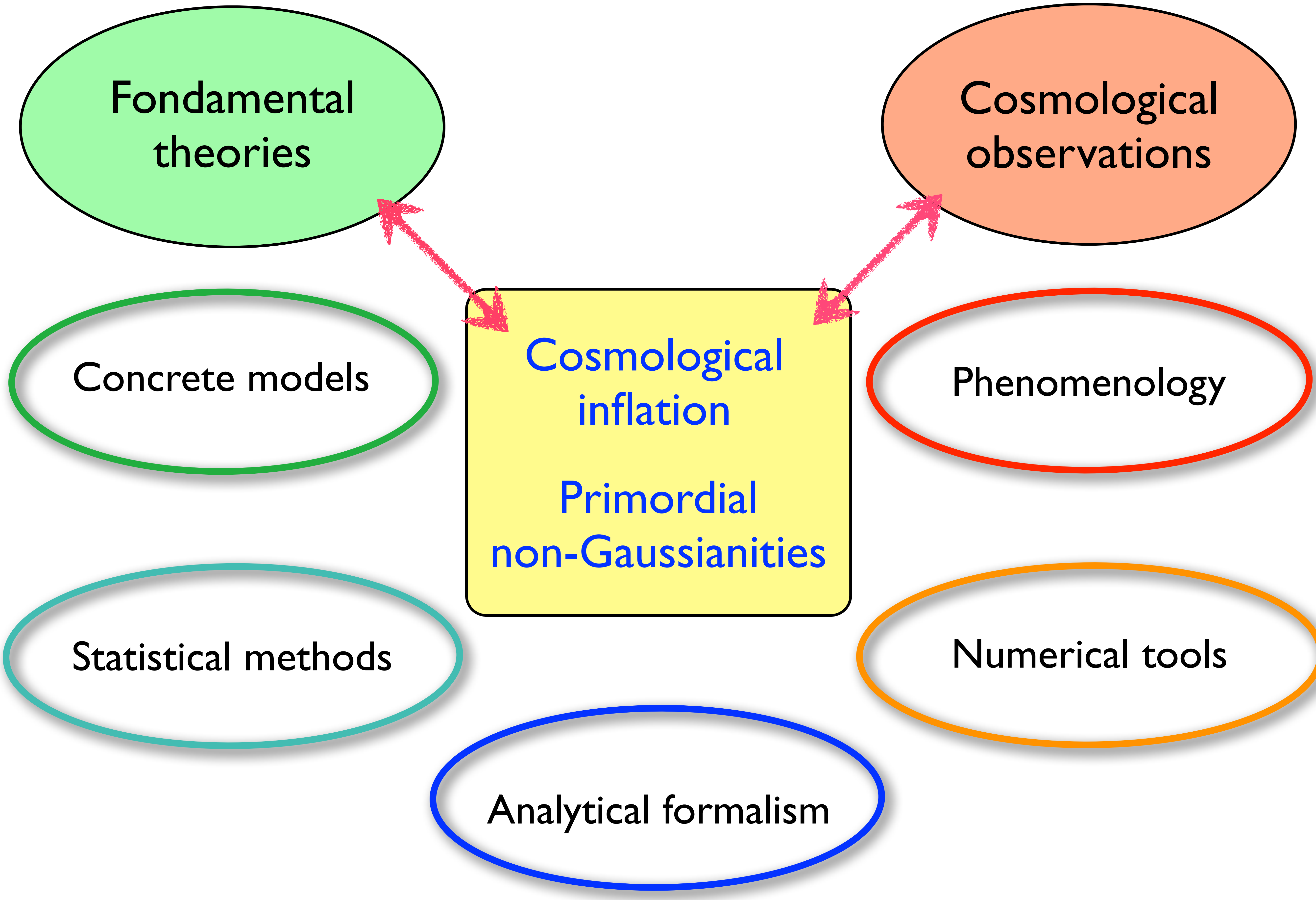
H. Funakoshi, S. RP JCAP 12

operator	correlation ($m^2\phi^2$)	correlation (DBI)
$\dot{\zeta}^3$	0.9992	0.9994
$\dot{\zeta}(\partial\zeta)^2$	0.99997	0.99995
$\zeta\dot{\zeta}^2$	0.999994	0.999990
$\zeta(\partial\zeta)^2$	0.999998	0.999995
$\dot{\zeta}\partial_i\zeta\partial^i(\partial^{-2}\dot{\zeta})$	0.99998	0.99997
$\partial^2\zeta(\partial_i\partial^{-2}\dot{\zeta})^2$	0.999990	0.99998



Other subjects I have worked on I could have developped...

- Orthogonal non-Gaussianities
- Trispectrum
- Effective Field Theory of inflation
- Inflation and modified gravity


Fondamental theories
Cosmological observations
Concrete models
Cosmological inflation
Primordial non-Gaussianities
Phenomenology
Statistical methods
Numerical tools
Analytical formalism

Some observational perspectives

- Primordial gravitational waves (COrE, CMBPol ...)

- Constraints on non-Gaussianities from Large Scale Structure (Euclid ...)

- Spectral distorsions of the CMB (Prism, Pixie)